Basic Mathematics

(22103-Common to All Branches)

SYLLABUS

Chapter No.	Name of chapter	Marks With Option
1	Logarithm	02
2	Determinant	06
3	Matrices	14
4	Partial Fraction	08
5	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles	14
6	Factorization & De-factorization Formulae.	12
7	Inverse Tri.Ratios.	04
8	Straight Line	12
9	Mensuration	10
10	Statistics	20
	Total Marks :-	102

BOARD THEORY PAPER PATTERN FOR ALL BRANCHES

Q.1		**Attempt any FIVE** **5*2=10**
	a)	Logarithm
	b)	Determinant
	c)	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles.
	d)	Mensuration
	e)	Mensuration
	f)	Statistics
	g)	Statistics
Q.2		**Attempt any THREE** **3*4=12**
	a)	Matrices
	b)	Partial Fraction
	c)	Determinant
	d)	Statistics
Q.3		**Attempt any THREE** **3*4=12**
	a)	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles.
	b)	Factorization & De-factorization Formulae.
	c)	Factorization & De-factorization Formulae.
	d)	Inverse Tri.Ratios.
Q.4		**Attempt any FOUR** **3*4=12**
	a)	Matrices
	b)	Partial Fraction
	c)	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles.
	d)	Factorization & De-factorization Formulae.
	e)	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles.

Q.5		**Attempt any TWO** **2*6=12**
	a)	**i)** Straight Line
		ii) Straight Line
	b)	**i)** Straight Line
		ii) Straight Line
	c)	**i)** Mensuration
		ii) Mensuration
Q.6		**Attempt any FOUR** **2*6=12**
	a)	**i)** Statistics
		ii) Statistics
	b)	**i)** Statistics
		ii) Statistics
	c)	Matrices

CLASS TEST - I
PAPER PATTERN

COURSE:- BASIC MATHEMATICS (22103) (Common to All Branches)

Syllabus:-

Unit No.	Name of the Unit	Course Outcome (CO)
1	**Logarithm**	**CO-103.1**
	Determinant	
	Matrices	
	Partial Fraction	**CO-103.2**
2	**Tri.Ratios of Allied,Compound, Multiple & Sub-multiple Angles.**	**CO-103.3**

Q.1	**Attempt any FOUR 4*2=8Marks**	**Course Outcome (CO)**
a)	Determinant	**CO-103.1**
b)	Determinant	**CO-103.1**
c)	Matrices	**CO-103.1**
d)	Partial Fraction	**CO-103.2**
e)	Logarithm	**CO-103.1**
f)	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles.	**CO-103.3**
Q.2	**Attempt any THREE 3*4=12 Marks**	
a)	Partial Fraction	**CO-103.2**
b)	Determinant	**CO-103.1**
c)	Matrices	**CO-103.1**
d)	Tri.Ratios of Allied, Compound, Multiple & Sub-multiple Angles.	**CO-103.3**

CLASS TEST - II
PAPER PATTERN

COURSE:- BASIC MATHEMATICS (22103) (Common to All Branches)

Syllabus:-

Unit No.	Name of the Unit	Course Outcome (CO)
2	**Tri.Ratios of Allied,Compound, Multiple & Sub-multiple Angles.**	**CO-103.3**
	Factorization & De-factorization Formulae.	
	Inverse Tri.Ratios.	
3	**Straight Line**	**CO-103.4**
4	**Mensuration**	**CO-103.5**
5	**Statistics**	**CO-103.6**

Q.1	**Attempt any FOUR 4*2=8Marks**	**Course Outcome (CO)**
a)	Tri.Ratios of Allied,Compound, Multiple & Sub-multiple Angles.	**CO-103.3**
b)	Tri.Ratios of Allied,Compound, Multiple & Sub-multiple Angles.	**CO-103.3**
c)	Inverse Tri.Ratios.	**CO-103.3**
d)	Straight Line	**CO-103.4**
e)	Mensuration	**CO-103.5**
f)	Statistics	**CO-103.6**
Q.2	**Attempt any THREE 3*4=12 Marks**	
a)	Factorization & De-factorization Formulae.	**CO-103.3**
b)	Statistics	**CO-103.6**
c)	Straight Line	**CO-103.4**
d)	Mensuration	**CO-103.5**

COURSE OUTCOME (CO)

COURSE:- BASIC MATHEMATICS (22103) (Common to All Branches)

PROGRAMME: - ALL

CO.NO	Course Outcome
CO-103.1	Apply the concepts of algebra to solve engineering related problems.
CO-103.2	Apply the concept of algebra to solve partial fraction.
CO-103.3	Utilize basic concepts of trigonometry to solve elementary engineering problems.
CO-103.4	Solve basic engineering problems under the condition of straight lines.
CO-103.5	Solve the problems based on measurement of regular closed figures and regular solids.
CO-103.6	Use basic concepts of statistics to solve engineering related problems.

LOGARITHM

Position in Question Paper **Total Marks-02**

Q.1. a) 2-Marks.

Types of Logarithm:

A. Common Logarithm:-

The logarithm to base 10 is called common logarithm. i.e. $log_{10}x$

B. Natural Logarithm:-

The logarithm to base e is called natural logarithm. i.e. $log_{e}x$

Laws of Logarithm

1) $log_{a}1 = 0$
2) $log_{a}a = 1$
3) $loga + logb = log(a.b)$
4) $loga - logb = log(\frac{a}{b})$
5) $loga^{b} = b.loga$

6) $$log_b a = \frac{loga}{logb}$$

7) $$a^{log_a y} = y$$

8) $$log(\frac{a}{b}) = -log(\frac{b}{a})$$

9) If $a^x = b$ then $log_a b = x$

Examples:-

Que. **Write the following terms in logarithmic form**

1) $5^3 = 125$

Ans. $5^3 = 125$

$log_5 125 = 3$

2) $3^4 = 81$

Ans. $3^4 = 81$

$log_3 81 = 4$

3) $7^0 = 1$

Ans. $7^0 = 1$

$log_7 1 = 0$

4) $5^{-2} = \frac{1}{25}$

Ans. $5^{-2} = \frac{1}{25}$

$log_5 \frac{1}{25} = -2$

5) $8^{-2} = \frac{1}{64}$

Ans. $8^{-2} = \frac{1}{64}$

$log_8 \frac{1}{64} = -2$

6) $9^{-2} = \frac{1}{81}$

Ans. $9^{-2} = \frac{1}{81}$

$log_9 \frac{1}{81} = -2$

7) $10^{-3} = 0.001$

Ans. $10^{-3} = 0.001$

$log_{10} 0.001 = -3$

8) $10^{-2} = 0.01$

Ans. $10^{-2} = 0.01$

$log_{10} 0.01 = -2$

9) $x^y = z$

Ans. $x^y = z$

$log_x z = y$

Que. **Write the following terms in exponential form**

1) $log_3 27 = 3$

Ans. $log_3 27 = 3$

$3^3 = 27$

2) $log_3 81 = 4$

Ans. $log_3 81 = 4$

$3^4 = 81$

3) $log_{3\sqrt{2}} 18 = 2$

Ans. $log_{3\sqrt{2}} 18 = 2$

$(3\sqrt{2})^2 = 18$

4) $log_4 \left(\frac{1}{16}\right) = -2$

Ans. $log_4 \left(\frac{1}{16}\right) = -2$

$4^{-2} = \frac{1}{16}$

5) $log_{0.01} 0.0001 = 2$

Ans. $log_{0.01} 0.0001 = 2$

$(0.01)^2 = 0.0001$

6) $log_{\sqrt{7}} 343 = 6$

Ans. $log_{\sqrt{7}} 343 = 6$

$(\sqrt{7})^6 = 343$

Que. **Evaluate without using log table**

1) $25^{log_5 8}$

Ans.

$$25^{\log_5 8} = [(5)^2]^{\log_5 8}$$
$$= 5^{2\log_5 8} = 5^{\log_5 8^2}$$
$$= 5^{\log_5 64}$$
$$25^{\log_5 8} = 64$$

2) $32^{log_2 5}$

Ans.

$$32^{\log_2 5} = [(2)^5]^{\log_2 5}$$
$$= 2^{5\log_2 5} = 2^{\log_2 5^5} = 2^{\log_2 3125}$$
$$32^{\log_2 5} = 3125$$

3) $8^{log_2 3}$

Ans.

$$8^{\log_2 3} = (2^3)^{\log_2 (3)} = 2^{3\log_2 (3)}$$
$$= 2^{\log_2 (3)^3} = 2^{\log_2 27}$$
$$8^{\log_2 3} = 27$$

4) $(625)^{log_5 7}$

Ans.

$$(625)^{\log_5 7} = \{(5)^4\}^{\log_5 7}$$
$$= (5)^{4 \times \log_5 7}$$
$$= (5)^{\log_5 (7)^4}$$

$$= (5)^{\log_5(2401)}$$

$$= 2401$$

5) $(18)^{log_{3\sqrt{2}}7}$

Ans.

$$(18)^{\log_{3\sqrt{2}}7} = \left\{(3\sqrt{2})^2\right\}^{\log_{3\sqrt{2}}7}$$

$$= (3\sqrt{2})^{2\times\log_{3\sqrt{2}}7}$$

$$= (3\sqrt{2})^{\log_{3\sqrt{2}}(7)^2}$$

$$= (3\sqrt{2})^{\log_{3\sqrt{2}}49}$$

$$= 49$$

6) $(12)^{log_{2\sqrt{3}}5}$

Ans.

$$\log_{12}(2\sqrt{3})^5 = \frac{\log(2\sqrt{3})^5}{\log 12}$$

$$= \frac{\log(2\sqrt{3})^5}{\log(2\sqrt{3})^2}$$

$$= \frac{5\times\log(2\sqrt{3})}{2\times\log(2\sqrt{3})} = \frac{5}{2}$$

Que. **Evaluate OR find the value of following**

1) $log_{81}3$

Ans.

$$\log_{81}3 = \frac{\log 3}{\log 81}$$

$$= \frac{\log 3}{\log(3)^4}$$

$$= \frac{\log 3}{4 \times \log 3}$$

$$= \frac{1}{4}$$

2) $log_3 81$ **[W-18 , W-17]**

Ans.

$$\log_3 81$$

$$= \log_3 (3)^4$$

$$= 4\log_3 3$$

$$= 4(1)$$

$$= 4$$

3) $log_3 243$

Ans.

$$\log_3 243 = \frac{\log 243}{\log 3}$$

$$= \frac{\log(3)^5}{\log 3}$$

$$= \frac{5 \times \log 3}{\log 3}$$

$$= 5$$

4) $log_{343}7$

Ans.

$$log_{343}7 = \frac{log7}{log343}$$

$$log_{343}7 = \frac{log7}{log7^3}$$

$$log_{343}7 = \frac{log7}{3log7}$$

$$log_{343}7 = \frac{1}{3}$$

5) $log_3 9$

Ans.

$$log_3 9 = \frac{log9}{log3}$$

$$log_3 9 = \frac{log3^2}{log3}$$

$$log_3 9 = \frac{2.log3}{log3}$$

$$log_3 9 = 2$$

6) $log_2 32$

Ans.

$$log_2 32 = \frac{log32}{log2}$$

$$log_2 32 = \frac{log 2^5}{log 2}$$

$$log_2 32 = \frac{5.log 2}{log 2}$$

$$log_2 32 = 5$$

7) $log_8 (\frac{1}{8})$

Ans. $$log_8 \left(\frac{1}{8}\right) = -log_8 8$$

$$log_8 \left(\frac{1}{8}\right) = -1$$

8) $log_{12} (2\sqrt{3}\,)^5$

Ans. $$log_{12} (2\sqrt{3}\,)^5 = 5log_{12} (2\sqrt{3}\,)$$

$$log_{12} (2\sqrt{3}\,)^5 = \frac{5.log(2\sqrt{3}\,)}{log 12}$$

$$log_{12} (2\sqrt{3}\,)^5 = \frac{5.log(2\sqrt{3}\,)}{log(2\sqrt{3})^2}$$

$$log_{12} (2\sqrt{3}\,)^5 = \frac{5.log(2\sqrt{3}\,)}{2log(2\sqrt{3}\,)}$$

$$log_{12} (2\sqrt{3}\,)^5 = \frac{5}{2}$$

9) $log_4 0.25$

Ans. $log_4 0.25 = log_4 \frac{1}{4}$

$log_4 0.25 = -log_4 4$

$log_4 0.25 = -1$

10) $log_{\sqrt{3}} \frac{1}{729}$

Ans. $log_{\sqrt{3}} \frac{1}{729} = -log_{\sqrt{3}} (\sqrt{3}\,)^{12}$

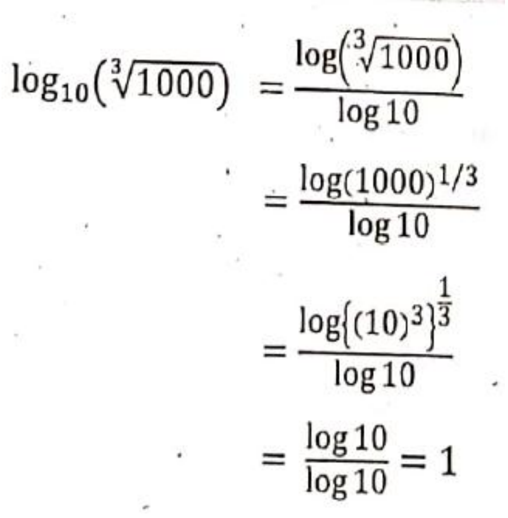
$log_{\sqrt{3}} \frac{1}{729} = -12 log_{\sqrt{3}} \sqrt{3}$

$log_{\sqrt{3}} \frac{1}{729} = -12.(1)$

$log_{\sqrt{3}} \frac{1}{729} = -12$

11) $log_{10} \sqrt[3]{100}$

Ans.



$$\log_{10}\left(\sqrt[3]{1000}\right) = \frac{\log\left(\sqrt[3]{1000}\right)}{\log 10}$$

$$= \frac{\log(1000)^{1/3}}{\log 10}$$

$$= \frac{\log\{(10)^3\}^{\frac{1}{3}}}{\log 10}$$

$$= \frac{\log 10}{\log 10} = 1$$

12) If $log_{10} 2 = 0.3010$ find $log_{10} 8$

Ans.

$$\begin{aligned} \log_{10} 8 &= \log_{10}(2)^3 \\ &= 3 \times \log_{10} 2 \\ &= 3 \times 0.3010 \\ &= 0.9030 \end{aligned}$$

Que. **Using logarithms to solve the following equations OR Find x if**

1) $2^x = 4$

Ans. $2^x = 4$

$log_2 4 = x$

$log_2 2^2 = x$

$2. log_2 2 = x$

$2(1) = x$

$2 = x$

2) $3^x = 9$

Ans. $3^x = 9$

$log_3 9 = x$

$log_3 3^2 = x$

$2. log_3 3 = x$

$2(1) = x$

$2 = x$

3) $4^x = 16$

Ans. $4^x = 16$

$log_4 16 = x$

$log_4 4^2 = x$

$2. log_4 4 = x$

$2(1) = x$

$2 = x$

4) $12^x = 144$

Ans. $12^x = 144$

$log_{12}144 = x$

$log_{12}12^2 = x$

$2. log_{12}12 = x$

$2(1) = x$

$2 = x$

5) $5^{-x} = \frac{1}{25}$

Ans. $5^{-x} = \frac{1}{25}$

$log_5 \frac{1}{25} = -x$

$-log_5 \frac{25}{1} = -x$

$-log_5 25 = -x$

$-log_5 5^2 = -x$

$-2. log_5 5 = -x$

$-2(1) = -x$

$-2 = -x$

$2 = x$

6) $7^{-x} = \frac{1}{49}$

Ans. $7^{-x} = \frac{1}{49}$

$log_7 \frac{1}{49} = -x$

$$-log_7\frac{49}{1} = -x$$

$$-log_7 49 = -x$$

$$-log_7 7^2 = -x$$

$$-2.log_7 7 = -x$$

$$-2(1) = -x$$

$$-2 = -x$$

$$2 = x$$

7) $9^{-x} = \frac{1}{81}$

Ans. $9^{-x} = \frac{1}{81}$

$$log_9\frac{1}{81} = -x$$

$$-log_9\frac{81}{1} = -x$$

$$-log_9 81 = -x$$

$$-log_9 9^2 = -x$$

$$-2.log_9 9 = -x$$

$$-2(1) = -x$$

$$-2 = -x$$

$$2 = x$$

8) $11^{-x} = \frac{1}{121}$

Ans. $11^{-x} = \frac{1}{121}$

$log_{11}\frac{1}{121} = -x$

$-log_{11}\frac{121}{1} = -x$

$-log_{11}121 = -x$

$-log_{11}11^2 = -x$

$-2. log_{11}11 = -x$

$-2(1) = -x$

$-2 = -x$

$2 = x$

9) $12^{-x} = \frac{1}{144}$

Ans. $12^{-x} = \frac{1}{144}$

$log_{12}\frac{1}{144} = -x$

$-log_{12}\frac{144}{1} = -x$

$-log_{12}144 = -x$

$-log_{12}12^2 = -x$

$-2. log_{12}12 = -x$

$-2(1) = -x$

$-2 = -x$

$2 = x$

10) $10^x = 5$

Ans. $10^x = 5$

$log_{10}5 = x$

$$\frac{log5}{log10} = x$$

$$\frac{log5}{log10} = x$$

$0.6989 = x$

11) $e^x = 16$

Ans. $e^x = 16$

$log_e16 = x$

$$\frac{log16}{loge} = x$$

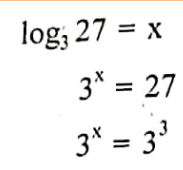
$$\frac{log16}{1} = x$$

$2.7725 = x$

Que. **Find *x* if**

1) $log_3 27 = x$

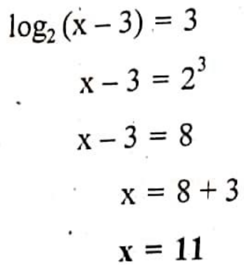
Ans.



$$\log_3 27 = x$$
$$3^x = 27$$
$$3^x = 3^3$$

2) $log_2(x-3) = 3$

Ans.



$$\log_2(x-3) = 3$$
$$x-3 = 2^3$$
$$x-3 = 8$$
$$x = 8+3$$
$$\mathbf{x = 11}$$

3) $log_2(7x+2) = 3$

Ans. $log_2(7x+2) = 3$

$2^3 = 7x+2$

$8 = 7x+2$

$6 = 7x$

$\frac{6}{7} = x$

4) $log_3(x+4) = 4$

Ans. $log_3(x+4) = 4$

$3^4 = x+4$

$81 = x+4$

$77 = x$

5) $log_3(x+6) = 2$ **[W-19]**

Ans. $\log_3(x+6)=2$

$\therefore x+6=3^2$

$\therefore x+6=9$

$\therefore x=3$

6) $log_3(x+5)=4$

Ans. $log_3(x+5)=4$

$3^4=x+5$

$81=x+5$

$76=x$

7) $log_2(x^2-6x+40)=5$

Ans. $log_2(x^2-6x+40)=5$

$2^5=x^2-6x+40$

$32=x^2-6x+40$

$-8=x^2-6x$

$0=x^2-6x+8$

$x=4\,,2$

8) $log_3(x-4)+log_3(x-2)=1$

Ans. $log_3(x-4)+log_3(x-2)=1$

$log_3[(x-4).(x-2)]=1$

$log_3[(x^2-6x+8)]=1$

$$3^1 = x^2 - 6x + 8$$

$$3 = x^2 - 6x + 8$$

$$0 = x^2 - 6x + 5$$

$$x = 5\,,1$$

9) $log_5(x-3) + log_5(x+4) = log_5 8$

Ans. $log_5(x-3) + log_5(x+4) = log_5 8$

$$log_5(x-3) + log_5(x+4) - log_5 8 = 0$$

$$log_5[(x-3).(x+4)] - log_5 8 = 0$$

$$log_5[(x^2 + x - 12)] - log_5 8 = 0$$

$$log_5[\left(\frac{x^2 + x - 12}{8}\right)] = 0$$

$$5^0 = \frac{x^2 + x - 12}{8}$$

$$1 = \frac{x^2 + x - 12}{8}$$

$$8 = x^2 + x - 12$$

$$0 = x^2 + x - 20$$

$$x = 4\,,-5$$

10) $log_{10}x + log_{10}(x-3) = 1$

Ans. $log_{10}x + log_{10}(x-3) = 1$

$$log_{10}[x(x-3)] = 1$$

$$log_{10}[x^2 - 3x] = 1$$

$$10^1 = x^2 - 3x$$

$10 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$x = 5, -2$

11) $log_2(x+5) + log_2(x-2) = 3$

Ans. $\log_2(x+5) + \log_2(x-2) = 3$

$\therefore \log_2[(x+5)(x-2)] = 3$

$\therefore (x+5)(x-2) = 2^3$

$\therefore x^2 - 2x + 5x - 10 = 8$

$\therefore x^2 + 3x - 10 = 8$

$\therefore x^2 + 3x - 10 - 8 = 0$

$\therefore x^2 + 3x - 18 = 0$

$x = -6$ OR $x = 3$

12) $log_2x - log_2(x-1) = 5$

Ans. $\log_2 x - \log_2(x-1) = 5$

$\log_2\left(\frac{x}{x-1}\right) = 5$

$\frac{x}{x-1} = 2^5$

$\frac{x}{x-1} = 32$

$x = 32(x-1)$

$$x = 32x - 32$$

$$x - 32x = -32$$

$$-31x = -32$$

$$31x = 32$$

$$x = \frac{32}{31}$$

13) $log_2(3x+7) = log_2(5x+1)$ **OR** $log_2(3x+7) - log_2(5x+1) = 0$

Ans. $log_2(3x+7) = log_2(5x+1)$

$$log_2(3x+7) - log_2(5x+1) = 0$$

$$log_2\left(\frac{3x+7}{5x+1}\right) = 0$$

$$2^0 = \frac{3x+7}{5x+1}$$

$$1 = \frac{3x+7}{5x+1}$$

$$5x+1 = 3x+7$$

$$2x = 8$$

$$x = 4$$

14) $log_x 2 + log_x 4 + log_x 8 = 6$

Ans. $log_x 2 + log_x 4 + log_x 8 = 6$

$$\frac{log2}{logx} + \frac{log4}{logx} + \frac{log8}{logx} = 6$$

$$\frac{log2 + log4 + log8}{logx} = 6$$

$$\frac{log(2.4.8)}{logx} = 6$$

$$\frac{log(64)}{logx} = 6$$

$$log64 = 6.logx$$

$$log64 = logx^6$$

$$64 = x^6$$

$$2^6 = x^6$$

$$2 = x$$

15) $log_2x + log_x2 = 2$

Ans. Given: $\log_2 x + \log_x 2 = 2$

$$\Rightarrow \quad \log_2 x + \frac{1}{\log_2 x} = 2$$

Let $\log_2 x = a$ say

$$\Rightarrow \quad \frac{a^2+1}{a} = 2$$

$$\Rightarrow \quad a^2 + 1 = 2a$$

$$\Rightarrow \quad a^2 - 2a + 1 = 0$$

$$\Rightarrow \quad (a-1)^2 = 0$$

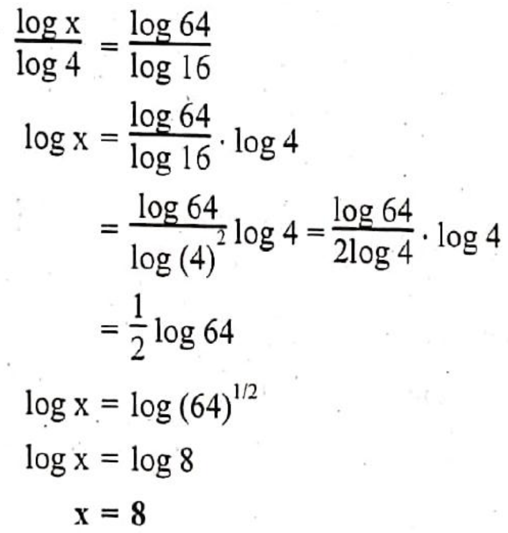
$$\Rightarrow \quad a = 1$$

$$\Rightarrow \quad \log_2 x = 1$$

$$\Rightarrow \quad x = 2^1 = 2$$

16) $\frac{logx}{log4} = \frac{log64}{log16}$

Ans. $$\frac{\log x}{\log 4} = \frac{\log 64}{\log 16}$$



$$\frac{\log x}{\log 4} = \frac{\log 64}{\log 16}$$

$$\log x = \frac{\log 64}{\log 16} \cdot \log 4$$

$$= \frac{\log 64}{\log (4)^2} \log 4 = \frac{\log 64}{2\log 4} \cdot \log 4$$

$$= \frac{1}{2} \log 64$$

$$\log x = \log (64)^{1/2}$$

$$\log x = \log 8$$

$$\mathbf{x = 8}$$

17) $\frac{logx}{log5} = \frac{log25}{log125}$

Ans. $\frac{logx}{log5} = \frac{log5^2}{log5^3}$

$$\frac{logx}{log5} = \frac{2.log5}{3.log5}$$

$$\frac{logx}{log5} = \frac{2}{3}$$

$$3.logx = 2.log5$$

$$logx^3 = log5^2$$

$$x^3 = 5^2$$

$$x^3 = 25$$

$$x = \sqrt[3]{25}$$

18) $\frac{4log3.logx}{log9} = log27$

Ans. $\frac{4log3.logx}{log9} = log27$

$$\frac{4log3.logx}{log3^2} = log27$$

$$\frac{4log3.logx}{2.log3} = log27$$

$$2.logx = log27$$

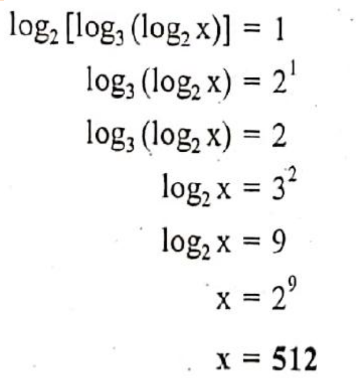
$$logx^2 = log27$$

$$x^2 = 27$$

$$x = \sqrt{27}$$

19) $log_2[\,log_3(log_2x)] = 1$

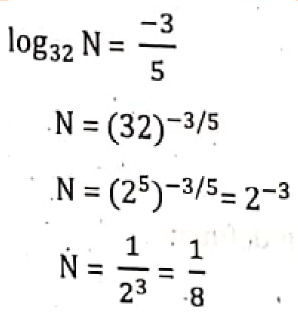
Ans.



$$\log_2[\log_3(\log_2 x)] = 1$$
$$\log_3(\log_2 x) = 2^1$$
$$\log_3(\log_2 x) = 2$$
$$\log_2 x = 3^2$$
$$\log_2 x = 9$$
$$x = 2^9$$
$$x = 512$$

20) $log_{32}x = \frac{-3}{5}$

Ans.



$$\log_{32} N = \frac{-3}{5}$$
$$N = (32)^{-3/5}$$
$$N = (2^5)^{-3/5} = 2^{-3}$$
$$N = \frac{1}{2^3} = \frac{1}{8}$$

21) $log_{16}x = \frac{-3}{4}$

Ans.

$$log_{16}x = \frac{-3}{4}$$

$$(16)^{-3/4} = x$$

$$\frac{1}{8} = x$$

Que. **Simplify**

1) $log_7(log_3 3)$

Ans. $log_7(log_3 3) = log_7(1)$

$log_7(log_3 3) = 0$

2) $(log_3 4)(log_4 81)$

Ans. $(\log_3 4)\ (\log_4 81)$

$$= \frac{\log 4}{\log 3} \times \frac{\log 81}{\log 4}$$

$$= \frac{\log 81}{\log 3} = \frac{\log 3^4}{\log 3} = \frac{4 \log 3}{\log 3} = 4$$

3) $\frac{log_8 81}{log_2 9}$

Ans. $\frac{log_8 81}{log_2 9} = \frac{\frac{log 81}{log 8}}{\frac{log 9}{log 2}}$

$\frac{log_8 81}{log_2 9} = \frac{log 81}{log 8} X \frac{log 2}{log 9}$

$\frac{log_8 81}{log_2 9} = \frac{log 9^2}{log 2^3} X \frac{log 2}{log 9}$

$\frac{log_8 81}{log_2 9} = \frac{2.log 9}{3.log 2} X \frac{log 2}{log 9}$

$\frac{log_8 81}{log_2 9} = \frac{2}{3}$

4) $log_2 14 - log_2 7$

Ans. $\log_2 14 - \log_2 7$

$$= \log_2\left(\frac{14}{7}\right) = \log_2(2) = 1$$

5) $log\frac{2}{3}+log\frac{4}{5}-log\frac{8}{15}$ **[S-18]**

Ans.

$$\log\left(\frac{2}{3}\right)+\log\left(\frac{4}{5}\right)-\log\left(\frac{8}{15}\right)=\log\left(\frac{2}{3}\times\frac{4}{5}\right)-\log\left(\frac{8}{15}\right).$$

:

$$=\log\left(\frac{8}{15}\right)-\log\left(\frac{8}{15}\right)$$

$$=0 \qquad OR \qquad =\log\left(\frac{\frac{8}{15}}{\frac{8}{15}}\right)=\log(1)=0$$

6) $log\frac{225}{32}-log\frac{25}{81}+log\frac{64}{729}$ **[SQP]**

Ans. $log\frac{225}{32}-log\frac{25}{81}+log\frac{64}{729}$

$$=log\left(\frac{\frac{225}{32}}{\frac{25}{81}}\right)+log\frac{64}{729}$$

$$=log\left(\frac{729}{32}\right)+log\frac{64}{729}$$

$$=log\left(\frac{729}{32}X\frac{64}{729}\right)$$

$$=log\,2$$

7) $log\frac{9}{14}-log\frac{15}{16}+log\frac{35}{24}$

Ans.

$$log\frac{9}{14}-log\frac{15}{16}+log\frac{35}{24}=log\left(\frac{\frac{9}{14}}{\frac{15}{16}}\right)+log\frac{35}{24}$$

$$log\frac{9}{14}-log\frac{15}{16}+log\frac{35}{24}=log\left(\frac{24}{35}\right)+log\frac{35}{24}$$

$$log\frac{9}{14}-log\frac{15}{16}+log\frac{35}{24}=log\left(\frac{24}{35}*\frac{35}{24}\right)$$

$$log\frac{9}{14}-log\frac{15}{16}+log\frac{35}{24}=log(1)$$

$$log\frac{9}{14} - log\frac{15}{16} + log\frac{35}{24} = 0$$

8) $$log\frac{15}{16} + log\frac{64}{81} - log\frac{4}{27} - log5$$

Ans.

$$log\frac{15}{16} + log\frac{64}{81} - log\frac{4}{27} - log5 = log(\frac{15}{16} * \frac{64}{81}) - log\frac{4}{27} - log5$$

$$log\frac{15}{16} + log\frac{64}{81} - log\frac{4}{27} - log5 = log(\frac{20}{27}) - log\frac{4}{27} - log5$$

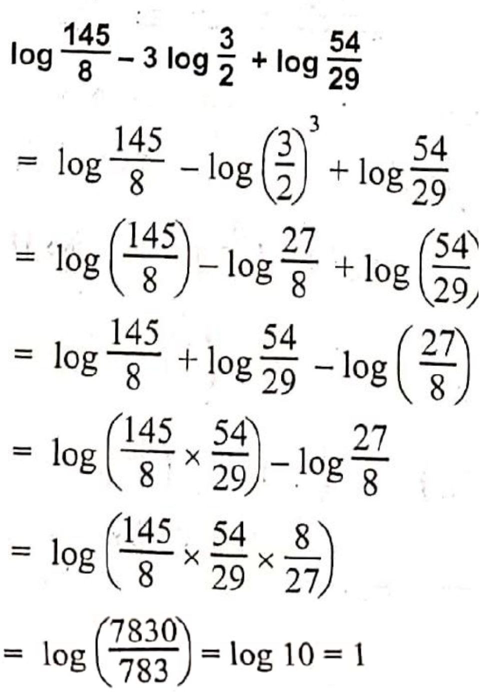
$$log\frac{15}{16} + log\frac{64}{81} - log\frac{4}{27} - log5 = log(\frac{\frac{20}{27}}{\frac{4}{27}}) - log5$$

$$log\frac{15}{16} + log\frac{64}{81} - log\frac{4}{27} - log5 = log(5) - log5$$

$$log\frac{15}{16} + log\frac{64}{81} - log\frac{4}{27} - log5 = 0$$

9) $$log\frac{145}{8} - 3\ log\frac{3}{2} + log\frac{54}{29}$$

Ans.



$$\log\frac{145}{8} - 3\log\frac{3}{2} + \log\frac{54}{29}$$

$$= \log\frac{145}{8} - \log\left(\frac{3}{2}\right)^3 + \log\frac{54}{29}$$

$$= \log\left(\frac{145}{8}\right) - \log\frac{27}{8} + \log\left(\frac{54}{29}\right)$$

$$= \log\frac{145}{8} + \log\frac{54}{29} - \log\left(\frac{27}{8}\right)$$

$$= \log\left(\frac{145}{8} \times \frac{54}{29}\right) - \log\frac{27}{8}$$

$$= \log\left(\frac{145}{8} \times \frac{54}{29} \times \frac{8}{27}\right)$$

$$= \log\left(\frac{7830}{783}\right) = \log 10 = 1$$

10) $$2log\frac{16}{15} + log\frac{25}{24} - log\frac{32}{27}$$

Ans.

$$2log\frac{16}{15}+log\frac{25}{24}-log\frac{32}{27}$$

$$=log(\frac{16}{15})^2+log\frac{25}{24}-log\frac{32}{27}$$

$$=log\frac{256}{225}+log\frac{25}{24}-log\frac{32}{27}$$

$$=log(\frac{256}{225}X\frac{25}{24})-log\frac{32}{27}$$

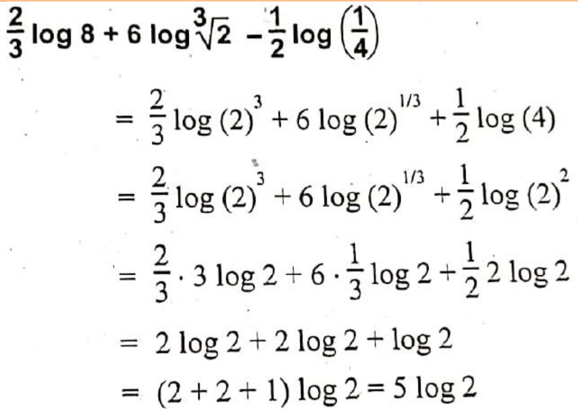
$$=log(\frac{256}{225}X\frac{25}{24})-log\frac{32}{27}$$

$$=log\frac{32}{27}-log\frac{32}{27}$$

$$=0$$

11) $\frac{2}{3}log8+6log\sqrt[3]{2}-\frac{1}{2}log\frac{1}{4}$

Ans.



$$\frac{2}{3}\log 8+6\log\sqrt[3]{2}-\frac{1}{2}\log\left(\frac{1}{4}\right)$$

$$=\frac{2}{3}\log(2)^3+6\log(2)^{1/3}+\frac{1}{2}\log(4)$$

$$=\frac{2}{3}\log(2)^3+6\log(2)^{1/3}+\frac{1}{2}\log(2)^2$$

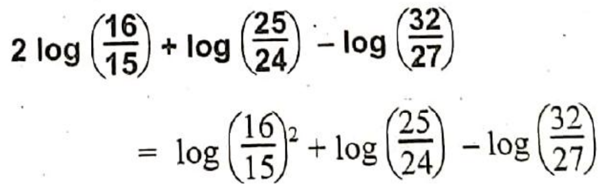
$$=\frac{2}{3}\cdot 3\log 2+6\cdot\frac{1}{3}\log 2+\frac{1}{2}2\log 2$$

$$=2\log 2+2\log 2+\log 2$$

$$=(2+2+1)\log 2=5\log 2$$

12) $2log\frac{3}{4}+log\left(13\frac{1}{3}\right)+log\left(7\frac{1}{2}\right)$

Ans.



$$2\log\left(\frac{16}{15}\right)+\log\left(\frac{25}{24}\right)-\log\left(\frac{32}{27}\right)$$

$$=\log\left(\frac{16}{15}\right)^2+\log\left(\frac{25}{24}\right)-\log\left(\frac{32}{27}\right)$$

$$= \log\left(\frac{256}{225}\right) + \log\left(\frac{25}{24}\right) + \log\left(\frac{27}{32}\right)$$

$$= \log\left(\frac{256}{225} \times \frac{25}{24} \times \frac{27}{32}\right) = \log(1) = 0$$

13) $\frac{1}{log_5 10} + \frac{1}{log_{20} 10}$

Ans. $\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{1}{\frac{log10}{log5}} + \frac{1}{\frac{log10}{log20}}$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{log5}{log10} + \frac{log20}{log10}$$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{log5 + log20}{log10}$$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{log(5 * 20)}{log10}$$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{log(100)}{log10}$$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{log(10)^2}{log10}$$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = \frac{2.log10}{log10}$$

$$\frac{1}{log_5 10} + \frac{1}{log_{20} 10} = 2$$

14) $2^{3log_2 3} + 12^{log_{2\sqrt{3}} 10}$

Ans. $2^{3log_2 3} + 12^{log_{2\sqrt{3}} 10} = 2^{log_2 3^3} + [(2\sqrt{3})^2]^{log_{2\sqrt{3}} 10}$

$$2^{3log_2 3} + 12^{log_{2\sqrt{3}} 10} = 3^3 + [(2\sqrt{3})]^{2log_{2\sqrt{3}} 10}$$

$$2^{3log_2 3} + 12^{log_{2\sqrt{3}} 10} = 3^3 + [(2\sqrt{3})]^{log_{2\sqrt{3}} 10^2}$$

$$2^{3log_2 3} + 12^{log_{2\sqrt{3}} 10} = 3^3 + 10^2$$

$$2^{3log_2 3} + 12^{log_{2\sqrt{3}} 10} = 27 + 100$$

Que. **Prove that**

1) $$log\left(\sqrt{x^2+1}+x\right)+log\left(\sqrt{x^2+1}-x\right)=0$$

Ans.

$$\log\left(x+\sqrt{x^2-1}\right)+\log\left(x-\sqrt{x^2-1}\right)$$

$$=\log\left\{\left(x+\sqrt{x^2-1}\right)\times\left(x-\sqrt{x^2-1}\right)\right\}$$

$$=\log\left\{(x)^2-\left(\sqrt{x^2-1}\right)^2\right\}$$

$$=\log\{x^2-(x^2-1)\}$$

$$=\log\{1\}$$

$$=0$$

2) $$\frac{1}{log_a abc}+\frac{1}{log_b abc}+\frac{1}{log_c abc}=1$$

Ans.

$$\frac{1}{log_a abc}+\frac{1}{log_b abc}+\frac{1}{log_c abc}=\frac{loga}{logabc}+\frac{logb}{logabc}+\frac{logc}{logabc}$$

$$\frac{1}{log_a abc}+\frac{1}{log_b abc}+\frac{1}{log_c abc}=\frac{loga+logb+logc}{logabc}$$

$$\frac{1}{log_a abc}+\frac{1}{log_b abc}+\frac{1}{log_c abc}=\frac{log(a*b*c)}{logabc}$$

$$\frac{1}{log_a abc}+\frac{1}{log_b abc}+\frac{1}{log_c abc}=1$$

3) $$\frac{1}{log_{ab}abc}+\frac{1}{log_{bc}abc}+\frac{1}{log_{ac}abc}=2$$

Ans.

$$\text{L.H.S.}=\frac{1}{\log_{ab}abc}+\frac{1}{\log_{bc}abc}+\frac{1}{\log_{ca}abc}$$

$$=\log_{abc}ab+\log_{abc}bc+\log_{abc}ca$$

$$=\log_{abc}(ab\cdot bc\cdot ca)$$

$$=\log_{abc}(abc)^2$$

$$=2\log_{abc}(abc)$$

$$=2\times 1$$

$$=2$$

$$=\text{R.H.S}$$

4) $\frac{1}{log_2 8}+\frac{1}{log_{64} 8}+\frac{1}{log_4 8}=3$

Ans.

$$\text{L.H.S.} = \frac{1}{\log_2 8}+\frac{1}{\log_{64} 8}+\frac{1}{\log_4 8}$$

$$= \frac{1}{\frac{\log 8}{\log 2}}+\frac{1}{\frac{\log 8}{\log 64}}+\frac{1}{\frac{\log 8}{\log 4}}$$

$$= \frac{\log 2}{\log 8}+\frac{\log 64}{\log 8}+\frac{\log 4}{\log 8}$$

$$= \frac{\log 2+\log 64+\log 4}{\log 8}$$

$$= \frac{\log(2\times 64\times 4)}{\log 8}=\frac{\log(512)}{\log 8}=\frac{\log 8^3}{\log 8}$$

$$= \frac{3\log 8}{\log 8}=3$$

$$= \text{R.H.S.}$$

5) $\frac{1}{log_3 6}+\frac{1}{log_8 6}+\frac{1}{log_9 6}=3$ **[S-19]**

Ans.

$$L.H.S=\frac{1}{\log_3 6}+\frac{1}{\log_8 6}+\frac{1}{\log_9 6}$$

$$=\frac{\log 3}{\log 6}+\frac{\log 8}{\log 6}+\frac{log 9}{\log 6}$$

$$=\frac{\log(3\times 8\times 9)}{\log 6}$$

$$=\frac{\log 216}{\log 6}$$

$$=\frac{\log 6^3}{\log 6}$$

$$=\frac{3\log 6}{\log 6}$$

$$=3=R.H.S$$

6) $$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=2$$

Ans.
$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=\frac{log6}{log24}+\frac{log12}{log24}+\frac{log8}{log24}$$

$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=\frac{log6+log12+log8}{log24}$$

$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=\frac{log(6*12*8)}{log24}$$

$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=\frac{log576}{log24}$$

$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=\frac{log24^2}{log24}$$

$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=\frac{2.log24}{log24}$$

$$\frac{1}{log_6 24}+\frac{1}{log_{12} 24}+\frac{1}{log_8 24}=2$$

7) $$log\left(\frac{p^2}{qr}\right)+log\left(\frac{q^2}{rp}\right)+log\left(\frac{r^2}{pq}\right)=0$$ **[SQP]**

Ans.
$$\text{L.H.S.} = \log\left(\frac{p^2}{qr}\right)+\log\left(\frac{q^2}{rp}\right)+\log\left(\frac{r^2}{pq}\right)$$

$$= \log\left[\frac{p^2}{qr}\times\frac{q^2}{rp}\times\frac{r^2}{pq}\right]$$

$$= \log\left[\frac{pp}{qr}\times\frac{q\cdot q}{rp}\times\frac{rr}{pq}\right] = \log[1] = 0$$

$$= \text{R.H.S.}$$

8) $$log\left(\frac{145}{49}\right)+log\left(\frac{14}{29}\right)-log\left(\frac{10}{7}\right)=0$$

Ans.
$$log\left(\frac{145}{49}\right)+log\left(\frac{14}{29}\right)-log\left(\frac{10}{7}\right)$$

$$=log\left(\frac{145}{49}*\frac{14}{29}\right)-log\left(\frac{10}{7}\right)$$

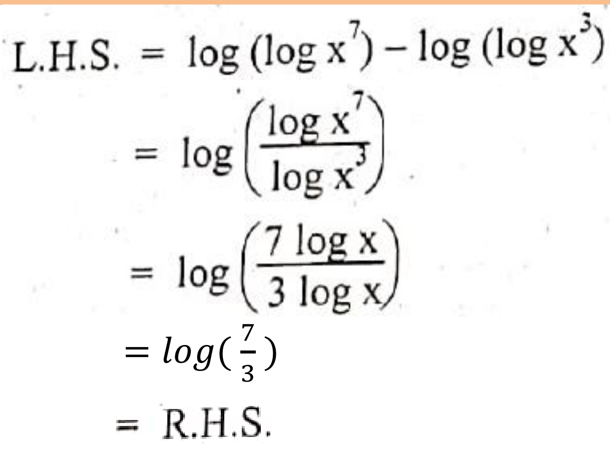
$$= log\left(\frac{145}{49}\right) + log\left(\frac{14}{29}\right) - log\left(\frac{10}{7}\right)$$

$$= log\left(\frac{10}{7}\right) - log\left(\frac{10}{7}\right)$$

$$= 0$$

9) $log(logx^7) - log(logx^3) = log(\frac{7}{3})$

Ans.



$$\text{L.H.S.} = \log(\log x^7) - \log(\log x^3)$$
$$= \log\left(\frac{\log x^7}{\log x^3}\right)$$
$$= \log\left(\frac{7\log x}{3\log x}\right)$$
$$= log(\frac{7}{3})$$
$$= \text{R.H.S.}$$

10) $log_y x . log_z y . log_x z = 1$

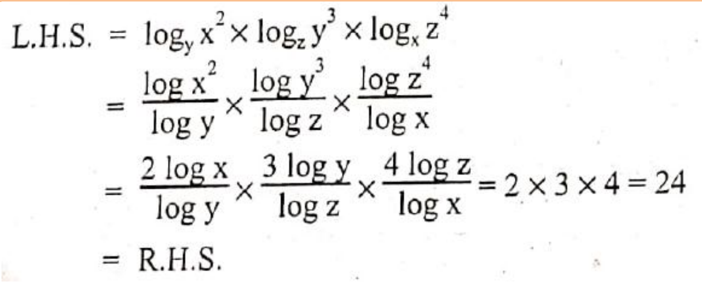
Ans. $log_y x . log_z y . log_x z$

$$= \frac{logx}{logy} . \frac{logy}{logz} . \frac{logz}{logx}$$

$$= 1$$

11) $log_y x^2 . log_z y^3 . log_x z^4 = 24$

Ans.



$$\text{L.H.S.} = \log_y x^2 \times \log_z y^3 \times \log_x z^4$$
$$= \frac{\log x^2}{\log y} \times \frac{\log y^3}{\log z} \times \frac{\log z^4}{\log x}$$
$$= \frac{2\log x}{\log y} \times \frac{3\log y}{\log z} \times \frac{4\log z}{\log x} = 2 \times 3 \times 4 = 24$$
$$= \text{R.H.S.}$$

12) $log_b a^3 . log_c b^3 . log_a c^3 = 27$

Ans. $log_b a^3 . log_c b^3 . log_a c^3 = 3log_b a . 3log_c b . 3log_a c$

$$log_b a^3 . log_c b^3 . log_a c^3 = \frac{3loga}{logb} . \frac{3.logb}{logc} . \frac{3.logc}{loga}$$

$$log_b a^3 . log_c b^3 . log_a c^3 = 3 * 3 * 3$$

$$log_b a^3 . log_c b^3 . log_a c^3 = 27$$

13) $log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = 1$

Ans. $log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = log_y (x)^{\frac{1}{3}} . log_z y^4 . log_x (z^3)^{\frac{1}{4}}$

$$log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = \frac{1}{3} * \ log_y (x) . 4 . log_z y . \frac{1}{4} * log_x (z^3)$$

$$log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = \frac{1}{3} * \frac{logx}{logy} . 4 * \frac{logy}{logz} . \frac{1}{4} * \frac{logz^3}{logx}$$

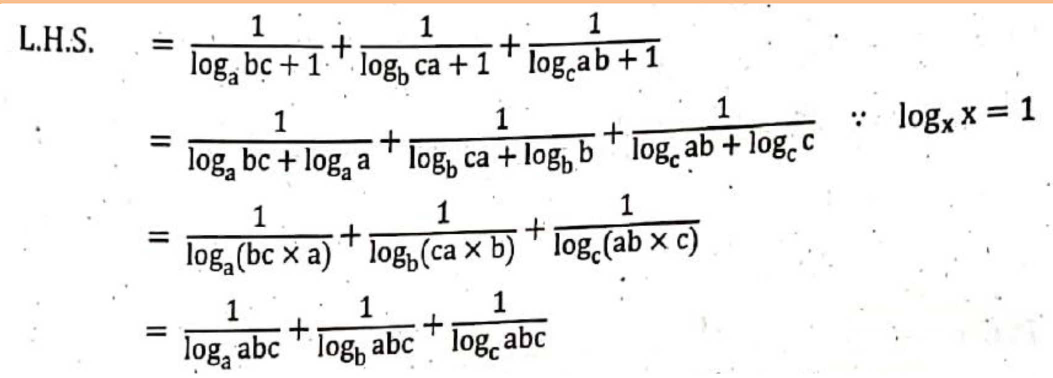
$$log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = \frac{1}{3} * \frac{logx}{logy} . 4 * \frac{logy}{logz} . \frac{1}{4} * \frac{3 * logz}{logx}$$

$$log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = \frac{1}{3} * 4 * \frac{1}{4} * 3$$

$$log_y \sqrt[3]{x} . log_z y^4 . log_x \sqrt[4]{z^3} = 1$$

14) $\frac{1}{1 + log_a bc} + \frac{1}{1 + log_b ca} + \frac{1}{1 + log_c ab} = 1$

Ans.



L.H.S. $= \frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$ $\because \log_x x = 1$

$= \frac{1}{\log_a (bc \times a)} + \frac{1}{\log_b (ca \times b)} + \frac{1}{\log_c (ab \times c)}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc}(a \cdot b \cdot c)$$

$$= \log_{abc}(abc)$$

$$= 1$$

$$= R.H.S.$$

15) $$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=2$$

Ans. $$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{1}{1+\frac{loga}{logbc}}+\frac{1}{1+\frac{logb}{logca}}+\frac{1}{1+\frac{logc}{logab}}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{1}{\frac{logbc+loga}{logbc}}+\frac{1}{\frac{logca+logb}{logca}}+\frac{1}{\frac{logab+logc}{logab}}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{logbc}{logbc+loga}+\frac{logca}{logca+logb}+\frac{logab}{logab+logc}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{logbc}{log(bc*a)}+\frac{logca}{log(ca*b)}+\frac{logab}{log(ab*c)}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{logbc}{log(abc)}+\frac{logca}{log(abc))}+\frac{logab}{log(abc)}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{logbc+logca+logab}{log(abc)}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{log(bc*ca*ab)}{log(abc)}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{log(abc)^2}{log(abc)}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=\frac{2*log(abc))}{log(abc)}$$

$$\frac{1}{1+log_{bc}a}+\frac{1}{1+log_{ca}b}+\frac{1}{1+log_{ab}c}=2$$

16) $log(\,1+2+3\,)=log1+log2+log3$

Ans.

$$\text{L.H.S.} = \log(1+2+3)$$
$$= \log(6)$$
$$= \log(1\times 2\times 3)$$
$$= \log(1)+\log(2)+\log(3)$$
$$= \text{R.H.S.}$$

17) $\frac{logx}{b-c} = \frac{logy}{c-a} = \frac{logz}{a-b}$ Show that $xyz = 1$

Ans. $\frac{logx}{b-c} = \frac{logy}{c-a} = \frac{logz}{a-b} = p$

$\frac{logx}{b-c} = p$ $\quad \therefore logx = pb - pc$

$\frac{logy}{c-a} = p$ $\quad \therefore logy = pc - pa$

$\frac{logz}{a-b} = p$ $\quad \therefore logz = pa - pb$

$\therefore logx + logy + logz = pb - pc + pc - pa + pa - pb$

$\therefore log(x * y * z) = 0$

$\therefore x * y * z = 0$

18) If $x^2 + y^2 = 7xy$ then show that $2\,log(\,x+y\,) = logx + logy + log9$

Ans. Given: $x^2+y^2 = 7xy$

Add 2xy to both sides we have

$\Rightarrow \quad x^2+y^2+2xy = 7xy+2xy$

$\Rightarrow \quad (x+y)^2 = 9xy$

Take logarithm of both sides we get

$$\log(x+y)^2 = \log(9xy)$$

$\Rightarrow \quad 2\cdot\log(x+y) = \log(9)+\log(x)+\log(y)$

$\Rightarrow \quad 2\cdot\log(x+y) = \log(x)+\log(y)+\log(9)$

TUTORIAL NO.1

LOGARITHM

1) **Evaluate:** $log_{81}3$

2) **Evaluate:** $log_{3}81$ **[W-18 , W-17]**

3) **Find *x* if** $log_x2 + log_x4 + log_x8 = 6$

4) **Simplify:** $log\frac{2}{3} + log\frac{4}{5} - log\frac{8}{15}$ **[S-18]**

5) **Simplify:** $log\frac{225}{32} - log\frac{25}{81} + log\frac{64}{729}$ **[SQP]**

6) **Prove that:** $\frac{1}{log_36} + \frac{1}{log_86} + \frac{1}{log_96} = 3$ **[S-19]**

7) **Prove that :** $log\left(\frac{p^2}{qr}\right) + log\left(\frac{q^2}{rp}\right) + log\left(\frac{r^2}{pq}\right) = 0$ **[SQP]**

8) **Prove that :** $log(\,logx^7\,) - log(\,logx^3\,) = log(\frac{7}{3})$

9) **Prove that :** $log_yx^2.log_zy^3.log_xz^4 = 24$

10) **Prove that:** $log(\,1 + 2 + 3\,) = log1 + log2 + log3$

DETERMINANT

Position in Question Paper **Total Marks-06**

Q.1. a) 2-Marks.

Q.2. c) 4-Marks.

A. DETERMINANT : - Definition :-

An arrangement of an element or object in a equal No. of horizontal lines called ROWS & vertical lines called COLUMN enclosed between two vertical bars or lines

Examples:-

Que. **Evaluate OR Expand OR Find the value of determinant**

1)
$$\begin{vmatrix} 1 & 7 & 0 \\ 2 & 1 & 9 \\ 6 & 0 & 8 \end{vmatrix}$$

Ans.

$$D = \begin{vmatrix} 1 & 7 & 0 \\ 2 & 1 & 9 \\ 6 & 0 & 8 \end{vmatrix}$$

$$D = 1\begin{vmatrix} 1 & 9 \\ 0 & 8 \end{vmatrix} - 7\begin{vmatrix} 2 & 9 \\ 6 & 8 \end{vmatrix} + 0\begin{vmatrix} 2 & 1 \\ 6 & 0 \end{vmatrix}$$

$$D = 1\,(8-0) - 7\,(16-54) + 0$$

$$D = 274$$

2) $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 2 \\ 3 & 1 & 6 \end{vmatrix}$ **[W-14]**

Ans.

$$\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 2 \\ 3 & 1 & 6 \end{vmatrix} = 2(24-2)-3(6-6)+5(1-12)$$

$$= -11$$

3) $\begin{vmatrix} 3 & -5 & -1 \\ 1 & 3 & 5 \\ -5 & 1 & 3 \end{vmatrix}$ **[W-12]**

Ans.

$$\begin{vmatrix} 3 & -5 & -1 \\ 1 & 3 & 5 \\ -5 & 1 & 3 \end{vmatrix} = 3(9-5)+5(3+25)-1(1+15)$$

$$= 136$$

4) $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -3 \end{vmatrix}$

Ans.

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -3 \end{vmatrix} = 1\begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix} - (-1)\begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(-3-(-1)) + 1(-6-(-1)) + 1(2-1)$$

$$= 1(-2) + 1(-5) + 1(1) = -2-5+1$$

$$D = -6$$

5) $\begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Ans.

$$D = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$D = \cos\theta \begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} - (-\sin\theta) \begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix} + (0) \begin{vmatrix} \sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix}$$

$$D = \cos\theta\,(\cos\theta - 0) + \sin\theta\,(\sin\theta - 0) + 0$$

$$D = \cos^2\theta + \sin^2\theta$$

$$D = 1$$

B. **Solve OR Find $'x'$ if**

Examples:-

1) $\begin{vmatrix} 2 & 3 & 1 \\ 6 & x & 2 \\ 4 & x & -2 \end{vmatrix} = 0$ **[S-14]**

Ans.

$$\begin{vmatrix} 2 & 3 & 1 \\ 6 & x & 2 \\ 4 & x & -2 \end{vmatrix} = 0$$

$$\therefore 2(-2x-2x)-3(-12-8)+1(6x-4x)=0$$

$$\therefore -8x+60+2x=0$$

$$\therefore -6x+60=0$$

$$\therefore -6x=-60 \qquad or \qquad 6x=60$$

$$\therefore x=10$$

2) $\begin{vmatrix} 1 & 1 & 1 \\ 3 & x & 3 \\ 1 & x & 2 \end{vmatrix} = 0$ **[S-17]**

Ans.

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & x & 3 \\ 1 & x & 2 \end{vmatrix} = 0$$

$$\therefore 1(2\times x - x\times 3) - 1(3\times 2 - 1\times 3) + 1(3\times x - 1\times x) = 0$$

$$\therefore -x - 3 + 2x = 0$$

$$\therefore x - 3 = 0$$

$$\therefore x = 3$$

3) $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$ **[S-16]**

Ans.

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$$

$$\therefore 1(18-12) - x(9-4) + x^2(3-2) = 0$$

$$\therefore 6 - 5x + x^2 = 0$$

$$\therefore (x-3)(x-2) = 0$$

$$\therefore x = 3 \text{ or } x = 2$$

4) $\begin{vmatrix} x & 0 & 0 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$ **[S-18]**

Ans.

$$\begin{vmatrix} x & 0 & 0 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$$

$$\therefore x(-2+4) = 0$$

$$\therefore 2x = 0$$

$$\therefore x = 0$$

5) $\begin{vmatrix} 0 & 7 & 2 \\ 11 & x & 10 \\ 4 & 8 & 1 \end{vmatrix} = 0$ **[W-13]**

Ans.

$$\begin{vmatrix} 0 & 7 & -2 \\ 11 & x & 10 \\ 4 & 8 & 1 \end{vmatrix} = 0$$

$$0 - 7(1\times 11 - 4\times 10) - 2(88 - 4x) = 0$$

$$-77+280-176+8x=0$$

$$27+8x=0$$

$$\therefore x=\frac{-27}{8} \qquad \text{OR}$$

$$\therefore x=-3.375$$

6) $\begin{vmatrix} x & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 4 \end{vmatrix} = 0$ **[W-18]**

Ans.

$$x\begin{vmatrix} -2 & 1 \\ -4 & 4 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 \\ -2 & 4 \end{vmatrix} - 4\begin{vmatrix} 3 & -2 \\ -2 & -4 \end{vmatrix} = 0$$

$$\therefore x(-8+4)-4(12+2)-4(-12-4)=0$$

$$\therefore -4x-56+64=0$$

$$\therefore -4x=-8$$

$$\therefore x=2$$

7) $\begin{vmatrix} 4 & 3 & 9 \\ 3 & 2 & 7 \\ 1 & 4 & x \end{vmatrix} = 0$ **[S-17]**

Ans.

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & x & 3 \\ 1 & x & 2 \end{vmatrix} = 0$$

$$\therefore 1(2\times x - x\times 3) - 1(3\times 2 - 1\times 3) + 1(3\times x - 1\times x) = 0$$

$$\therefore -x-3+2x=0$$

$$\therefore x-3=0$$

$$\therefore x=3$$

8) $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$ **[S-19, S-15, SQP]** **OR**

Find missing term $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & - \end{vmatrix} = 0$ **[W-15]**

Ans. Let consider the missing term x

$$\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$$

$$\therefore 4(-2x-28)-3(3x-77)+9(12+22)=0$$

$$\therefore -8x-112-9x+231+306=0$$

$$\therefore -17x+425=0$$

$$\therefore x=\frac{425}{17}$$

$$\therefore \boxed{x=25}$$

9) $\begin{vmatrix} x & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$ **[S-19]** **OR**

Find the value of $'p'$ **if** $\begin{vmatrix} p & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$ **[W-17]**

Ans. $\begin{vmatrix} x & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$

$$\therefore x(-2+4)-4(3+2)-4(-12-4)=0$$

$\therefore 2x-20+64=0$

$\therefore 2x+44=0$

$\therefore x=-22$

10) **Find the value of $'K'$ if** $\begin{vmatrix} 2 & -K & 7 \\ 3 & -4 & 13 \\ 8 & -11 & 33 \end{vmatrix}=0$ **[S-13]**

Ans.

$$\begin{vmatrix} 2 & -k & 7 \\ 3 & -4 & 13 \\ 8 & -11 & 33 \end{vmatrix}=0$$

$$2(-4\times 33+11\times 13)+k(3\times 33-13\times 8)+7(-33+32)=0$$

$$22+k(-5)-7=0$$

$$15-5k=0$$

$$15=5k$$

$$k=3$$

C. Application Of Determinant(Crammer's Rule):-

$$x = \frac{D_x}{D}\ , \qquad y = \frac{D_y}{D}\ , \qquad z = \frac{D_z}{D}$$

Examples:-

Que. **Solve the following equations by Crammer's Rule / Determinant method.**

1) $x + y + z = 3$ $\qquad x - y + z = 1$ $\qquad x + y - 2z = 0$

[S-19 , S-17]

Ans.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 1(2-1) - 1(-2-1) + 1(1+1)$$

$$= 6$$

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 3(2-1) - 1(-2-0) + 1(1-0)$$

$$= 6$$

$$D_y = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{vmatrix}$$

$$= 1(-2-0) - 3(-2-1) + 1(0-1)$$

$$= 6$$

$$D_z = \begin{vmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$=1(0-1)-1(0-1)+3(1+1)$$

$$=6$$

$$x=\frac{D_x}{D}=\frac{6}{6}=1$$

$$y=\frac{D_y}{D}=\frac{6}{6}=1$$

$$z=\frac{D_z}{D}=\frac{6}{6}=1$$

2) $3x+y+z=4$ $\quad 2x-3y+z=7$ $\quad x+y+3z=6$

[S-18]

Ans.

$$D=\begin{vmatrix} 3 & 1 & 1 \\ 2 & -3 & 1 \\ 1 & 1 & 3 \end{vmatrix}=3(-9-1)-1(6-1)+1(2+3)=-30$$

$$D_x=\begin{vmatrix} 4 & 1 & 1 \\ 7 & -3 & 1 \\ 6 & 1 & 3 \end{vmatrix}=4(-9-1)-1(21-6)+1(7+18)=-30$$

$$\therefore x=\frac{D_x}{D}=\frac{-30}{-30}=1$$

$$D_y=\begin{vmatrix} 3 & 4 & 1 \\ 2 & 7 & 1 \\ 1 & 6 & 3 \end{vmatrix}=3(21-6)-4(6-1)+1(12-7)=30$$

$$\therefore y=\frac{D_y}{D}=\frac{30}{-30}=-1$$

$$D_z = \begin{vmatrix} 3 & 1 & 4 \\ 2 & -3 & 7 \\ 1 & 1 & 6 \end{vmatrix} = 3(-18-7)-1(12-7)+4(2+3) = -60$$

$$\therefore z = \frac{D_z}{D} = \frac{-60}{-30} = 2$$

3) $x - y - 2z = 1$ $\quad 2x + 3y + 4z = 4$ $\quad 3x - 2y - 6z = 5$

[W-17,S-13]

Ans.

$$\text{Let } D = \begin{vmatrix} 1 & -1 & -2 \\ 2 & 3 & 4 \\ 3 & -2 & -6 \end{vmatrix}$$

$$= 1(-18+8)+1(-12-12)-2(-4-9)$$

$$= -8$$

$$D_x = \begin{vmatrix} 1 & -1 & -2 \\ 4 & 3 & 4 \\ 5 & -2 & -6 \end{vmatrix}$$

$$= 1(-18+8)+1(-24-20)-2(-8-15)$$

$$= -8$$

$$D_y = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 4 & 4 \\ 3 & 5 & -6 \end{vmatrix}$$

$$= 1(-24-20)-1(-12-12)-2(10-12)$$

$$= -16$$

$$D_y = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 4 \\ 3 & -2 & 5 \end{vmatrix}$$

$$= 1(15+8)+1(10-12)+1(-4-9)$$

$$= 8$$

$$x = \frac{D_x}{D} = \frac{-8}{-8} = 1$$

$$y = \frac{D_y}{D} = \frac{-16}{-8} = 2$$

$$z = \frac{D_z}{D} = \frac{8}{-8} = -1$$

4) $3x + 3y - z = 11$ $2x - y + 2z = 9$ $4x + 3y + 2z = 25$

[W-17]

Ans.

$$D = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$$

$$= 3(-2-6) - 3(4-8) - 1(6+4)$$

$$= -22$$

$$D_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$$

$$= 11(-2-6) - 3(18-50) - 1(27+25)$$

$$= -44$$

$$\therefore x = \frac{D_x}{D} = \frac{-44}{-22} = 2$$

$$D_y = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$$

$$= 3(18-50) - 11(4-8) - 1(50-36)$$

$$= -66$$

$$\therefore y = \frac{D_y}{D} = \frac{-66}{-22} = 3$$

$$D_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$$

$$= 3(-25-27) - 3(50-36) + 11(6+4)$$

$$= -88$$

$$\therefore z = \frac{D_z}{D} = \frac{-88}{-22} = 4$$

5) $x+y=4-z$ $y+z=1-2x$ $x+z=y$

[S-15]

Ans.

$$x+y+z=4$$
$$2x+y+z=1$$
$$x-y+z=0$$

$$D=\begin{vmatrix}1 & 1 & 1\\ 2 & 1 & 1\\ 1 & -1 & 1\end{vmatrix}=1(1+1)-1(2-1)+1(-2-1)$$
$$=-2$$

$$D_x=\begin{vmatrix}4 & 1 & 1\\ 1 & 1 & 1\\ 0 & -1 & 1\end{vmatrix}=4(1+1)-1(1-0)+1(-1-0)$$
$$=6$$

$$D_y=\begin{vmatrix}1 & 4 & 1\\ 2 & 1 & 1\\ 1 & 0 & 1\end{vmatrix}=1(1-0)-4(2-1)+1(0-1)$$
$$=-4$$

$$D_z=\begin{vmatrix}1 & 1 & 4\\ 2 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=1(0+1)-1(0-1)+4(-2-1)$$
$$=-10$$

$$\therefore x=\frac{D_x}{D}=\frac{6}{-2}=-3$$
$$y=\frac{D_y}{D}=\frac{-4}{-2}=2$$
$$z=\frac{D_z}{D}=\frac{-10}{-2}=5$$

6) $x+z=4$ $y+z=2$ $x+y=0$

[S-19]

Ans.

$$D=\begin{vmatrix}1&0&1\\0&1&1\\1&1&0\end{vmatrix}=1(0-1)-0(0-1)+1(0-1)=-2$$

$$D_x=\begin{vmatrix}4&0&1\\2&1&1\\0&1&0\end{vmatrix}=4(0-1)-0(0-0)+1(2-0)=-2$$

$$\therefore x=\frac{D_x}{D}=\frac{-2}{-2}=1$$

$$D_y=\begin{vmatrix}1&4&1\\0&2&1\\1&0&0\end{vmatrix}=1(0-0)-4(0-1)+1(0-2)=2$$

$$\therefore y=\frac{D_y}{D}=\frac{2}{-2}=-1$$

$$D_z=\begin{vmatrix}1&0&4\\0&1&2\\1&1&0\end{vmatrix}=1(0-2)-0(0-2)+4(0-1)=-6$$

$$\therefore z=\frac{D_z}{D}=\frac{-6}{-2}=3$$

7) $x+y=0$ $y+z=2$ $x+z=4$

[W-18,W-12]

Ans.

$$D=\begin{vmatrix}1&1&0\\0&1&1\\1&0&1\end{vmatrix}=1(1-0)-1(0-1)+0=1+1=2$$

$$D_x = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 0 - 1(2-4) + 0 = 2$$

$$\therefore x = \frac{D_x}{D} = \frac{2}{2} = 1$$

$$D_y = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1(2-4) - 0 + 0 = -2$$

$$\therefore y = \frac{D_y}{D} = \frac{-2}{2} = -1$$

$$D_z = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 4 \end{vmatrix} = 1(4-0) - 1(0-2) + 0 = 4 + 2 = 6$$

$$\therefore z = \frac{D_z}{D} = \frac{6}{2} = 3$$

8) $2x + 3y = 5$ $y - 3z = -2$ $z + 3x = 4$

[W-16]

Ans.

$$D = \begin{vmatrix} 2 & 3 & 0 \\ 0 & 1 & -3 \\ 3 & 0 & 1 \end{vmatrix} = 2(1+0) - 3(0+9) = -25$$

$$D_x = \begin{vmatrix} 5 & 3 & 0 \\ -2 & 1 & -3 \\ 4 & 0 & 1 \end{vmatrix} = 5(1+0) - 3(-2+12) = -25$$

$$D_y = \begin{vmatrix} 2 & 5 & 0 \\ 0 & -2 & -3 \\ 3 & 4 & 1 \end{vmatrix} = 2(-2+12) - 5(0+9) = -25$$

$$D_z = \begin{vmatrix} 2 & 3 & 5 \\ 0 & 1 & -2 \\ 3 & 0 & 4 \end{vmatrix} = 2(4+0)-3(0+6)+5(0-3) = -25$$

$$\therefore x = \frac{D_x}{D} = \frac{-25}{-25} = 1$$

$$\therefore y = \frac{D_y}{D} = \frac{-25}{-25} = 1$$

$$\therefore z = \frac{D_z}{D} = \frac{-25}{-25} = 1$$

9) $x + y = 3$ $y + z = 5$ $x + z = 4$

[S-16]

Ans.

$$D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0)-1(0-1)+0(0-1) = 2$$

$$D_x = \begin{vmatrix} 3 & 1 & 0 \\ 5 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 3(1-0)-1(5-4)+0 = 2$$

$$D_y = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1(5-4)-3(0-1)+0 = 4$$

$$D = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 1 & 0 & 4 \end{vmatrix} = 1(4-0)-1(0-5)+3(0-1) = 6$$

$$\therefore x=\frac{D_x}{D}=\frac{-25}{-25}=1$$

$$\therefore y=\frac{D_y}{D}=\frac{-25}{-25}=1$$

$$\therefore z=\frac{D_z}{D}=\frac{-25}{-25}=1$$

10) $x+y+z=2$ $y+z=1$ $x+z=3$

[W-18]

Ans.

$$D=\begin{vmatrix}1&1&1\\0&1&1\\1&0&1\end{vmatrix}$$

$$=1(1-0)-1(0-1)+1(0-1)=1$$

$$D_x=\begin{vmatrix}2&1&1\\1&1&1\\3&0&1\end{vmatrix}$$

$$=2(1-0)-1(1-3)+1(0-3)=1$$

$$\therefore x=\frac{D_x}{D}=\frac{1}{1}=1$$

$$D_y=\begin{vmatrix}1&2&1\\0&1&1\\1&3&1\end{vmatrix}$$

$$=1(1-3)-2(0-1)+1(0-1)=-1$$

$$\therefore y=\frac{D_y}{D}=\frac{-1}{1}=-1$$

$$D_z = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= 1(3-0) - 1(0-1) + 2(0-1) = 2$$

$$\therefore z = \frac{D_z}{D} = \frac{2}{1} = 2$$

11) $\frac{5}{x+2} + \frac{3}{y+1} = 2 \qquad \frac{10}{x+2} - \frac{3}{y+1} = 1$

[W-15]

Ans.

$$\text{Let } \frac{1}{x+2} = p \text{ and } \frac{1}{y+1} = q$$

$$\therefore 5p + 3q = 2$$

$$10p - 3q = 1$$

$$D = \begin{vmatrix} 5 & 3 \\ 10 & -3 \end{vmatrix} = -15 - 30 = -45$$

$$D_p = \begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} = -6 - 3 = -9$$

$$D_q = \begin{vmatrix} 5 & 2 \\ 10 & 1 \end{vmatrix} = 5 - 20 = -15$$

$$\therefore p = \frac{D_p}{D} = \frac{-9}{-45} = \frac{1}{5}$$

$$q = \frac{D_q}{D} = \frac{-15}{-45} = \frac{1}{3}$$

$$\text{But } \frac{1}{x+2} = p$$

$$\frac{1}{x+2} = \frac{1}{5}$$

$$\therefore x + 2 = 5$$

$$x = 3$$

$$\frac{1}{y+1}=q$$

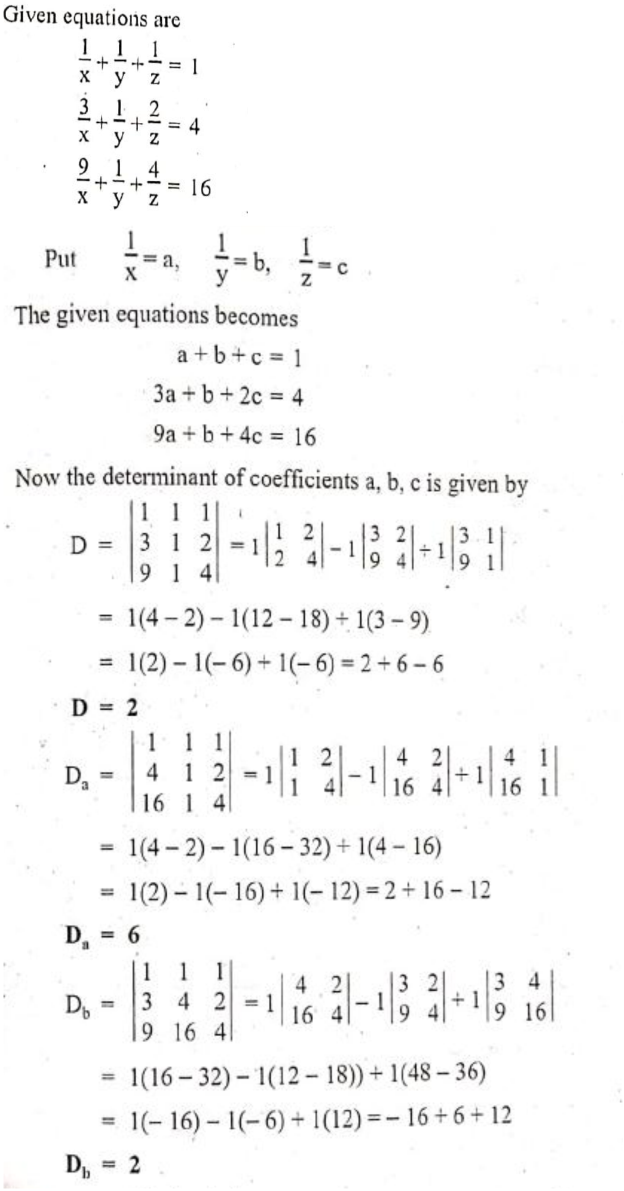
$$\frac{1}{y+1}=\frac{1}{3}$$

$$y+1=3$$

$$y=2$$

12) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ $\frac{3}{x}+\frac{1}{y}+\frac{2}{z}=4$ $\frac{9}{x}+\frac{1}{y}+\frac{4}{z}=16$

Ans.



Given equations are

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$$

$$\frac{3}{x}+\frac{1}{y}+\frac{2}{z}=4$$

$$\frac{9}{x}+\frac{1}{y}+\frac{4}{z}=16$$

Put $\frac{1}{x}=a,\quad \frac{1}{y}=b,\quad \frac{1}{z}=c$

The given equations becomes

$$a+b+c=1$$

$$3a+b+2c=4$$

$$9a+b+4c=16$$

Now the determinant of coefficients a, b, c is given by

$$D=\begin{vmatrix}1&1&1\\3&1&2\\9&1&4\end{vmatrix}=1\begin{vmatrix}1&2\\2&4\end{vmatrix}-1\begin{vmatrix}3&2\\9&4\end{vmatrix}+1\begin{vmatrix}3&1\\9&1\end{vmatrix}$$

$$=1(4-2)-1(12-18)+1(3-9)$$

$$=1(2)-1(-6)+1(-6)=2+6-6$$

$$\mathbf{D=2}$$

$$D_a=\begin{vmatrix}1&1&1\\4&1&2\\16&1&4\end{vmatrix}=1\begin{vmatrix}1&2\\1&4\end{vmatrix}-1\begin{vmatrix}4&2\\16&4\end{vmatrix}+1\begin{vmatrix}4&1\\16&1\end{vmatrix}$$

$$=1(4-2)-1(16-32)+1(4-16)$$

$$=1(2)-1(-16)+1(-12)=2+16-12$$

$$\mathbf{D_a=6}$$

$$D_b=\begin{vmatrix}1&1&1\\3&4&2\\9&16&4\end{vmatrix}=1\begin{vmatrix}4&2\\16&4\end{vmatrix}-1\begin{vmatrix}3&2\\9&4\end{vmatrix}+1\begin{vmatrix}3&4\\9&16\end{vmatrix}$$

$$=1(16-32)-1(12-18))+1(48-36)$$

$$=1(-16)-1(-6)+1(12)=-16+6+12$$

$$\mathbf{D_b=2}$$

$$D_c = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 4 \\ 9 & 1 & 16 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 1 & 16 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ 9 & 16 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix}$$

$$= 1(16-4) - 1(48-36) + 1(3-9)$$

$$= 1(12) - 1(12) + 1(-6) = 12 - 12 - 6$$

$$D_c = -6$$

$$a = \frac{D_a}{D} = \frac{6}{2} = 3$$

$$b = \frac{D_b}{D} = \frac{2}{2} = 1$$

$$c = \frac{D_c}{D} = \frac{-6}{2} = -3$$

$$a = 3, \quad b = 1, \quad c = -3$$

$$\frac{1}{x} = a$$

$$\frac{1}{x} = 3$$

$$1 = 3x$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$\frac{1}{y} = b$$

$$\therefore \frac{1}{y} = 1$$

$$\therefore 1 = y$$

$$\therefore y = 1$$

$$\frac{1}{z} = c$$

$$\therefore \frac{1}{z} = -3$$

$$\therefore 1 = -3z$$

$$\therefore -3z = 1$$

$$\therefore z = \frac{-1}{3}$$

13) $4yz + 3zx + 5xy = 10xyz$ $5yz + 6xz + 7xy = 0$ $3yz + 2xz + xy = 0$

Ans.

$$4yz + 3zx + 5xy = 10xyz$$

$$5yz + 6zx + 7xy = 0$$

$$3yz + 2zx + xy = 0$$

Divide the equations xyz on both sides.

$$\frac{4yz}{xyz} + \frac{3zx}{xyz} + \frac{5xy}{xyz} = \frac{10xyz}{xyz}$$

$$\frac{5yz}{xyz} + \frac{6zx}{xyz} + \frac{7xy}{xyz} = \frac{0}{xyz}$$

$$\frac{3yz}{xyz} + \frac{2zx}{xyz} + \frac{xy}{xyz} = \frac{0}{xyz}$$

Then

$$\therefore \quad \frac{4}{x}+\frac{3}{y}+\frac{5}{z}=10$$

$$\therefore \quad \frac{5}{x}+\frac{6}{y}+\frac{7}{z}=0$$

$$\therefore \quad \frac{3}{x}+\frac{2}{y}+\frac{1}{z}=0$$

To convert them into linear form.

Put $\frac{1}{x}=a,\ \frac{1}{y}=b,\ \frac{1}{z}=c$

$$\therefore \quad 4a+3b+5c=10$$

$$5a+6b+7c=0$$

$$3a+2b+c=0$$

Now the determinant of coefficients a, b, c is given by

Here $$D=\begin{vmatrix}4&3&5\\5&6&7\\3&2&1\end{vmatrix}=4\begin{vmatrix}6&7\\2&1\end{vmatrix}-3\begin{vmatrix}5&7\\3&1\end{vmatrix}+5\begin{vmatrix}5&6\\3&2\end{vmatrix}$$

$$=4(6-14)-3(5-21)+5(10-18)$$

$$=4(-8)-3(-16)+5(-8)$$

$$=-32+48-40$$

$$\mathbf{D=-24}$$

$$D_a=\begin{vmatrix}10&3&5\\0&6&7\\0&2&1\end{vmatrix}$$

$$=10\begin{vmatrix}6&7\\2&1\end{vmatrix}-3\begin{vmatrix}0&7\\0&1\end{vmatrix}+5\begin{vmatrix}0&6\\0&2\end{vmatrix}$$

$$=10(6-14)-3(0-0)+5(0-0)$$

$$=10(-8)-3(0)+5(0)=-80-0+0$$

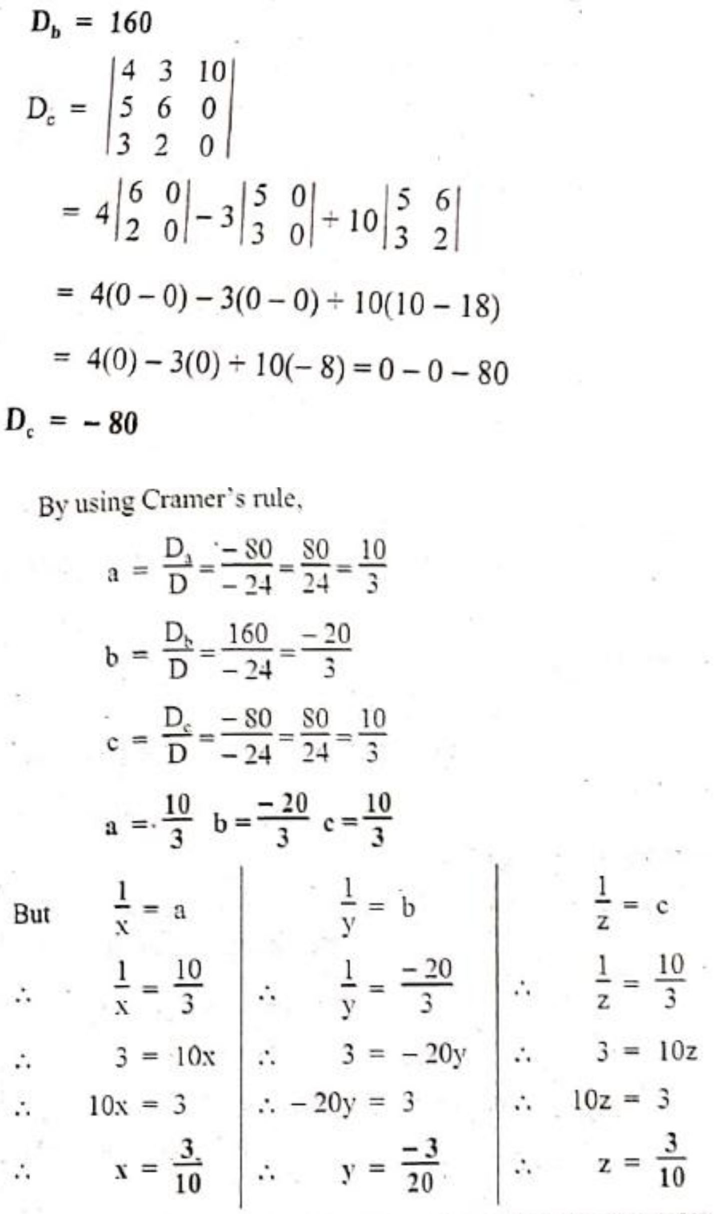
$$\mathbf{D_a=-80}$$

$$D_b=\begin{vmatrix}4&10&5\\5&0&7\\3&0&1\end{vmatrix}$$

$$=4\begin{vmatrix}0&7\\0&1\end{vmatrix}-10\begin{vmatrix}5&7\\3&1\end{vmatrix}+5\begin{vmatrix}5&0\\3&0\end{vmatrix}$$

$$=4(0-0)-10(5-21)+5(0-0)$$

$$=4(0)-10(-160)+5(0)=0+160+0$$



$$D_b = 160$$

$$D_c = \begin{vmatrix} 4 & 3 & 10 \\ 5 & 6 & 0 \\ 3 & 2 & 0 \end{vmatrix}$$

$$= 4\begin{vmatrix} 6 & 0 \\ 2 & 0 \end{vmatrix} - 3\begin{vmatrix} 5 & 0 \\ 3 & 0 \end{vmatrix} + 10\begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$$= 4(0-0) - 3(0-0) + 10(10-18)$$

$$= 4(0) - 3(0) + 10(-8) = 0 - 0 - 80$$

$$D_c = -80$$

By using Cramer's rule,

$$a = \frac{D_a}{D} = \frac{-80}{-24} = \frac{80}{24} = \frac{10}{3}$$

$$b = \frac{D_b}{D} = \frac{160}{-24} = \frac{-20}{3}$$

$$c = \frac{D_c}{D} = \frac{-80}{-24} = \frac{80}{24} = \frac{10}{3}$$

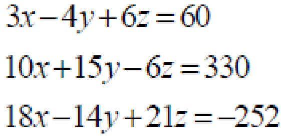
$$a = \frac{10}{3} \quad b = \frac{-20}{3} \quad c = \frac{10}{3}$$

But $\frac{1}{x} = a$	$\frac{1}{y} = b$	$\frac{1}{z} = c$
$\therefore \frac{1}{x} = \frac{10}{3}$	$\therefore \frac{1}{y} = \frac{-20}{3}$	$\therefore \frac{1}{z} = \frac{10}{3}$
$\therefore 3 = 10x$	$\therefore 3 = -20y$	$\therefore 3 = 10z$
$\therefore 10x = 3$	$\therefore -20y = 3$	$\therefore 10z = 3$
$\therefore x = \frac{3}{10}$	$\therefore y = \frac{-3}{20}$	$\therefore z = \frac{3}{10}$

14) $\frac{x}{4} - \frac{y}{3} + \frac{z}{2} = 5$ $\frac{x}{3} + \frac{y}{2} - \frac{z}{5} = 11$ $\frac{x}{7} - \frac{y}{9} + \frac{z}{6} = -2$

[W-14]

Ans.



$$3x - 4y + 6z = 60$$
$$10x + 15y - 6z = 330$$
$$18x - 14y + 21z = -252$$

$$\therefore D=\begin{vmatrix}3 & -4 & 6\\ 10 & 15 & -6\\ 18 & -14 & 21\end{vmatrix}=3(315-84)+4(210+108)+6(-140-270)$$

$$=-495$$

$$D_y=\begin{vmatrix}3 & 60 & 6\\ 10 & 330 & -6\\ 18 & -252 & 21\end{vmatrix}=3(6930-1512)-60(210+108)+6(-2520-5940)$$

$$=-53586$$

$$D_z=\begin{vmatrix}3 & -4 & 60\\ 10 & 15 & 330\\ 18 & -14 & -252\end{vmatrix}=3(-3780+4620)+4(-2520-5940)+60(-140-270)$$

$$=-55920$$

$$\therefore y=\frac{D_y}{D}=\frac{-53586}{-495}=108.255$$

$$z=\frac{D_z}{D}=\frac{-55920}{-495}=112.970$$

D. Engineering Application:-

Examples:-

Que. Solve the following equations by Crammer's Rule / Determinant method.

1) The voltage in an electric circuit are related by the following equation .Find the values of V_1 , V_2 , V_3 by using crammers rule

$V_1 + V_2 + V_3 = 9$ $V_1 - V_2 + V_3 = 3$ $V_1 + V_2 - V_3 = 1$

[W-19,S-18 , S-14]

Ans.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1(1-1)-1(-1-1)+1(1+1)=4$$

$$D_{V_1} = \begin{vmatrix} 9 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 9(1-1)-1(-3-1)+1(3+1)=8$$

$$\therefore V_1 = \frac{D_{V_1}}{D} = \frac{8}{4} = 2$$

$$D_{V_2} = \begin{vmatrix} 1 & 9 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1(-3-1)-9(-1-1)+1(1-3)=12$$

$$\therefore V_2 = \frac{D_{V_2}}{D} = \frac{12}{4} = 3$$

$$D_{V_3} = \begin{vmatrix} 1 & 1 & 9 \\ 1 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1(-1-3)-1(1-3)+9(1+1)=16$$

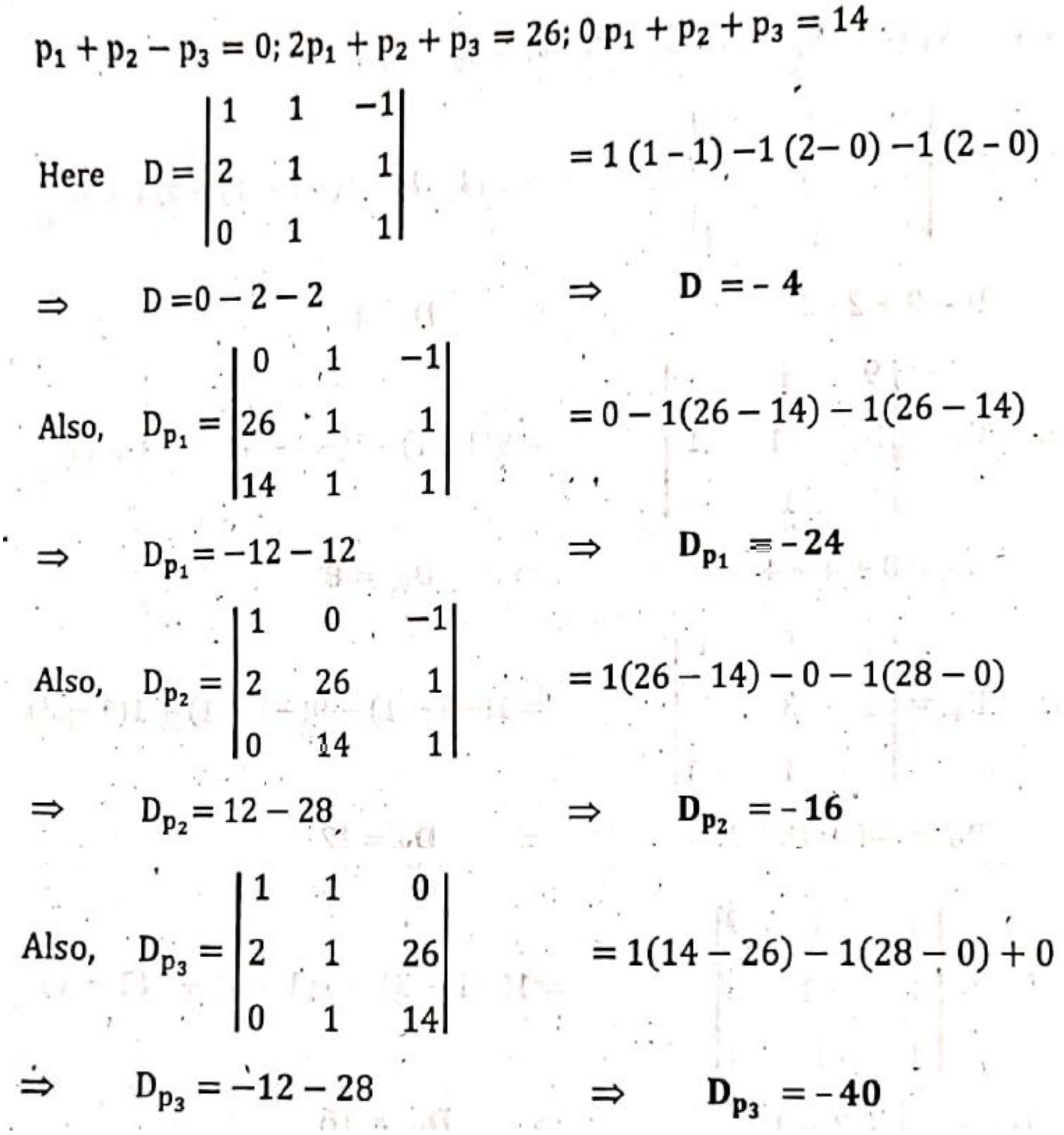
$$\therefore V_3 = \frac{D_{V_3}}{D} = \frac{16}{4} = 4$$

2) The following equations are obtained as a result of experiment. Find the values of P_1 , P_2 , P_3 by using crammers rule

$P_1 + P_2 - P_3 = 0$ $2P_1 + P_2 + P_3 = 26$ $P_2 + P_3 = 14$

[SQP]

Ans.



$$p_1 + p_2 - p_3 = 0; 2p_1 + p_2 + p_3 = 26; 0\,p_1 + p_2 + p_3 = 14$$

$$\text{Here} \quad D = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1\,(1-1) - 1\,(2-0) - 1\,(2-0)$$

$$\Rightarrow \quad D = 0 - 2 - 2 \qquad \Rightarrow \quad \mathbf{D = -4}$$

$$\text{Also,} \quad D_{p_1} = \begin{vmatrix} 0 & 1 & -1 \\ 26 & 1 & 1 \\ 14 & 1 & 1 \end{vmatrix} = 0 - 1(26-14) - 1(26-14)$$

$$\Rightarrow \quad D_{p_1} = -12 - 12 \qquad \Rightarrow \quad \mathbf{D_{p_1} = -24}$$

$$\text{Also,} \quad D_{p_2} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 26 & 1 \\ 0 & 14 & 1 \end{vmatrix} = 1(26-14) - 0 - 1(28-0)$$

$$\Rightarrow \quad D_{p_2} = 12 - 28 \qquad \Rightarrow \quad \mathbf{D_{p_2} = -16}$$

$$\text{Also,} \quad D_{p_3} = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 26 \\ 0 & 1 & 14 \end{vmatrix} = 1(14-26) - 1(28-0) + 0$$

$$\Rightarrow \quad D_{p_3} = -12 - 28 \qquad \Rightarrow \quad \mathbf{D_{p_3} = -40}$$

$\therefore$ By Cramer's rule,

$$p_1 = \frac{D_{p1}}{D} = \frac{-24}{-4} = 6$$

$$p_2 = \frac{D_{p2}}{D} = \frac{-16}{-4} = 4$$

$$p_3 = \frac{D_{p3}}{D} = \frac{-40}{-4} = 10$$

E. AREA OF TRIANGLE

$$A=\frac{1}{2}X\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

Note:-

1) **If the value of Determinant is Zero then the given points are Collinear points.**
2) **If the value of Determinant is Non-Zero then the given points are Non Collinear points**

Examples:-

Que. **Find the area of the triangle whose vertices are**

1) (4 , 3) (1 , 4) (2 , 3) **[W-18]**

Ans.

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}4 & 3 & 1\\ 1 & 4 & 1\\ 2 & 3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[4(4-3)-3(1-2)+1(3-8)\right]$$

$$=1$$

2) (3 , 1) (−1 , 3) (−3 , −2) **[S-18]**

Ans.

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}3 & 1 & 1\\ -1 & 3 & 1\\ -3 & -2 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[3(3+2)-1(-1+3)+1(2+9)\right]$$

$$\underline{}$$

$$A=12$$

3) $(-3,1)\ (1,-3)\ (2,3)$ **[W-19]**

Ans.

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$\therefore A=\frac{1}{2}\begin{vmatrix}-3 & 1 & 1\\ 1 & -3 & 1\\ 2 & 3 & 1\end{vmatrix}$$

$$\therefore A=\frac{1}{2}\left[-3(-3-3)-1(1-2)+1(3+6)\right]$$

$$\therefore A=14$$

4) $(4,7)\ (1,3)\ (5,1)$ **[SQP]**

Ans.

$$=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}4 & 7 & 1\\ 1 & 3 & 1\\ 5 & 1 & 1\end{vmatrix}$$

$$=\frac{1}{2}\{4(3-1)-7(1-5)+1(1-15)\}$$

$$=\frac{1}{2}\{8+28-14\}$$

$$=\frac{1}{2}\{22\}$$

$$=11\text{sq. units.}$$

5) (6 ,6) (2 ,3) (4 ,7)

Ans.

$$=\frac{1}{2}\begin{vmatrix}6 & 6 & 1\\ 2 & 3 & 1\\ 4 & 7 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[6\begin{vmatrix}3 & 1\\ 7 & 1\end{vmatrix}-6\begin{vmatrix}2 & 1\\ 4 & 1\end{vmatrix}+1\begin{vmatrix}2 & 3\\ 4 & 7\end{vmatrix}\right]$$

$$=\frac{1}{2}[6(3-7)-6(2-4)+1(14-12)]$$

$$=\frac{1}{2}[6(-4)-6(-2)+1(2)]$$

$$=\frac{1}{2}[-24+12+2]$$

$$=\frac{1}{2}[-10]=-5$$

$$=5\text{ sq. units}\qquad(\because\text{ Area is always positive})$$

6) $(-8,-2)(-4,-6)(-1,5)$

Ans.

$$=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}-8 & -2 & 1\\ -4 & -6 & 1\\ -1 & 5 & 1\end{vmatrix}$$

$$=\frac{1}{2}\{-8(-6-5)+2(-4+1)+1(-20-6)\}$$

$$=\frac{1}{2}\{88-6-26\}$$

$$=\frac{1}{2}\{56\}$$

7) $(2,1)(1,4)(-3,\ 2)$

Ans.

$$=\frac{1}{2}\begin{vmatrix}2 & 1 & 1\\ 1 & 4 & 1\\ -3 & 2 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[2\begin{vmatrix}4 & 1\\ 2 & 1\end{vmatrix}-1\begin{vmatrix}1 & 1\\ -3 & 1\end{vmatrix}\div 1\begin{vmatrix}1 & 4\\ -3 & 2\end{vmatrix}\right]$$

$$=\frac{1}{2}[2(4-2)-1(1-(-3))\div 1(2-(-12))]$$

$$= \frac{1}{2}[2(2) - 1(4) + 1(14)]$$

$$= \frac{1}{2}[4 - 4 + 14]$$

$$= \frac{1}{2}[14]$$

$$= 7 \text{ Sq. units}$$

8) $(-1, 5)(3, 1)(5, 7)$

Ans.

$$= \frac{1}{2}\begin{vmatrix} -1 & 5 & 1 \\ 3 & 1 & 1 \\ 5 & 7 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-1\begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} - 5\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 5 & 7 \end{vmatrix}\right]$$

$$= \frac{1}{2}[-1(1-7) - 5(3-5) + 1(21-5)]$$

$$= \frac{1}{2}[-1(-6) - 5(-2) + 1(16)]$$

$$= \frac{1}{2}[6 + 10 + 16]$$

$$= \frac{1}{2}[32]$$

$$= 16 \text{ Sq. units}$$

9) $(2, 3)(5, 7)(-3, 4)$

Ans.

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 5 & 7 & 1 \\ -3 & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2\begin{vmatrix} 7 & 1 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 5 & 1 \\ -3 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 7 \\ -3 & 4 \end{vmatrix}\right]$$

$$= \frac{1}{2}[2(7-4) - 3(5-(-3)) + 1(20-(-21))]$$

$$= \frac{1}{2}[2(3) - 3(8) + 1(41)]$$

$$= \frac{1}{2}[6 - 24 + 41]$$

$$= \frac{1}{2}[23] = \frac{23}{2} \text{ OR}$$

$$= 11.5 \text{ Sq. units}$$

10) $(1, 2)\ (-6, 1)\ (0, 8)$

Ans.

$$= \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ -6 & 1 & 1 \\ 0 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 2\begin{vmatrix} -6 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} -6 & 1 \\ 0 & 8 \end{vmatrix}\right]$$

$$= \frac{1}{2}[1(1-8) - 2(-6-0) + 1(-48-0)]$$

$$= \frac{1}{2}[1(-7) - 2(-6) + 1(-48)]$$

$$= \frac{1}{2}[-7 + 12 - 48]$$

$$= \frac{1}{2}[-43]$$

$$= +\frac{43}{2} \text{ Sq. units OR } 21.5 \text{ Sq. units}$$

11) Show that the points $(8, 1)$ $(3, -4)$ $(2, -5)$ are collinear using determinant method. **[W-17]**

Ans.

$$=\begin{vmatrix} 8 & 1 & 1 \\ 3 & -4 & 1 \\ 2 & -5 & 1 \end{vmatrix}$$

$$=8(-4+5)-1(3-2)+1(-15+8)$$

$$=0$$

$\therefore$ Points are collinear

12) If the area of the triangle formed by the points $(-7, 8)$ $(9, -12)$ $(13, y_1)$ is 248 Sq.unit. Find y_1

Ans.

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$248 = \frac{1}{2}\begin{vmatrix} -7 & 8 & 1 \\ 9 & -12 & 1 \\ 13 & y_1 & 1 \end{vmatrix}$$

$$248 = \frac{1}{2}\left[-7\begin{vmatrix} -12 & 1 \\ y_1 & 1 \end{vmatrix} - 8\begin{vmatrix} 9 & 1 \\ 13 & 1 \end{vmatrix} + 1\begin{vmatrix} 9 & -12 \\ 13 & y_1 \end{vmatrix}\right]$$

$$= \frac{1}{2}[-7(-12-y_1)-8(9-13)+1(9\,y_1+156)]$$

$$248 = \frac{1}{2}[84+7\,y_1+32+9y_1+156]$$

$$248 \times 2 = 272 + 16\, y_1$$

$$496 = 272 + 16\, y_1$$

$$496 - 272 = 16\, y_1$$

$$224 = 16\, y_1$$

$$y_1 = \frac{224}{16} \qquad y_1 = 14$$

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DEPARTMENT OF SCIENCE AND HUMANITY

ACADEMIC YEAR-2020-2021

TUTORIAL NO.2

DETERMINANT

1) Evaluate $\begin{vmatrix} 3 & -5 & -1 \\ 1 & 3 & 5 \\ -5 & 1 & 3 \end{vmatrix}$ **[W-12]**

2) Find the value of $'x'$ if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$ **[S-19, S-15, SQP]**

3) Find the value of $'x'$ if $\begin{vmatrix} 1 & 1 & 1 \\ 3 & x & 3 \\ 1 & x & 2 \end{vmatrix} = 0$ **[S-18]**

4) Solve by using crammers rule $x - y - 2z = 1$, $2x + 3y + 4z = 4$, $3x - 2y - 6z = 5$ **[W-17,S-13]**

5) Solve by using crammers rule $x + y = 0$, $y + z = 2$, $x + z = 4$ **[W-18,W-12]**

6) The voltage in an electric circuit are related by the following equation .Find the values of V_1 , V_2 , V_3 by using crammers rule $V_1 + V_2 + V_3 = 9$, $V_1 - V_2 + V_3 = 3$, $V_1 + V_2 - V_3 = 1$ **[W-19,S-18 , S-14]**

7) The following equations are obtained as a result of experiment. Find the values of P_1 , P_2 , P_3 by using crammers rule

$P_1 + P_2 - P_3 = 0$, $2P_1 + P_2 + P_3 = 26$, $P_2 + P_3 = 14$ **[SQP]**

8) Find the area of triangle whose vertices are $(4,3)$ $(1,4)$ $(2,3)$ **[W-18]**

9) Find the area of triangle whose vertices are $(3,1)$ $(-1,3)$ $(-3,-2)$ **[S-18]**

10) Show that the points $(8,1)$ $(3,-4)$ $(2,-5)$ are collinear using determinant method **[W-17]**

MATRICES

Position in Question Paper Total Marks-14

Q.2. a) 4-Marks.

Q.4. a) 4-Marks.

Q.6. c) 6-Marks.

Matrix: - Definition:-

An arrangement of an element or object in a horizontal lines called ROWS & vertical lines called COLUMN enclosed between two closed brackets or Square brackets.

Types of Matrices:-

1) **Row matrix:-** A matrix of order 1xn is called Row matrix **OR**

A matrix containing only one row & any number of columns is called row matrix.

e.g.:- i) $[1 \quad 2 \quad 3 \quad 4 \quad 5]_{1x5}$ is row matrix.

ii) $[1]_{1x1}$ is row matrix.

iii) $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}_{4x1}$ is not row matrix.

2) **Column matrix:-** A matrix of order n x1 is called column matrix **OR**

A matrix containing any number of row & only one column is called column matrix.

e.g.:- i) $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}_{4x1}$ is column matrix.

ii) $[1]_{1x1}$ is column matrix.

iii) $[1 \quad 2 \quad 3 \quad 4 \quad 5]_{1x5}$is not column matrix.

3) **Square matrix: -** A matrix having equal number of rows & columns.

e.g.:- i) $\begin{bmatrix}1 & 2\\3 & -7\end{bmatrix}_{2x2}$ is Square matrix.

ii) If A= $[1]_{1x1}$ is Square matrix.

iii) $\begin{bmatrix}1 & 2\\-5 & 9\\1 & -6\end{bmatrix}_{3x2}$ is not Square matrix.

4) **Rectangular Matrix:-** A matrix in which No of rows ≠ No. of column.

e.g.:- i) $\begin{bmatrix}1 & 2 & 3 & 4 & 5\end{bmatrix}_{1x5}$is rectangular matrix.

ii) $\begin{bmatrix}1\end{bmatrix}_{1x1}$is not rectangular matrix.

5) **Upper triangular matrix: -** A square matrix in which all the elements below the principal diagonal are zero is called an upper triangular matrix.

e.g.:- i) $\begin{bmatrix}1 & 2 & 3\\0 & -5 & 6\\0 & 0 & 2\end{bmatrix}_{3x3}$ is upper triangular matrix.

ii) $\begin{bmatrix}1 & 2 & 3\\0 & 0 & 6\\0 & 0 & 0\end{bmatrix}_{3x3}$ is upper triangular matrix.

iii) $\begin{bmatrix}1 & 2 & 3\\1 & 0 & 6\\0 & 0 & 0\end{bmatrix}_{3x3}$ is not upper triangular matrix

iv) $\begin{bmatrix}1 & 2\\0 & -7\end{bmatrix}_{2x2}$ is upper triangular matrix.

v) $\begin{bmatrix}1 & 2\\2 & -7\end{bmatrix}_{2x2}$ is not upper triangular matrix.

6) **Lower triangular matrix:-** A square matrix in which all the elements above the principal diagonal are zero is called an upper triangular matrix.

e.g.:- i) $\begin{bmatrix}1 & 0 & 0\\2 & -5 & 0\\3 & 6 & 2\end{bmatrix}_{3x3}$ is lower triangular matrix.

ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}_{3x3}$ is lower triangular matrix.

iii) $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3x3}$ is not lower triangular matrix

iv) $\begin{bmatrix} 1 & 0 \\ 0 & -7 \end{bmatrix}_{2x2}$ is lower triangular matrix.

v) $\begin{bmatrix} 1 & 2 \\ 0 & -7 \end{bmatrix}_{2x2}$ is not lower triangular matrix.

7) **Triangular matrix:-** A matrix which is either an upper triangular or lower triangular is called triangular matrix.

e.g.:- i) $\begin{bmatrix} 1 & 2 \\ 0 & -7 \end{bmatrix}_{2x2}$ is triangular matrix.

ii) $\begin{bmatrix} 1 & 0 \\ 0 & -7 \end{bmatrix}_{2x2}$ is lower triangular matrix.

8) **Diagonal matrix: -** A square matrix in which all non- diagonal elements are zero & at least one principal diagonal element is non-zero is called a diagonal matrix.

e.g.:- i) $[1]_{1x1}$ is diagonal matrix.

ii) $\begin{bmatrix} 0 & 0 \\ 0 & -7 \end{bmatrix}_{2x2}$ is diagonal matrix.

iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3x3}$ is diagonal matrix.

iv) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3x3}$ is not diagonal matrix.

9) **Scalar matrix:-** A diagonal matrix in which all the principal diagonal (diagonal) elements are equal & all non-diagonal elements are zeros is called Scalar matrix.

e.g.:- i) $[1]_{1x1}$ is Scalar matrix.

ii) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3x3}$ is not Scalar matrix.

iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3x3}$ is not Scalar matrix.

iv) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$ is not Scalar matrix.

10) **Identity matrix: -** A diagonal matrix in which all the principal diagonal (diagonal) elements are '1' (unity) & all non-diagonal elements are zeros is called Scalar matrix.

e.g.:- i) $[1\ ]_{1x1}$is Identity matrix.

ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2x2}$ is Identity matrix.

iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$ is Identity matrix.

iv) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$ is not Identity matrix.

11) **Symmetric matrix:-** A square matrix $A = A^T$.

e.g.:- i) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2x2}$ is Symmetric matrix.

ii) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$ is Symmetric matrix.

iii) $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$ is not Symmetric matrix.

12) **Skew-Symmetric matrix:-** A square matrix $A = -A^T$ & all diagonal elements are zeros.

e.g.:- i) $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}_{3x3}$ is skew-Symmetric matrix

Note: $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = symmetric + skew - symetric$

13) **Idempotent matrix:-** A square matrix A is said to be idempotent matrix if $A^2 = A$.

e.g.:- i) A= $\begin{bmatrix} 1 & 0 \\ 5 & 0 \end{bmatrix}_{2x2}$ is Idempotent matrix.

14) **Involuntary matrix:-** A square matrix A is said to be Involuntary matrix if $A^2 = I$

15) **Nilpotent matrix:-** A square matrix A is said to be nilpotent matrix if there exist a positive integer m such that $A^m = Null\ matrix = Zero\ matrix = 0$

16) **Zero matrix OR Null matrix:-** A square matrix containing all zero elements is called zero matrix OR Null matrix.

e.g.:- i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2x2}$ is Zero matrix OR Null matrix.

ii) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3x3}$ is Zero matrix OR Null matrix.

17) **Orthogonal matrix:-** A square matrix A is said to be orthogonal matrix if

$A.A^T = I$ **OR** $A^T.A = I$

18) **Singular matrix OR Non-invertible matrix:-** A square matrix A is called singular matrix if $|A| = 0$

19) **Non-singular matrix OR invertible matrix:-** A square matrix A is called non-singular matrix if $|A| \neq 0$

20) **Transpose of matrix:-** The interchange of rows into column or column into rows is called transpose of matrix.

e.g.:- i) $$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3} \text{ then } A^T = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$$

ALGEBRA OF MATRICES :-

1) **EQUALITY.**

2) **ADDITION.**

3) **SUBTRACTION.**

4) **SCALAR MULTIPLICATION.**

5) **MULTIPLICATION.**

A. EQUALITY, ADDITION, SUBTRACTION, SCALAR MULTIPLICATION OF MATRIX :-

Examples:-

1) If $\begin{bmatrix}3 & -6\\4 & 2\end{bmatrix}+\begin{bmatrix}2 & 3\\-2 & 1\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}$ find $a , b , c , d.$ **[W-15]**

Ans.

$$\begin{bmatrix}3 & -6\\4 & 2\end{bmatrix}+\begin{bmatrix}2 & 3\\-2 & 1\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}$$

$$\therefore \begin{bmatrix}5 & -3\\2 & 3\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}$$

$$\therefore a=5 \ , \ b=-3 \ , \ c=2 \ , \ d=3$$

2) Find the value of a & b if $\begin{bmatrix}a-4b & 5\\6 & -a+b\end{bmatrix}=\begin{bmatrix}11 & 5\\6 & -5\end{bmatrix}$ **[S-16,W-14]**

Ans.

$$\begin{bmatrix}a-4b & 5\\6 & -a+b\end{bmatrix}=\begin{bmatrix}11 & 5\\6 & -5\end{bmatrix}$$

$$\therefore a-4b=11$$

$$\underline{-a+b=-5}$$

$$\therefore -3b=6$$

$$\therefore \boxed{b=-2}$$

$$\therefore \boxed{a=3}$$

3) If $A=\begin{bmatrix}2 & 3\\4 & 7\end{bmatrix}$, $B=\begin{bmatrix}1 & 3\\4 & 6\end{bmatrix}$ find the matrix $3A-2B$ **[S-14,W-12]**

Ans.

$$3A-2B=3\begin{bmatrix}2&3\\4&7\end{bmatrix}-2\begin{bmatrix}1&3\\4&6\end{bmatrix}$$

$$=\begin{bmatrix}6&9\\12&21\end{bmatrix}-\begin{bmatrix}2&6\\8&12\end{bmatrix}$$

$$=\begin{bmatrix}4&3\\4&9\end{bmatrix}$$

4) If $A=\begin{bmatrix}2&3\\4&5\end{bmatrix}$, $B=\begin{bmatrix}3&7\\1&9\end{bmatrix}$ find the matrix $2A+3B$ **[W-13]**

Ans.

$$2A+3B=2\begin{bmatrix}2&3\\4&5\end{bmatrix}+3\begin{bmatrix}3&7\\1&9\end{bmatrix}$$

$$=\begin{bmatrix}4&6\\8&10\end{bmatrix}+\begin{bmatrix}9&21\\3&27\end{bmatrix}$$

$$=\begin{bmatrix}13&27\\11&37\end{bmatrix}$$

5) If $A=\begin{bmatrix}5&3\\-1&1\end{bmatrix}$, $B=\begin{bmatrix}2&-1\\3&2\end{bmatrix}$ find the matrix $2A-3B$ **[SQP]**

Ans.

$A=\begin{bmatrix}5&3\\-1&1\end{bmatrix}$ and $B=\begin{bmatrix}2&-1\\3&2\end{bmatrix}$

$$2A-3B=2\begin{bmatrix}5&3\\-1&1\end{bmatrix}-3\begin{bmatrix}2&-1\\3&2\end{bmatrix}=\begin{bmatrix}10&6\\-2&2\end{bmatrix}-\begin{bmatrix}6&-3\\9&6\end{bmatrix}$$

$$2A-3B=\begin{bmatrix}4&9\\-11&-4\end{bmatrix}$$

6) If $X=\begin{bmatrix}1&2\\-3&4\end{bmatrix}$, $Y=\begin{bmatrix}4&5\\1&-3\end{bmatrix}$ find the matrix $3X+Y$ **[SQP]**

Ans.

$$3X+Y=3\begin{bmatrix}1&2\\-3&4\end{bmatrix}+\begin{bmatrix}4&5\\1&-3\end{bmatrix}$$

$$=\begin{bmatrix} 3\times1 & 3\times2 \\ 3\times(-3) & 3\times4 \end{bmatrix}+\begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix}+\begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 3+4 & 6+5 \\ -9+1 & 12+(-3) \end{bmatrix}$$

$$3X+Y=\begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$$

7) If $A=\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$ Find the matrix $2A+B$ **[S-18]**

Ans.

$$2A+B=2\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}+\begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 4 \\ -6 & 8 \end{bmatrix}+\begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 6 & 9 \\ -5 & 5 \end{bmatrix}$$

8) If $A=\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ Find $2A+3B-5I$ where I the unit matrix of order two. **[W-16]**

Ans.

$$2A+3B-5I=2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}+3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}-5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix}+\begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}-\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 15 \\ 2 & 24 \end{bmatrix}$$

9) If $A=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$ find the matrix B Such that $2A+3B=0$ **[W-14]**

Ans. $2A+3B=0$

$\therefore 3B=-2A$

$$=-2\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 2\\ -4 & -8\end{bmatrix}$$

$$\therefore B=\frac{1}{3}\begin{bmatrix}-6 & 2\\ -4 & -8\end{bmatrix}$$

10) If $A=\begin{bmatrix}3 & -1\\ 0 & 4\end{bmatrix}$ find the matrix B such that $2A+B=0$ **[W-18]**

Ans. $2A+B=0$ $\quad\quad \therefore B=-2A$

$$\therefore B=-2\begin{bmatrix}3 & -1\\ 0 & 4\end{bmatrix}$$

$$\therefore B=\begin{bmatrix}-6 & 2\\ 0 & -8\end{bmatrix}$$

11) If $A=\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}$ find the matrix X Such that $2A+X=3B$

[S-17]

Ans. $\because 2A+X=3B$

$\therefore X=3B-2A$

$$=3\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}-2\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$$

$$= \begin{bmatrix} 9 & -6 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 5 & -4 \\ -11 & 6 \end{bmatrix}$$

12) If $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ Find matrix X if $2X + 3A - 4B = I$

[S-18,W-15]

Ans. $2X + 3A - 4B = I$

$$2X + 3\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2X + \begin{bmatrix} 9 & -3 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -12 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2X + \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix}$$

$$2X = \begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$

13) If $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$ Find matrix X such that $A + 2X = \begin{bmatrix} 3 & 6 \\ 0 & 1 \end{bmatrix}$ **[W-17]**

Ans.

$$A+2X=\begin{bmatrix}3&6\\0&1\end{bmatrix}$$

$$\therefore 2X=\begin{bmatrix}3&6\\0&1\end{bmatrix}-A$$

$$\therefore 2X=\begin{bmatrix}3&6\\0&1\end{bmatrix}-\begin{bmatrix}1&-2\\4&3\end{bmatrix}$$

$$\therefore 2X=\begin{bmatrix}2&8\\-4&-2\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}2&8\\-4&-2\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}1&4\\-2&-1\end{bmatrix}$$

14) Find X such that $\begin{bmatrix}4&5\\-3&6\end{bmatrix}+X=\begin{bmatrix}10&-1\\0&-6\end{bmatrix}$ **[S-15]**

Ans.

$$\therefore X=\begin{bmatrix}10&-1\\0&-6\end{bmatrix}-\begin{bmatrix}4&5\\-3&6\end{bmatrix}$$

$$=\begin{bmatrix}10-4&-1-5\\0+3&-6-6\end{bmatrix}$$

$$=\begin{bmatrix}6&-6\\3&-12\end{bmatrix}$$

15) Find A if $2A+3\begin{bmatrix}1&3\\2&5\end{bmatrix}=\begin{bmatrix}5&7\\6&3\end{bmatrix}$ **[S-19,S-13]**

Ans.

$$2A+3\begin{bmatrix}1&3\\2&5\end{bmatrix}=\begin{bmatrix}5&7\\6&3\end{bmatrix}$$

$$2A+\begin{bmatrix}3&9\\6&15\end{bmatrix}=\begin{bmatrix}5&7\\6&3\end{bmatrix}$$

$$2A=\begin{bmatrix}5&7\\6&3\end{bmatrix}-\begin{bmatrix}3&9\\6&15\end{bmatrix}$$

$$2A=\begin{bmatrix}2&-2\\0&-12\end{bmatrix}$$

$$A=\begin{bmatrix}1&-1\\0&-6\end{bmatrix}$$

16) If $A=\begin{bmatrix}1&3&2\\-1&2&0\\4&0&3\end{bmatrix}$, $B=\begin{bmatrix}1&0&0\\1&2&0\\1&0&3\end{bmatrix}$, $C=\begin{bmatrix}2&1&2\\2&2&1\\1&2&2\end{bmatrix}$ then find the matrix D such that $2A-3B-D=C$ **[S-13]**

Ans. Given, $2A-3B-D=C$

$$D=2A-3B-C$$

$$D=2\begin{bmatrix}1&3&2\\-1&2&0\\4&0&3\end{bmatrix}-3\begin{bmatrix}1&0&0\\1&2&0\\1&0&3\end{bmatrix}-\begin{bmatrix}2&1&2\\2&2&1\\1&2&2\end{bmatrix}$$

$$D=\begin{bmatrix}2&6&4\\-2&4&0\\8&0&6\end{bmatrix}-\begin{bmatrix}3&0&0\\3&6&0\\3&0&9\end{bmatrix}-\begin{bmatrix}2&1&2\\2&2&1\\1&2&2\end{bmatrix}$$

$$D=\begin{bmatrix}-3&5&2\\-7&-4&-1\\4&-2&-5\end{bmatrix}$$

17) Prove that the matrix $\begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$ is a non- singular matrix. **[S-15,S-13]**

Ans.

Consider, $\begin{vmatrix} 1 & 4 \\ 6 & 9 \end{vmatrix}$

$=9-24$

$=-15 \neq 0$

$\therefore$ Given matrix is non-singular matrix.

18) If $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & -3 \\ 0 & -1 & 1 \end{bmatrix}$ find $|A|$ & verify that A is singular or non- singular matrix. **[S-17]**

Ans.

$$|A| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -3 \\ 0 & -1 & 1 \end{vmatrix}$$

$=2(1-3)+1(4-0)+3(-4-0)$

$=-4+4-12$

$=-12 \neq 0$

$\therefore$ A is non-singular matrix

19) If $A = \begin{bmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$ find whether A is singular or non-singular matrix.

[S-16]

Ans.

$$A = \begin{bmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 7 & 0 & 2 \\ 1 & 2 & 6 \\ 4 & 5 & 3 \end{vmatrix}$$

$$= 7(6-30) - 0(3-24) + 2(5-8) = -174 \neq 0$$

$\therefore$ Matrix A is non singular

20) If $A = \begin{bmatrix} x & 2 & -5 \\ 3 & 1 & 2y \end{bmatrix}, B = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$ and if $3A = B$ find x & y

[S-14]

Ans.

$$Given \quad 3A = B$$

$$\therefore 3\begin{bmatrix} x & 2 & -5 \\ 3 & 1 & 2y \end{bmatrix} = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3x & 6 & -15 \\ 9 & 3 & 6y \end{bmatrix} = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$$

$$\therefore 3x = 2y+5 \quad and \quad 6y = -6$$

$$\therefore x = 1 \quad and \quad y = -1$$

21) Find x, y, z $\begin{bmatrix} 2+x & -1 & 3 \\ 0 & y & z \\ 4 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1+x & 2 & 3 \\ 0 & 1+y & 4 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 6 \\ 0 & -1 & 6 \\ 6 & 4 & 8 \end{bmatrix}$

[W-15]

Ans.

$$\begin{bmatrix} 3+2x & 1 & 6 \\ 0 & 1+2y & z+4 \\ 6 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 6 \\ 0 & -1 & 6 \\ 6 & 4 & 8 \end{bmatrix}$$

$$\because 3+2x=6$$

$$2x=3$$

$$\therefore x=\frac{3}{2}$$

$$1+2y=-1$$

$$2y=-2$$

$$\therefore y=-1$$

$$z+4=6$$

$$\therefore z=2$$

22) If $A=\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}, B=\begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix}$ verify that $A+B=B+A$ **[S-16]**

Ans.

$$A+B=\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}+\begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix}=\begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 4 & 2 \end{bmatrix}$$

$$B+A=\begin{bmatrix} -1 & -1 \\ 3 & 2 \\ 4 & -2 \end{bmatrix}+\begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}=\begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 4 & 2 \end{bmatrix}$$

B. MATRIX MULTIPLICATION :-

Examples:-

1) If $A=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & -5\\ 0 & 1\end{bmatrix}$ find AB **[W-16]**

Ans.

$$AB=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}1 & -5\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & -5+2\\ 2+0 & -10+3\end{bmatrix}$$

$$=\begin{bmatrix}1 & -3\\ 2 & -7\end{bmatrix}$$

2) If $A=\begin{bmatrix}5 & 4\\ 4 & 3\end{bmatrix}$, $B=\begin{bmatrix}-3 & 4\\ 4 & -5\end{bmatrix}$ verify that $AB=BA$ **[SQP]**

Ans.

$$A=\begin{bmatrix}5 & 4\\ 4 & 3\end{bmatrix} \text{ and } B=\begin{bmatrix}-3 & 4\\ 4 & -5\end{bmatrix}$$

$$AB=\begin{bmatrix}5 & 4\\ 4 & 3\end{bmatrix}\cdot\begin{bmatrix}-3 & 4\\ 4 & -5\end{bmatrix}$$

$$AB=\begin{bmatrix}-15+16 & 20-20\\ -12+12 & 16-15\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}-3 & 4\\ 4 & -5\end{bmatrix}\cdot\begin{bmatrix}5 & 4\\ 4 & 3\end{bmatrix}$$

$$BA=\begin{bmatrix}-15+16 & -12+12\\ 20-20 & 16-15\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

3) If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$ verify that $AB \neq BA$ **[S-18]**

Ans.

$$AB = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-15 & -6-10 \\ 2+0 & -4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -16 \\ 2 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 & -5+0 \\ 9+4 & -15+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -5 \\ 13 & -15 \end{bmatrix}$$

$$AB \neq BA$$

4) If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ show that $AB = BA = I$ **[S-19]**

Ans.

$$AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\therefore BA=\begin{bmatrix}6-5 & 15-15\\ -2+2 & -5+6\end{bmatrix}$$

$$\therefore BA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\therefore AB=BA=I$$

5) If $A=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ $,B=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$, $C=\begin{bmatrix}4 & 5\\ 2 & 3\end{bmatrix}$ then show that $AB=AC$ **[S-13]**

Ans.

$$AB=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$$

$$=\begin{bmatrix}12 & 16\\ 12 & 16\end{bmatrix}$$

$$AC=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}4 & 5\\ 2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}12 & 16\\ 12 & 16\end{bmatrix}$$

$$\therefore \mathrm{AB=AC}$$

6) If $A=\begin{bmatrix}3 & 4 & -2\\ 2 & 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & -1\\ 3 & 4\\ -0 & 2\end{bmatrix}$ find AB **[W-18,S-15]**

Ans.

$$\therefore AB=\begin{bmatrix}3 & 4 & -2\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 4\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}6+12-0 & -3+16-4\\ 4+3+0 & -2+4+0\end{bmatrix}$$

$$=\begin{bmatrix}18 & 9\\ 7 & 2\end{bmatrix}$$

$$\therefore AB=\begin{bmatrix}3 & 4 & -2\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 4\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}6+12-0 & -3+16-4\\ 4+3+0 & -2+4+0\end{bmatrix}$$

$$=\begin{bmatrix}18 & 9\\ 7 & 2\end{bmatrix}$$

7) If $A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$, $B=\begin{bmatrix}1\\ 9\\ 8\end{bmatrix}$ find AB **[W-12]**

Ans.

$$AB=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}1\\ 9\\ 8\end{bmatrix}$$

$$=\begin{bmatrix}1+18+24\\ 4+45+48\end{bmatrix}$$

$$=\begin{bmatrix}43\\ 97\end{bmatrix}$$

8) If $A=\begin{bmatrix}4 & 2\\ 8 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & 6\\ -4 & -12\end{bmatrix}$ show that AB is null matrix.**[W-12]**

Ans.

$$AB=\begin{bmatrix}4 & 2\\ 8 & 4\end{bmatrix}\begin{bmatrix}2 & 6\\ -4 & -12\end{bmatrix}$$

$$=\begin{bmatrix}8-8 & 24-24\\ 16-16 & 48-48\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

9) If $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$ show that A^2 is null matrix. **[S-18,W-17,W-16]**

Ans. $A^2 = A.A$

$$= \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} . \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 8-8 \\ -2+2 & -4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore A^2$ is a null matrix.

10) If $A = \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}$ show that A^2 is not null matrix. **[S-17]**

Ans.

$$A^2 = A.A = \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 9-9 & 27-27 \\ -3+3 & -9+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore A^2$ is null matrix

11) If $A = \begin{bmatrix} 3 & 9 \\ -1 & -9 \end{bmatrix}$ show that A^2 not is null matrix. **[SQP]**

Ans. $A^2 = A.A = \begin{bmatrix} 3 & 9 \\ -1 & -9 \end{bmatrix} . \begin{bmatrix} 3 & 9 \\ -1 & -9 \end{bmatrix}$

$$= \begin{bmatrix} 9-9 & 27-81 \\ -3+9 & -9+81 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -54 \\ 6 & 72 \end{bmatrix}$$

$\therefore A^2$ not is null matrix

12) If $A = \begin{bmatrix} 1 & -5 \\ 6 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ find the matrix $AB - 2I$ where I identity matrix. **[S-14]**

Ans.

$$\therefore AB - 2I = \begin{bmatrix} 1 & -5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+5 \\ 6+0 & 0-4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 6 & -4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 6 & -4 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 \\ 6 & -6 \end{bmatrix}$$

13) If $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ show that $A^2 - 3A = 2I$. **[W-13]**

Ans. $A^2 - 3A = A.A - 3A$

$$= \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 8+4 \\ 2+1 & 4+1 \end{bmatrix} - \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 12 \\ 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$=2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$=2I$$

14) If $A=\begin{bmatrix}1 & -2 & 3\\2 & 3 & -1\\-3 & 1 & 2\end{bmatrix}$ Find $A^2-3A+9I$ where I is unit matrix. **[W-14]**

Ans. $A^2-3A+9I$

$$=\begin{bmatrix}1 & -2 & 3\\2 & 3 & -1\\-3 & 1 & 2\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\2 & 3 & -1\\-3 & 1 & 2\end{bmatrix}-3\begin{bmatrix}1 & -2 & 3\\2 & 3 & -1\\-3 & 1 & 2\end{bmatrix}+9\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-4-9 & -2-6+3 & 3+2+6\\2+6+3 & -4+9-1 & 6-3-2\\-3+2-6 & 6+3+2 & -9-1+4\end{bmatrix}-\begin{bmatrix}3 & -6 & 9\\6 & 9 & -3\\-9 & 3 & 6\end{bmatrix}+\begin{bmatrix}9 & 0 & 0\\0 & 9 & 0\\0 & 0 & 9\end{bmatrix}$$

$$=\begin{bmatrix}-12 & -5 & 11\\11 & 4 & 1\\-7 & 11 & -6\end{bmatrix}-\begin{bmatrix}3 & -6 & 9\\6 & 9 & -3\\-9 & 3 & 6\end{bmatrix}+\begin{bmatrix}9 & 0 & 0\\0 & 9 & 0\\0 & 0 & 9\end{bmatrix}$$

$$=\begin{bmatrix}-12-3+9 & -5+6+0 & 11-9+0\\11-6+0 & 4-9+9 & 1+3+0\\-7+9+0 & 11-3+0 & -6-6+9\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 1 & 2\\5 & 4 & 4\\2 & 8 & -3\end{bmatrix}$$

15) If $A=\begin{bmatrix}1 & 2 & 6\\7 & 4 & 10\\1 & 3 & 5\end{bmatrix}$ Find A^2-3A+I. **[W-13]**

Ans.

$$A^2-3A+I=A.A-3A+I$$

$$=\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}-3\begin{bmatrix}1&2&6\\7&4&10\\1&3&5\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}1+14+6&2+8+18&6+20+30\\7+28+10&14+16+30&42+40+50\\1+21+5&2+12+15&6+30+25\end{bmatrix}-\begin{bmatrix}3&6&18\\21&12&30\\3&9&15\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}21&28&56\\45&60&132\\27&29&61\end{bmatrix}-\begin{bmatrix}3&6&18\\21&12&30\\3&9&15\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}21-3+1&28-6+0&56-18+0\\45-21+0&60-12+1&132-30+0\\27-3+0&29-9+0&61-15+1\end{bmatrix}$$

$$=\begin{bmatrix}19&22&38\\24&49&102\\24&20&47\end{bmatrix}$$

16) Find x & y, if $\left\{4\begin{bmatrix}1&2&0\\2&-1&3\end{bmatrix}-2\begin{bmatrix}1&3&-1\\2&-3&4\end{bmatrix}\right\}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$ **[S-18]**

Ans.

$$\left\{4\begin{bmatrix}1&2&0\\2&-1&3\end{bmatrix}-2\begin{bmatrix}1&3&-1\\2&-3&4\end{bmatrix}\right\}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\left\{\begin{bmatrix}4&8&0\\8&-4&12\end{bmatrix}-\begin{bmatrix}2&6&-2\\4&-6&8\end{bmatrix}\right\}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 2 \\ 4 & 2 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 4+0-2 \\ 8+0-4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore x = 2, y = 4$$

17) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that

$(A + 2B)C = X$ find x, y, z **[W-17]**

Ans.

$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left(\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 4 \\ 2 & 8 & 10 \\ 4 & 2 & 0 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left(\begin{bmatrix} 7 & 3 & 6 \\ 5 & 8 & 11 \\ 7 & 3 & 2 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 7+6+18 \\ 5+16+33 \\ 7+6+6 \end{bmatrix} = \begin{bmatrix} 31 \\ 54 \\ 19 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

by equating

$$x = 31, y = 54, z = 19$$

18) Find x, y, z, if $\left\{\begin{bmatrix}1 & 3 & 2\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix} + 2\begin{bmatrix}3 & 0 & 2\\1 & 4 & 5\\2 & 1 & 0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$ **[S-17]**

Ans.

$$\left\{\begin{bmatrix}1 & 3 & 2\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix} + 2\begin{bmatrix}3 & 0 & 2\\1 & 4 & 5\\2 & 1 & 0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix}1 & 3 & 2\\2 & 0 & 1\\3 & 1 & 2\end{bmatrix} + \begin{bmatrix}6 & 0 & 4\\2 & 8 & 10\\4 & 2 & 0\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix}7 & 3 & 6\\4 & 8 & 11\\7 & 3 & 2\end{bmatrix}\right\}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore \begin{bmatrix}7+6+18\\4+16+33\\7+6+6\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore \begin{bmatrix}31\\53\\19\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore x = 31 \quad , \quad y = 53 \quad , \quad z = 19$$

19) If $\left\{3\begin{bmatrix}3 & 1\\4 & 0\\3 & -3\end{bmatrix} - 2\begin{bmatrix}0 & 2\\-2 & 3\\-5 & 4\end{bmatrix}\right\}\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix}$ find x, y, z **[S-16,W-12,SQP]**

Ans.

$$\left\{3\begin{bmatrix}3&1\\4&0\\3&-3\end{bmatrix}-2\begin{bmatrix}0&2\\-2&3\\-5&4\end{bmatrix}\right\}\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore\left\{\begin{bmatrix}9&3\\12&0\\9&-9\end{bmatrix}-\begin{bmatrix}0&4\\-4&6\\-10&8\end{bmatrix}\right\}\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}\quad or\quad \begin{bmatrix}9-0&3-4\\12+4&0-6\\9+10&-9-8\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore\begin{bmatrix}9&-1\\16&-6\\19&-17\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore\begin{bmatrix}-9-2\\-16-12\\-19-34\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore\begin{bmatrix}-11\\-28\\-53\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore x=-11,\quad y=-28,\quad z=-53$$

20) Find the values of x & y if $\begin{bmatrix}1&2\\3&2\end{bmatrix}\begin{bmatrix}x&5&-3\\2&y&5\end{bmatrix}=\begin{bmatrix}5&-3&7\\7&7&1\end{bmatrix}$ **[SQP]**

Ans. $\begin{bmatrix}1&2\\3&2\end{bmatrix}\begin{bmatrix}x&5&-3\\2&y&5\end{bmatrix}=\begin{bmatrix}5&-3&7\\7&7&1\end{bmatrix}$

$$\begin{bmatrix}x+4&5+2y&-3+10\\3x+4&15+2y&-9+10\end{bmatrix}=\begin{bmatrix}5&-3&7\\7&7&1\end{bmatrix}$$

$$\begin{bmatrix}x+4&5+2y&7\\3x+4&15+2y&1\end{bmatrix}=\begin{bmatrix}5&-3&7\\7&7&1\end{bmatrix}$$

$x + 4 = 5$	$5 + 2y = -3$	$3x + 4 = 7$	$15 + 2y = 7$
$x = 5 - 4$	$2y = -3 - 5$	$3x = 7 - 4$	$2y = 7 - 15$
$x = 1$	$2y = -8$	$3x = 3$	$2y = -8$
	$y = -4$	$x = 1$	$y = -4$

21) If $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ find A^2 **[W-18,W-14]**

Ans.

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -2-3+4 & 2+2-3 \\ -4-6+8 & 2+9-8 & -2-6+6 \\ -8-8+12 & 4+12-12 & -4-8+9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

22) If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ Prove that $A^2 = I$ **[S-19,W-18,S-18]**

Ans.

$$A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$$

$$A^2=AA=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}0+4-3 & 0-3+3 & 0+4-4\\ 0-12+12 & 4+9-12 & -4-12+16\\ 0-12+12 & 3+9-12 & -3-12+16\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

23) If $A=\begin{bmatrix}0 & 1 & -1\\ 3 & -2 & 3\\ 2 & -2 & 3\end{bmatrix}$ Prove that $A^2=I$ **[S-14]**

Ans.

$$A^2=A\cdot A=\begin{bmatrix}0 & 1 & -1\\ 3 & -2 & 3\\ 2 & -2 & 3\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\ 3 & -2 & 3\\ 2 & -2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}0+3-2 & 0-2+2 & 0+3-3\\ 0-6+6 & 3+4-6 & -3-6+9\\ 0-6+6 & 2+4-6 & -2-6+9\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=I$$

24) If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ find A^2 **[W-13]**

OR

If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ find $A^2 - 8A$. **[S-19,W-18]**

OR

If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ Show that $A^2 - 8A$ is scalar matrix.**[S-19,S-18,S-14]**

Ans.

$$A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+16+16 & 8+8+16 & 8+16+8 \\ 8+8+16 & 16+4+16 & 16+8+8 \\ 8+16+8 & 16+8+8 & 16+16+4 \end{bmatrix}$$

$$= \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix}$$

$$8A = 8\begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix}$$

$$A^2 - 8A = \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} - \begin{bmatrix} 16 & 32 & 32 \\ 32 & 16 & 32 \\ 32 & 32 & 16 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$\therefore A^2 - 8A$ is a scalar matrix

25) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$

verify that $A(B+C) = AB + AC$ **[W-18,W-14,SQP]**

Ans.

$$B + C = \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\therefore A(B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -1+8 & 2+6 \\ 2+12 & -4+9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3+4 & 1+0 \\ 6+6 & -2+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$$

$$\therefore A(B+C) = AB + AC$$

26) If $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 7 & -5 \\ 0 & 5 \end{bmatrix}$

verify that $(AB)C = A(BC)$ **[W-12]**

Ans.

$$AB=\begin{bmatrix}2&-2\\3&1\end{bmatrix}\begin{bmatrix}-1&5\\4&-3\end{bmatrix}$$

$$=\begin{bmatrix}-10&16\\1&12\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-10&16\\1&12\end{bmatrix}\begin{bmatrix}7&-5\\0&5\end{bmatrix}$$

$$=\begin{bmatrix}-70&130\\7&55\end{bmatrix}$$

$$BC=\begin{bmatrix}-1&5\\4&-3\end{bmatrix}\begin{bmatrix}7&-5\\0&5\end{bmatrix}$$

$$=\begin{bmatrix}-7&30\\28&-35\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}2&-2\\3&1\end{bmatrix}\begin{bmatrix}-7&30\\28&-35\end{bmatrix}$$

$$=\begin{bmatrix}-70&130\\7&55\end{bmatrix}$$

$$\therefore \boxed{(AB)C=A(BC)}$$

27) If $A=\begin{bmatrix}3&1&-1\\3&1&2\end{bmatrix}$, $B=\begin{bmatrix}1&1\\2&0\\3&-1\end{bmatrix}$, $C=\begin{bmatrix}1\\3\end{bmatrix}$ Verify that $A(BC)=(AB)C$

[W-17]

Ans.

$$\text{L.H.S} = A(BC) = \begin{bmatrix}3&1&-1\\3&1&2\end{bmatrix}\left\{\begin{bmatrix}1&1\\2&0\\3&-1\end{bmatrix}.\begin{bmatrix}1\\3\end{bmatrix}\right\}$$

$$A(BC) = \begin{bmatrix}3&1&-1\\3&1&2\end{bmatrix}\begin{bmatrix}1+3\\2+0\\3-3\end{bmatrix}$$

$$A(BC) = \begin{bmatrix}3&1&-1\\3&1&2\end{bmatrix}\begin{bmatrix}4\\2\\0\end{bmatrix}$$

$$A(BC) = \begin{bmatrix}12+2-0\\12+2+0\end{bmatrix}$$

$$A(BC) = \begin{bmatrix}14\\14\end{bmatrix}$$

$$\text{R.H.S} = (AB)C = \left\{\begin{bmatrix}3&1&-1\\3&1&2\end{bmatrix}.\begin{bmatrix}1&1\\2&0\\3&-1\end{bmatrix}\right\}\begin{bmatrix}1\\3\end{bmatrix}$$

$$(AB)C = \begin{bmatrix}3+2-3&3+0+1\\3+2+6&3+0-2\end{bmatrix}.\begin{bmatrix}1\\3\end{bmatrix}$$

$$(AB)C = \begin{bmatrix}2&4\\11&1\end{bmatrix}.\begin{bmatrix}1\\3\end{bmatrix}$$

$$(AB)C = \begin{bmatrix}2+12\\11+3\end{bmatrix}$$

$$(AB)C = \begin{bmatrix}14\\14\end{bmatrix}$$

$$\therefore A(BC) = (AB)C$$

28) If $A = \begin{bmatrix}1&-2\\-3&-1\end{bmatrix}, B = \begin{bmatrix}4&2&-5\\1&0&3\end{bmatrix}, C = \begin{bmatrix}6&-7&0\\-1&2&5\\1&0&3\end{bmatrix}$

verify that $(AB)C = A(BC)$ **[S-15]**

Ans.

$$A=\begin{bmatrix}1&-2\\-3&-1\end{bmatrix},\quad B=\begin{bmatrix}4&2&-5\\1&0&3\end{bmatrix},\quad C=\begin{bmatrix}6&-7&0\\-1&2&5\\1&0&3\end{bmatrix}$$

$$AB=\begin{bmatrix}1&-2\\-3&-1\end{bmatrix}\begin{bmatrix}4&2&-5\\1&0&3\end{bmatrix}$$

$$=\begin{bmatrix}4-2&2-0&-5-6\\-12-1&-6-0&15-3\end{bmatrix}$$

$$=\begin{bmatrix}2&2&-11\\-13&-6&12\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}2&2&-11\\-13&-6&12\end{bmatrix}\begin{bmatrix}6&-7&0\\-1&2&5\\1&0&3\end{bmatrix}$$

$$=\begin{bmatrix}12-2-11&-14+4-0&0+10-33\\-78+6+12&91-12+0&0-30+36\end{bmatrix}$$

$$=\begin{bmatrix}-1&-10&-23\\-60&79&6\end{bmatrix}$$

$$BC=\begin{bmatrix}4&2&-5\\1&0&3\end{bmatrix}\begin{bmatrix}6&-7&0\\-1&2&5\\1&0&3\end{bmatrix}$$

$$=\begin{bmatrix}24-2-5&-28+4-0&0+10-15\\6-0+3&-7+0+0&0+0+9\end{bmatrix}$$

$$=\begin{bmatrix}17&-24&-5\\9&-7&9\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}1 & -2\\ -3 & -1\end{bmatrix}\begin{bmatrix}17 & -24 & -5\\ 9 & -7 & 9\end{bmatrix}$$

$$=\begin{bmatrix}17-18 & -24+14 & -5-18\\ -51-9 & 72+7 & 15-9\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -10 & -23\\ -60 & 79 & 6\end{bmatrix}$$

$$\therefore (AB)C=A(BC)$$

29) If $A=\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 3 & -2\end{bmatrix}$ whether AB is singular or non-singular matrix? **[W-17]**

Ans.

$$AB=\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & -2\end{bmatrix}$$

$$\therefore AB=\begin{bmatrix}2+3 & 4-2\\ 0+9 & 0-6\end{bmatrix}$$

$$\therefore AB=\begin{bmatrix}5 & 2\\ 9 & -6\end{bmatrix}$$

$$\therefore |AB|=\begin{vmatrix}5 & 2\\ 9 & -6\end{vmatrix}=-30-18=-48$$

$$\therefore |AB|\neq 0$$

$\therefore AB$ is non-singular matrix

30) If $A=\begin{bmatrix}-2 & 0 & 1\\ 1 & 2 & 3\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$ Show that AB is non-singular matrix.

[SQP]

Ans.

$$AB=\begin{bmatrix}-2 & 0 & 1\\ 1 & 2 & 3\end{bmatrix}\begin{bmatrix}0 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$$

$$AB = \begin{bmatrix} 0+0+1 & -2+0+1 \\ 0+4+3 & 1+6+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -1 \\ 7 & 10 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 1 & -1 \\ 7 & 10 \end{vmatrix} = 10-(-7) = 17 \neq 0$$

$\therefore$ AB is non-singular matrix.

31) If $A = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$ whether AB is singular or non-singular matrix. **[W-19]**

Ans.

$$AB = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 \\ 18 & 33 \end{bmatrix}$$

$$\text{Consider } |AB| = \begin{vmatrix} -4 & 2 \\ 18 & 33 \end{vmatrix}$$

$$= -132 - 36$$

$$= -168 \neq 0$$

$\therefore AB$ is non singular matrix

32) If $A+I = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix}$ obtain the matrix $(A+I)(A-I)$. **[W-16]**

Ans.

$$A=\begin{bmatrix}1&3&4\\-1&1&3\\-2&-3&1\end{bmatrix}-I$$

$$=\begin{bmatrix}1&3&4\\-1&1&3\\-2&-3&1\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}0&3&4\\-1&0&3\\-2&-3&0\end{bmatrix}$$

$$A-I=\begin{bmatrix}0&3&4\\-1&0&3\\-2&-3&0\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}-1&3&4\\-1&-1&3\\-2&-3&-1\end{bmatrix}$$

$$(A+I)(A-I)=\begin{bmatrix}1&3&4\\-1&1&3\\-2&-3&1\end{bmatrix}\begin{bmatrix}-1&3&4\\-1&-1&3\\-2&-3&-1\end{bmatrix}$$

$$=\begin{bmatrix}-1-3-8&3-3-12&4+9-4\\1-1-6&-3-1-9&-4+3-3\\2+3-2&-6+3-3&-8-9-1\end{bmatrix}$$

$$=\begin{bmatrix}-12&-12&9\\-6&-13&-4\\3&-6&-18\end{bmatrix}$$

C. TRANSPOSE OF A MATRIX :-

Examples:-

1) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$ find $A^T + B^T$ & $A^T - B^T$ **[W-15]**

Ans.

$$A^T + B^T = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3 & 6 \\ -1 & 3 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$$

2) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ Verify that $(A + B)^T = A^T + B^T$ **[S-14]**

Ans.

$$A + B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A + B)^T = \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$$

$$\therefore A^T + B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$$

$$(A + B)^T = A^T + B^T$$

3) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}$, $\mathrm{B} = \begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$ Verify that $(A+B)' = A' + B'$

[S-19]

Ans.

$$A=\begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}, B=\begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$$

$$\therefore A+B=\begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}+\begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$$

$$\therefore A+B=\begin{bmatrix} 1 & 5 & 3 \\ 5 & 8 & 0 \end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix} 1 & 5 \\ 5 & 8 \\ 3 & 0 \end{bmatrix}$$

$$A'+B'=\begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix}+\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$A'+B'=\begin{bmatrix} 1 & 5 \\ 5 & 8 \\ 3 & 0 \end{bmatrix}$$

$$\therefore (A+B)' = A'+B'$$

4) If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ Verify that $(AB)^T = B^T A^T$

[S-16,W-13, SQP]

Ans.

$$AB=\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$AB=\begin{bmatrix}6-3 & -2 & 4-3\\ 3+5 & -1 & 2+5\end{bmatrix}$$

$$AB=\begin{bmatrix}3 & -2 & 1\\ 8 & -1 & 7\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}3 & 8\\ -2 & -1\\ 1 & 7\end{bmatrix}$$

$$B^T A^T=\begin{bmatrix}3 & 1\\ -1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\ -3 & 5\end{bmatrix}$$

$$B^T A^T=\begin{bmatrix}3 & 8\\ -2 & -1\\ 1 & 7\end{bmatrix}$$

$$\therefore (AB)^T=B^T A^T$$

5) If $A=\begin{bmatrix}1 & -3\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}1 & 0 & 1\\ 2 & -1 & 3\end{bmatrix}$ Verify that $(AB)^T=B^T A^T$ **[W-12]**

Ans.

$$AB=\begin{bmatrix}1 & -3\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ 2 & -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-5 & 3 & -8\\ 0 & 1 & -1\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}-5 & 0\\ 3 & 1\\ -8 & -1\end{bmatrix}$$

$$B^T A^T=\begin{bmatrix}1 & 2\\ 0 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-5 & 0\\ 3 & 1\\ -8 & -1\end{bmatrix}$$

6) If $A=\begin{bmatrix}2 & 3 & -1\\ 1 & 0 & 4\end{bmatrix}$, $B=\begin{bmatrix}-3 & 7\\ -5 & 6\\ -4 & 4\end{bmatrix}$ Verify that $(AB)'=B'A'$ **[S-13]**

Ans.

$$\text{Consider } AB=\begin{bmatrix}2 & 3 & -1\\ 1 & 0 & 4\end{bmatrix}\begin{bmatrix}-3 & 7\\ -5 & 6\\ -4 & 4\end{bmatrix}$$

$$AB=\begin{bmatrix}-6-15+4 & 14+18-4\\ -3-16 & 7+16\end{bmatrix}$$

$$AB=\begin{bmatrix}-17 & 28\\ -19 & 23\end{bmatrix}$$

$$(AB)'=\begin{bmatrix}-17 & -19\\ 28 & 23\end{bmatrix}$$

$$B'A'=\begin{bmatrix}-3 & -5 & -4\\ 7 & 6 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 0\\ -1 & 4\end{bmatrix}$$

$$B'A'=\begin{bmatrix}-17 & -19\\ 28 & 23\end{bmatrix}$$

$$\therefore (AB)'=B'A'$$

7) If $A=\begin{bmatrix}2 & 5 & 6\\ 0 & 1 & 2\end{bmatrix}$, $B=\begin{bmatrix}6 & 1\\ 0 & 4\\ 5 & 7\end{bmatrix}$ Verify that $(AB)^T=B^TA^T$ **[W-17]**

OR

If $A = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$ find $(AB)^T$ **[W-17]**

Ans.

$$AB = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 12+0+30 & 2+20+42 \\ 0+0+10 & 0+4+14 \end{bmatrix}$$

$$AB = \begin{bmatrix} 42 & 64 \\ 10 & 18 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 42 & 10 \\ 64 & 18 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 6 & 0 & 5 \\ 1 & 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 5 & 1 \\ 6 & 2 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 12+0+30 & 0+0+10 \\ 2+20+42 & 0+4+14 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 42 & 10 \\ 64 & 18 \end{bmatrix}$$

$$\therefore (AB)^T = B^T A^T$$

8) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ Verify that $(AB)^T = B^T A^T$

[W-19,S-18]

Ans.

$$AB=\begin{bmatrix}1&2&-1\\3&0&2\\4&5&0\end{bmatrix}\begin{bmatrix}1&0&0\\2&1&0\\0&1&3\end{bmatrix}$$

$$=\begin{bmatrix}1+4+0&0+2-1&0+0-3\\3+0+0&0+0+2&0+0+6\\4+10+0&0+5+0&0+0+0\end{bmatrix}$$

$$AB=\begin{bmatrix}5&1&-3\\3&2&6\\14&5&0\end{bmatrix}$$

$$(AB)^T=\begin{bmatrix}5&3&14\\1&2&5\\-3&6&0\end{bmatrix}$$

$$B^TA^T=\begin{bmatrix}1&2&0\\0&1&1\\0&0&3\end{bmatrix}\begin{bmatrix}1&3&4\\2&0&5\\-1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}1+4+0&3+0+0&4+10+0\\0+2-1&0+0+2&0+5+0\\0+0-3&0+0+6&0+0+0\end{bmatrix}$$

$$=\begin{bmatrix}5&3&14\\1&2&5\\-3&6&0\end{bmatrix}$$

$$(AB)^T=B^TA^T$$

9) Express the matrix A as the sum of symmetric & skew-symmetric matrices

$$A=\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}$$ **[W-18, S-17,S-15]**

Ans.

$$A=\frac{1}{2}\left(A+A^T\right)+\frac{1}{2}\left(A-A^T\right)$$

$$=\frac{1}{2}\left\{\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}+\begin{bmatrix}-1&2&5\\7&3&0\\1&4&5\end{bmatrix}\right\}+\frac{1}{2}\left\{\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}-\begin{bmatrix}-1&2&5\\7&3&0\\1&4&5\end{bmatrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}-2 & 9 & 6\\ 9 & 6 & 4\\ 6 & 4 & 10\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0 & 5 & -4\\ -5 & 0 & 4\\ 4 & -4 & 0\end{bmatrix}$$

10) Show that matrix $A=\begin{bmatrix}cos\theta & 0 & sin\theta\\ 0 & 1 & 0\\ -sin\theta & 0 & cos\theta\end{bmatrix}$ is orthogonal matrix. **[W-16]**

Ans.

$$A=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix} \qquad \therefore\ A^T=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$$

$$A.A^T=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+0+\sin^2\theta & 0+0+0 & -\cos\theta\sin\theta+\sin\theta\cos\theta\\ 0+0+0 & 0+1+0 & 0+0+0\\ -\sin\theta\cos\theta+0+\cos\theta\sin\theta & 0+0+0 & \sin^2\theta+0+\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$\therefore\ A$ is an orthogonal matrix.

TUTORIAL NO.3

MATRICES

1) Find the value of a & b if $\begin{bmatrix} a-4b & 5 \\ 6 & -a+b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$ **[S-16,W-14]**

2) If $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ Find matrix X if $2X + 3A - 4B = I$

[S-18,W-15]

3) Find A if $2A + 3\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 6 & 3 \end{bmatrix}$ **[S-19,S-13]**

4) If $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$ show that A^2 is null matrix. **[S-18,W-17,W-16]**

5) If $\left\{ 3\begin{bmatrix} 3 & 1 \\ 4 & 0 \\ 3 & -3 \end{bmatrix} - 2\begin{bmatrix} 0 & 2 \\ -2 & 3 \\ -5 & 4 \end{bmatrix} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ find x, y, z **[S-16,W-12,SQP]**

6) If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ Prove that $A^2 = I$ **[S-19,W-18,S-18]**

7) If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ Show that $A^2 - 8A$ is scalar matrix. **[S-19,S-18,S-14]**

8) If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

verify that $(AB)^T = B^T A^T$ **[S-16,W-13,SQP]**

9) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$

verify that $A(B + C) = AB + AC$ **[W-18,W-14,SQP]**

10) Express the matrix A as the sum of symmetric & skew-symmetric matrices

$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ **[W-18, S-17,S-15]**

D. ADJOINT OF A MATRIX :-

Sign for matrix $2X2$:- $\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

Sign for matrix $3X3$:- $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

Examples:-

Que. **Find the adjiont of the matrix**

1) $A = \begin{bmatrix} 4 & -6 \\ 1 & 7 \end{bmatrix}$ **[W-14]**

Ans.

$$Let \quad A = \begin{bmatrix} 4 & -6 \\ 1 & 7 \end{bmatrix}$$

$$\therefore A_{11} = 7 \qquad A_{12} = -1$$

$$A_{21} = 6 \qquad A_{22} = 4$$

$$\therefore C(A) = \begin{bmatrix} 7 & -1 \\ 6 & 4 \end{bmatrix}$$

$$\therefore adj(A) = \begin{bmatrix} 7 & 6 \\ -1 & 4 \end{bmatrix}$$

2) $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$ **[W-13]**

Ans.

$$\text{Matrix of Minors} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$$

$$\text{Matrix of Cofactors} = \begin{bmatrix} 1 & -2 \\ -5 & 6 \end{bmatrix}$$

$$\text{Adj. } A = \begin{bmatrix} 1 & -5 \\ -2 & 6 \end{bmatrix}$$

3) $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ **[S-19]**

Ans.

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 1 & 5 \\ -1 & -1 & -1 \\ 7 & -5 & -13 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} -3 & -1 & 5 \\ 1 & -1 & 1 \\ 7 & 5 & -13 \end{bmatrix}$$

$$AdjA = \begin{bmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{bmatrix}$$

4) $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$

Ans.

Ans.: Here $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$ $\therefore$ det. $A = |A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix}$

We know, $\text{cof}(a_{ij}) = (-1)^{i+j}.\text{minor}(a_{ij})$.

$$\therefore \quad A_{11} = \text{cof}(1) = (-1)^{1+1}\begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} = -28 + 30 = 2$$

$$A_{12} = \text{cof}(0) = (-1)^{1+2}\begin{vmatrix} 3 & 5 \\ 0 & -7 \end{vmatrix} = -(-21 - 0) = 21$$

$$A_{13} = \text{cof}(-1) = (-1)^{1+3}\begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix} = -18 - 0 = -18$$

$$A_{21} = \text{cof}(3) = (-1)^{2+1}\begin{vmatrix} 0 & -1 \\ -6 & -7 \end{vmatrix} = -(-0-6) = 6$$

$$A_{22} = \text{cof}(4) = (-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 0 & -7 \end{vmatrix} = -7+0 = -7$$

$$A_{23} = \text{cof}(5) = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 0 & -6 \end{vmatrix} = -(-6-0) = 6$$

$$A_{31} = \text{cof}(0) = (-1)^{3+1}\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} = 0+4 = 4$$

$$A_{32} = \text{cof}(-6) = (-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} = -(5+3) = -8$$

$$A_{33} = \text{cof}(-7) = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = 4-0 = 4$$

Thus the matrix formed by the cofactors of the elements of |A| is

$$\text{cof}(A) = \begin{bmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & -8 & 4 \end{bmatrix}$$

$$\therefore \quad \text{adj}A = (\text{cof}A)' = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$$

5) $A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

Ans.

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$A_{11} = \text{Co-factor of } a_{11} = +\begin{vmatrix} 4 & 4 \\ 2 & 1 \end{vmatrix} = (4-8) = -4$$

$$A_{12} = \text{Co-factor of } a_{12} = -\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = -(2-12) = -(-10) = 10$$

$$A_{13} = \text{Co-factor of } a_{13} = +\begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = +(4-12) = -8$$

$$A_{21} = \text{Co-factor of } a_{21} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = -(-1) = 1$$

$$A_{22} = \text{Co-factor of } a_{22} = +\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} = +(-1-3) = -4$$

$$A_{23} = \text{Co-factor of } a_{23} = -\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = -(-2-3) = -(-5) = 5$$

$$A_{31} = \text{Co-factor of } a_{31} = +\begin{vmatrix} 1 & 1 \\ 4 & 4 \end{vmatrix} = +(4-4) = 0$$

$$A_{32} = \text{Co-factor of } a_{32} = -\begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix} = -(-4-2) = -(-6) = 6$$

$$A_{33} = \text{Co-factor of } a_{33} = +\begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix} = -4-2 = -6$$

$$\text{Co-factor of matrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -4 & 10 & -8 \\ 1 & -4 & 5 \\ 0 & 6 & -6 \end{bmatrix}$$

$$\text{adj } A = \text{Transpose of cofactor of matrix}$$

$$\text{adj } A = \begin{bmatrix} -4 & 1 & 0 \\ 10 & -4 & 6 \\ -8 & 5 & -6 \end{bmatrix}$$

6) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

Ans.

Here $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$$\therefore \quad \text{det. } A = |A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix}$$

We know, $\text{cof}(a_{ij}) = (-1)^{i+j}.\text{ minor}(a_{ij})$.

$$\therefore \quad A_{11} = \text{cof}(1) = (-1)^{1+1}\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1$$

$$A_{12} = \text{cof}(2) = (-1)^{1+2}\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = -(12 - 15) = -(-3) = 3$$

$$A_{13} = \text{cof}(3) = (-1)^{1+3}\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = 10 - 12 = -2$$

$$A_{21} = \text{cof}(2) = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -(12 - 15) = -(-3) = 3$$

$$A_{22} = \text{cof}(4) = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} = 6 - 9 = -3$$

$$A_{23} = \text{cof}(5) = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = -(5 - 6) = 1$$

$$A_{31} = \text{cof}(3) = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2$$

$$A_{32} = \text{cof}(5) = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = -(5 - 6) = 1$$

$$A_{33} = \text{cof}(6) = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0$$

Thus the matrix formed by the cofactors of the elements of $|A|$ is

$$\text{cof}(A) = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad \text{adj}A = (\text{cof}A)' = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

E. INVERSE OF A MATRIX BY ADJOINT METHOD :-

$$A^{-1} = \frac{1}{|A|}.(adjA)$$

Examples:-

Que. **Find A^{-1} by Adjoint method OR Find inverse by Adjoint method if**

1) $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$

Ans.

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= 8 - 9 = -1 \neq 0$$

$\therefore$ A is non-singular matrix. $\therefore$ A^{-1} is exists

A_{11} = Co-factor of $a_{11} = +2$

A_{12} = Co-factor of $a_{12} = -3$

A_{21} = Co-factor of $a_{21} = -3$

A_{22} = Co-factor of $a_{22} = +4$

$$\text{Co-factor of matrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-1}\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

2) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ **[S-19]**

Ans.

$$A=\begin{bmatrix}1&1&1\\1&1&-1\\1&-1&0\end{bmatrix}$$

$$\therefore |A|=\begin{vmatrix}1&1&1\\1&1&-1\\1&-1&0\end{vmatrix}$$

$$=1(0-1)-1(0+1)+1(-1-1)$$

$$=-4\neq 0$$

$$\therefore A^{-1}\text{exists}$$

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}1&-1\\-1&0\end{vmatrix}&\begin{vmatrix}1&-1\\1&0\end{vmatrix}&\begin{vmatrix}1&1\\1&-1\end{vmatrix}\\ \begin{vmatrix}1&1\\-1&0\end{vmatrix}&\begin{vmatrix}1&1\\1&0\end{vmatrix}&\begin{vmatrix}1&1\\1&-1\end{vmatrix}\\ \begin{vmatrix}1&1\\1&-1\end{vmatrix}&\begin{vmatrix}1&1\\1&-1\end{vmatrix}&\begin{vmatrix}1&1\\1&1\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}-1&1&-2\\1&-1&-2\\-2&-2&0\end{bmatrix}$$

$$\text{Matrix of cofactors }=\begin{bmatrix}-1&-1&-2\\-1&-1&2\\-2&2&0\end{bmatrix}$$

$$\text{Adj}.A=\begin{bmatrix}-1&-1&-2\\-1&-1&2\\-2&2&0\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{Adj}.A$$

$$A^{-1}=\frac{1}{-4}\begin{bmatrix}-1&-1&-2\\-1&-1&2\\-2&2&0\end{bmatrix}$$

3) $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$ **[W-18,W-16]**

Ans.

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1(24-25)-2(12-15)+3(10-12)$$

$$= -1+6-6$$

$$|A| = -1 \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -1 & -3 & -2 \\ -3 & -3 & -1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\text{Adj}.A = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}.A$$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

4) $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ **[W-17]**

Ans.

$$|A| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3+0)-2(-1-0)-2(2-0)=1 \neq 0 \therefore A^{-1} \text{ exists}$$

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 3 & -1 & 2 \\ -2 & 1 & -2 \\ 6 & -2 & 5 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$$

$$\text{Adj}.A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}.adj.A = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

5) $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ **[S-17]**

Ans.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\therefore \left|A\right| = \begin{vmatrix} 2 & -1 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 2(0+4)+1(1-4)+0(-1-0)$$

$$= 5 \neq 0$$

$\therefore A^{-1}$ exists

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} 0 & 4 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & -1 \\ -1 & 2 & -1 \\ -4 & 8 & 1 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 4 & 3 & -1 \\ 1 & 2 & 1 \\ -4 & -8 & 1 \end{bmatrix}$$

$$\text{Adj}.A = \begin{bmatrix} 4 & 1 & -4 \\ 3 & 2 & -8 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\left|A\right|}\text{Adj}.A$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & 1 & -4 \\ 3 & 2 & -8 \\ -1 & 1 & 1 \end{bmatrix}$$

6) $A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$ **[W-15]**

Ans.

$$\text{Let } A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$$

$$\therefore |A|=\begin{vmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{vmatrix}$$

$$|A|=3(-3+4)+3(2-0)+4(-2-0)$$

$$=3+6-8$$

$$|A|=1\neq 0$$

$\therefore A^{-1}$ exists

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}-3&4\\-1&1\end{vmatrix}&\begin{vmatrix}2&4\\0&1\end{vmatrix}&\begin{vmatrix}2&-3\\0&-1\end{vmatrix}\\\begin{vmatrix}-3&4\\-1&1\end{vmatrix}&\begin{vmatrix}3&4\\0&1\end{vmatrix}&\begin{vmatrix}3&-3\\0&-1\end{vmatrix}\\\begin{vmatrix}-3&4\\-3&4\end{vmatrix}&\begin{vmatrix}3&4\\2&4\end{vmatrix}&\begin{vmatrix}3&-3\\2&-3\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}1&2&-2\\1&3&-3\\0&4&-3\end{bmatrix}$$

$$\text{Matrix of cofactors }=\begin{bmatrix}1&-2&-2\\-1&3&3\\0&-4&-3\end{bmatrix}$$

$$\text{Adj}A=\begin{bmatrix}1&-1&0\\-2&3&-4\\-2&3&-3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{Adj}A$$

$$A^{-1}=\frac{1}{1}\begin{bmatrix}1&-1&0\\-2&3&-4\\-2&3&-3\end{bmatrix}$$

7) $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$ **[S-15]**

Ans.

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$$

$$\therefore |A| = 1(2-12) - 2(-1-3) + 4(-4-2) = -26$$

$$\therefore A^{-1} \text{ exists.}$$

Matrix of Cofactor of A is,

$$C(A) = \begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} \\ -\begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 4 & -6 \\ 14 & -3 & -2 \\ -2 & -7 & 4 \end{bmatrix}$$

$$adj(A) = \begin{bmatrix} -10 & 14 & -2 \\ 4 & -3 & -7 \\ -6 & -2 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj(A)$$

$$= \frac{1}{-26} \begin{bmatrix} -10 & 14 & -2 \\ 4 & -3 & -7 \\ -6 & -2 & 4 \end{bmatrix}$$

8) $A=\begin{bmatrix}2 & -1 & -3\\ 3 & -4 & -2\\ 5 & 2 & 4\end{bmatrix}$ **[W-13]**

Ans.

$$\text{Consider, } |A|=\begin{vmatrix}2 & -1 & -3\\ 3 & -4 & -2\\ 5 & 2 & 4\end{vmatrix}$$

$$=2(-16+4)+1(12+10)-3(6+20)$$

$$=-24+22-78$$

$$=-80$$

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}4 & -2\\ 2 & 4\end{vmatrix} & \begin{vmatrix}3 & -2\\ 5 & 4\end{vmatrix} & \begin{vmatrix}3 & -4\\ 5 & 2\end{vmatrix}\\ \begin{vmatrix}-1 & -3\\ 2 & 4\end{vmatrix} & \begin{vmatrix}2 & -3\\ 5 & 4\end{vmatrix} & \begin{vmatrix}2 & -1\\ 5 & 2\end{vmatrix}\\ \begin{vmatrix}-1 & -3\\ -4 & -2\end{vmatrix} & \begin{vmatrix}2 & -3\\ 3 & -2\end{vmatrix} & \begin{vmatrix}2 & -1\\ 3 & -4\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}-16+4 & 12+10 & 6+20\\ -4+6 & 8+15 & 4+5\\ 2-12 & -4+9 & -8+3\end{bmatrix}=\begin{bmatrix}-12 & 22 & 26\\ 2 & 23 & 9\\ -10 & 5 & -5\end{bmatrix}$$

$$\text{Matrix of cofactor}=\begin{bmatrix}-12 & -22 & 26\\ -2 & 23 & -9\\ -10 & -5 & -5\end{bmatrix}$$

$$\text{Adj}.A=\begin{bmatrix}-12 & -2 & -10\\ -22 & 23 & -5\\ 26 & -9 & -5\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}.adj.A=\frac{1}{-80}\begin{bmatrix}-12 & -2 & -10\\ -22 & 23 & -5\\ 26 & -9 & -5\end{bmatrix}$$

F. Solution of simultaneous equation by matrix- inversion Method:-

$$X = A^{-1}B = \frac{1}{|A|}.(adjA).B$$

Que. **Solve the following equations by matrix inversion method**

1) $5x + y = 13$ $3x + 2y = 5$

[S-14]

Ans.

$$5x + y = 13$$
$$3x + 2y = 5$$
$$\therefore A = \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$
$$\therefore |A| = \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} = 10 - 3 = 7$$
$$C(A) = \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix}$$
$$\therefore adj(A) = \begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$$
$$\therefore A^{-1} = \frac{1}{|A|} adj(A)$$
$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$$
$\therefore$ the solution is,
$$X = A^{-1}B$$
$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 13 \\ 5 \end{bmatrix}$$
$$= \frac{1}{7}\begin{bmatrix} 21 \\ -14 \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$
$$\therefore x = 3, \quad y = -2$$

2) $x + y + z = 6$ $3x - y + 3z = 10$ $5x + 5y - 4z = 3$

[S-19]

Ans.

$$\text{Let } A=\begin{bmatrix}1 & 1 & 1\\ 3 & -1 & 3\\ 5 & 5 & -4\end{bmatrix},\quad B=\begin{bmatrix}6\\ 10\\ 3\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & 1 & 1\\ 3 & -1 & 3\\ 5 & 5 & -4\end{vmatrix}$$

$$|A|=1(4-15)-1(-12-15)+1(15+5)$$

$$\therefore |A|=36\neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}-1 & 3\\ 5 & -4\end{vmatrix} & \begin{vmatrix}3 & 3\\ 5 & -4\end{vmatrix} & \begin{vmatrix}3 & -1\\ 5 & 5\end{vmatrix}\\ \begin{vmatrix}1 & 1\\ 5 & -4\end{vmatrix} & \begin{vmatrix}1 & 1\\ 5 & -4\end{vmatrix} & \begin{vmatrix}1 & 1\\ 5 & 5\end{vmatrix}\\ \begin{vmatrix}1 & 1\\ -1 & 3\end{vmatrix} & \begin{vmatrix}1 & 1\\ 3 & 3\end{vmatrix} & \begin{vmatrix}1 & 1\\ 3 & -1\end{vmatrix}\end{bmatrix}$$

$$\text{Matrix of minors}=\begin{bmatrix}-11 & -27 & 20\\ -9 & -9 & 0\\ 4 & 0 & -4\end{bmatrix}$$

$$\text{Matrix of cofactors}=\begin{bmatrix}-11 & 27 & 20\\ 9 & -9 & 0\\ 4 & 0 & -4\end{bmatrix}$$

$$\text{Adj}\,A=\begin{bmatrix}-11 & 9 & 4\\ 27 & -9 & 0\\ 20 & 0 & -4\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{Adj}\,A$$

$$A^{-1}=\frac{1}{36}\begin{bmatrix}-11 & 9 & 4\\ 27 & -9 & 0\\ 20 & 0 & -4\end{bmatrix}$$

$$\therefore X=A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36}\begin{bmatrix} -11 & 9 & 4 \\ 27 & -9 & 0 \\ 20 & 0 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36}\begin{bmatrix} -66+90+12 \\ 162-90+0 \\ 120+0-12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36}\begin{bmatrix} 36 \\ 72 \\ 108 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore \ x=1 , \ y=2 , \ z=3.$

3) $x + y + z = 3$ $\quad x + 2y + 3z = 4$ $\quad x + 4y + 9z = 6$

[S-19,W-17, S-17, S-13]

Ans. $x+y+z=3 , x+2y+3z=4 , x+4y+9z=6$

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$$

$\therefore |A| = 1(18-12)-1(9-3)+1(4-2)$

$\therefore |A| = 2 \neq 0$

$\therefore A^{-1}$ exists

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 & 2 \\ 5 & 8 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$AdjA = \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}.A$$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 18-20+6 \\ -18+32-12 \\ 6-12+6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore\ x = 2\ ,\ y = 1\ ,\ z = 0$

4) $x + 3y + 2z = 6$ $3x - 2y + 5z = 5$ $2x - 3y + 6z = 7$

[W-18,W-15,SQP]

Ans.

$$\text{Let } A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix}\ ,\quad B = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}\quad ,\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{vmatrix}$$

$$|A| = 1(-12+15) - 3(18-10) + 2(-9+4)$$

$$|A| = -31$$

$$\therefore |A| \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix} -2 & 5 \\ -3 & 6 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 3 & -2 \\ 2 & -3 \end{vmatrix} \\ \begin{vmatrix} 3 & 2 \\ -3 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} \\ \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 & -5 \\ 24 & 2 & -9 \\ 19 & -1 & -11 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 3 & -8 & -5 \\ -24 & 2 & 9 \\ 19 & 1 & -11 \end{bmatrix}$$

$$Adj.A = \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}.A$$

$$= \frac{1}{-31}\begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-31}\begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-31}\begin{bmatrix} 18-120+133 \\ -48+10+7 \\ -30+45-77 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-31}\begin{bmatrix} 31 \\ -31 \\ -62 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore \ x = -1 , \ y = 1 , \ z = 2$$

5) $x + 3y + 3z = 12$ $x + 4y + 4z = 15$ $x + 3y + 4z = 13$

[S-18,SQP]

Ans. Let $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{vmatrix}$$

$$|A| = 1(16-12) - 3(4-4) + 3(3-4)$$

$$|A| = 4 - 0 - 3$$

$$\therefore |A| = 1 \neq 0$$

$\therefore A^{-1}$ exists

$$\text{Matrix of minors} = \begin{bmatrix} \begin{vmatrix}4 & 4\\3 & 4\end{vmatrix} & \begin{vmatrix}1 & 4\\1 & 4\end{vmatrix} & \begin{vmatrix}1 & 4\\1 & 3\end{vmatrix} \\ \begin{vmatrix}3 & 3\\3 & 4\end{vmatrix} & \begin{vmatrix}1 & 3\\1 & 4\end{vmatrix} & \begin{vmatrix}1 & 3\\1 & 3\end{vmatrix} \\ \begin{vmatrix}3 & 3\\4 & 4\end{vmatrix} & \begin{vmatrix}1 & 3\\1 & 4\end{vmatrix} & \begin{vmatrix}1 & 3\\1 & 4\end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & -1 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 4 & 0 & -1 \\ -3 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{Adj} A = \begin{bmatrix} 4 & -3 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj} A$$

$$= \frac{1}{1}\begin{bmatrix} 4 & -3 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 & -3 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 48-45+0 \\ 0+15-13 \\ -12+0+13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore\ x = 3\ ,\ y = 2\ ,\ z = 1.$

6) $3x + y + 2z = 3$ $2x - 3y - z = -3$ $x + 2y + z = 4$

[S-18, S-16, S-15 , SQP]

Ans.

$$\text{Let } A=\begin{bmatrix}3&1&2\\2&-3&-1\\1&2&1\end{bmatrix}$$

$$|A|=3(-3+2)-1(2+1)+2(4+3)$$

$$\therefore |A|=8$$

$$A=\begin{bmatrix}3&1&2\\2&-3&-1\\1&2&1\end{bmatrix}$$

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}-3&-1\\2&1\end{vmatrix}&\begin{vmatrix}2&-1\\1&1\end{vmatrix}&\begin{vmatrix}2&-3\\1&2\end{vmatrix}\\ \begin{vmatrix}1&2\\2&1\end{vmatrix}&\begin{vmatrix}3&2\\1&1\end{vmatrix}&\begin{vmatrix}3&1\\1&2\end{vmatrix}\\ \begin{vmatrix}1&2\\-3&-1\end{vmatrix}&\begin{vmatrix}3&2\\2&-1\end{vmatrix}&\begin{vmatrix}3&1\\2&-3\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}-1&3&7\\-3&1&5\\5&-7&-11\end{bmatrix}$$

$$\text{Matrix of cofactors}=\begin{bmatrix}-1&-3&7\\3&1&-5\\5&7&-11\end{bmatrix}$$

$$\text{Adj}.A=\begin{bmatrix}-1&3&5\\-3&1&7\\7&-5&-11\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{Adj}.A$$

$$\therefore A^{-1}=\frac{1}{8}\begin{bmatrix}-1&3&5\\-3&1&7\\7&-5&-11\end{bmatrix}$$

$$X=A^{-1}B$$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{8}\begin{bmatrix}-1&3&5\\-3&1&7\\7&-5&-11\end{bmatrix}\begin{bmatrix}3\\-3\\4\end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -3-9+20 \\ -9-3+28 \\ 21+15-44 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\therefore\ x=1,\ y=2,\ z=-1$

7) $2x+3y-z=-3$ $5x+y+3z=10$ $4x+3y-2z=-3$

[W-17]

Ans.

$$\text{Let } A=\begin{bmatrix} 2 & 3 & -1 \\ 5 & 1 & 3 \\ 4 & 3 & -2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B=\begin{bmatrix} -3 \\ 10 \\ -3 \end{bmatrix}$$

$$\text{Consider, } |A|=\begin{vmatrix} 2 & 3 & -1 \\ 5 & 1 & 3 \\ 4 & 3 & -2 \end{vmatrix}$$

$$=2(-2-9)-3(-10-12)-1(15-4)$$

$$=33\neq 0 \therefore A^{-1}\text{exists}$$

$$\text{Matrix of minors }=\begin{bmatrix} \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 5 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} \end{bmatrix}=\begin{bmatrix} -11 & -22 & 11 \\ -3 & 0 & -6 \\ 10 & 11 & -13 \end{bmatrix}$$

$$\text{Matrix of cofactors} = \begin{bmatrix} -11 & 22 & 11 \\ 3 & 0 & 6 \\ 10 & -11 & -13 \end{bmatrix}$$

$$AdjA = \begin{bmatrix} -11 & 3 & 10 \\ 22 & 0 & -11 \\ 11 & 6 & -13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}.adj.A \;= \frac{1}{33}\begin{bmatrix} -11 & 3 & 10 \\ 22 & 0 & -11 \\ 11 & 6 & -13 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{33}\begin{bmatrix} -11 & 3 & 10 \\ 22 & 0 & -11 \\ 11 & 6 & -13 \end{bmatrix}\begin{bmatrix} -3 \\ 10 \\ -3 \end{bmatrix}$$

$$= \frac{1}{33}\begin{bmatrix} 33+30-30 \\ -66+0+33 \\ -33+60+39 \end{bmatrix} = \frac{1}{33}\begin{bmatrix} 33 \\ -33 \\ 66 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

8) $x+y+z=3$ $\quad$ $3x-2y+3z=4$ $\quad$ $5x+5y+z=11$

[W-19,W-16,W-12]

Ans.

$$\therefore A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 5 & 5 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 4 \\ 11 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 5 & 5 & 1 \end{vmatrix} = 1(-2-15)-1(3-15)+1(15+10)$$

$$= 20$$

$$\therefore adj(A) = \begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj(A)$$

$$= \frac{1}{20}\begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}$$

$\therefore$ the solution is,

$$X = A^{-1}B$$

$$= \frac{1}{20}\begin{bmatrix} -17 & 4 & 5 \\ 12 & -4 & 0 \\ 25 & 0 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 11 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, \quad y = 1, \quad z = 1$$

9) $x + 2y + 3z = 1$ $\quad 2x + 3y + 2z = 2$ $\quad 3x + 2y + 4z = 1$

[W-14]

Ans.

$$A=\begin{bmatrix}1&2&3\\2&3&2\\3&2&4\end{bmatrix}\qquad X=\begin{bmatrix}x\\y\\z\end{bmatrix}\qquad K=\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$\therefore |A|=1(12-4)-2(8-6)+3(2-9)=-11$$

$$C(A)=\begin{bmatrix}\begin{vmatrix}3&2\\2&4\end{vmatrix}&-\begin{vmatrix}2&2\\3&4\end{vmatrix}&\begin{vmatrix}2&3\\3&2\end{vmatrix}\\-\begin{vmatrix}2&3\\2&4\end{vmatrix}&\begin{vmatrix}1&3\\3&4\end{vmatrix}&-\begin{vmatrix}1&2\\3&2\end{vmatrix}\\\begin{vmatrix}2&3\\3&2\end{vmatrix}&-\begin{vmatrix}1&3\\2&2\end{vmatrix}&\begin{vmatrix}1&2\\2&3\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}8&-2&-5\\-2&-5&4\\-5&4&-1\end{bmatrix}$$

$$\therefore adj(A)=\begin{bmatrix}8&-2&-5\\-2&-5&4\\-5&4&-1\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{-11}\begin{bmatrix}8&-2&-5\\-2&-5&4\\-5&4&-1\end{bmatrix}$$

$$\therefore X=A^{-1}K=\frac{1}{-11}\begin{bmatrix}8&-2&-5\\-2&-5&4\\-5&4&-1\end{bmatrix}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$=\frac{1}{-11}\begin{bmatrix}-1\\-8\\2\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{11}\\ \frac{8}{11}\\ -\frac{2}{11}\end{bmatrix}$$

$$\therefore x=\frac{1}{11} \qquad y=\frac{8}{11} \qquad z=-\frac{2}{11}$$

10) $2x+y=3$ $2y+3z=4$ $2x+2z=8$

[W-13]

Ans.

$$\text{Let } A=\begin{bmatrix}2&1&0\\0&2&3\\2&0&2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}3\\4\\8\end{bmatrix}$$

$$\text{Consider, } |A|=\begin{vmatrix}2&1&0\\0&2&3\\2&0&2\end{vmatrix}$$

$$=2(4-0)-1(0-6)+0$$

$$=8+6$$

$$=14\neq 0$$

$$\therefore A^{-1}\text{exists}$$

$$\text{Matrix of minors}=\begin{bmatrix}\begin{vmatrix}2&3\\0&2\end{vmatrix}&\begin{vmatrix}0&3\\2&2\end{vmatrix}&\begin{vmatrix}0&2\\2&0\end{vmatrix}\\ \begin{vmatrix}1&0\\0&2\end{vmatrix}&\begin{vmatrix}2&0\\2&2\end{vmatrix}&\begin{vmatrix}2&1\\2&0\end{vmatrix}\\ \begin{vmatrix}1&0\\2&3\end{vmatrix}&\begin{vmatrix}2&0\\0&3\end{vmatrix}&\begin{vmatrix}2&1\\0&2\end{vmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}4 & -6 & -4\\ 2 & 4 & -2\\ 3 & 6 & 4\end{bmatrix}$$

$$\text{matrix of cofactors} = \begin{bmatrix}4 & 6 & -4\\ -2 & 4 & 2\\ 3 & -6 & 4\end{bmatrix}$$

$$\text{Adj}\,A = \begin{bmatrix}4 & -2 & 3\\ 6 & 4 & -6\\ -4 & 2 & 4\end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{14}\begin{bmatrix}4 & -2 & 3\\ 6 & 4 & -6\\ -4 & 2 & 4\end{bmatrix}\begin{bmatrix}3\\ 4\\ 8\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{14}\begin{bmatrix}12-8+24\\ 18+16-48\\ -12+8+32\end{bmatrix} = \frac{1}{14}\begin{bmatrix}28\\ -14\\ 28\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}2\\ -1\\ 2\end{bmatrix}$$

TUTORIAL NO.4

MATRICES

1) Find the adjiont of the matrix $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$ **[W-13]**

2) Find the adjiont of the matrix $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ **[S-19]**

3) Find A^{-1} by Adjoint method $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ **[W-18,W-16]**

4) Find A^{-1} by Adjoint method $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ **[W-15]**

5) Solve the equation by matrix inversion method $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ **[S-19,W-17,S-17,S-13]**

6) Solve the equation by matrix inversion method $x + 3y + 2z = 6$, $3x - 2y + 5z = 5$, $2x - 3y + 6z = 7$ **[W-18,W-15,SQP]**

7) Solve the equation by matrix inversion method $x + 3y + 3z = 12$, $x + 4y + 4z = 15$, $x + 3y + 4z = 13$ **[S-18,SQP]**

8) Solve the equation by matrix inversion method $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$ **[S-18, S-16,S- 15,SQP]**

9) Solve the equation by matrix inversion method $2x + 3y - z = -3$, $5x + y + 3z = 10$, $4x + 3y - 2z = -3$ **[W-17]**

10) Solve the equation by matrix inversion method $x + y + z = 3$, $3x - 2y + 3z = 4$, $5x + 5y + z = 11$ **[W-19,W-16,W-12]**

PARTIAL FRACTION

Position in Question Paper **Total Marks-08**

Q.2. b) 4-Marks.

Q.4. b) 4-Marks.

Proper fraction :-

In the rational fraction if the degree of the numerator is smaller than the degree of denominator is called Proper Fraction

e.g.:- i) $\dfrac{x+5}{x^2-x}$ ii) $\dfrac{2x+3}{x^2-2x-3}$

Improper fraction :-

In the rational fraction if the degree of the numerator is greater than or equal to the degree of denominator is called Improper Fraction

e.g.:- i) $\dfrac{x^4}{x^3+1}$ ii) $\dfrac{x^3}{x^3+1}$

$$\boldsymbol{Function = Q + \frac{R}{D}}$$

A. SIMPLE NON REPEATED LINEAR FACTORS :-

Examples:-

Que. **Resolve into partial fraction**

1) $\frac{x-2}{x(x-1)}$ **[S-19]**

Ans.

$$\frac{x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$\therefore x-2 = (x-1)A + xB$$

Put $x = 0$

$$\therefore 0-2 = A(0-1) + B(0)$$

$$\therefore A = 2$$

Put $x = 1$

$$1-2 = A(1-1) + B(1)$$

$$\therefore B = -1$$

$$\therefore \frac{x-2}{x(x-1)} = \frac{2}{x} + \frac{(-1)}{x-1}$$

2) $\frac{x+4}{x(x+1)}$ **[S-18,S-17]**

Ans.

$$\frac{x+4}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\therefore x+4 = A(x+1) + B(x)$$

Put $x = 0$ $\boxed{A = 4}$

Put $x = -1$ $\boxed{B = -3}$

$$\therefore \frac{x+4}{x(x+1)} = \frac{4}{x} + \frac{-3}{x+1}$$

3) $\frac{x+5}{x^2-x}$ **[S-15,SQP]**

Ans.

$$\frac{x+5}{x^2-x}=\frac{x+5}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$$

$$\therefore \boxed{x+5=(x-1)A+xB}$$

$Put\ x=0$

$\therefore 0+5=(0-1)A+0$

$\therefore 5=-A$

$\therefore \boxed{-5=A}$

$Put\ x-1=0 \qquad \therefore x=1$

$\therefore 1+5=0A+B$

$\therefore \boxed{6=B}$

$$\therefore \boxed{\frac{x+5}{x^2-x}=\frac{-5}{x}+\frac{6}{x-1}}$$

4) $\frac{1}{x^2+x}$ **[W-17] OR** $\frac{1}{x(x+1)}$ **[W-16]**

Ans.

$$\therefore \frac{1}{x(x+1)}=\frac{A}{x}+\frac{B}{x+1}$$

$1=(x+1)A+xB$

Put $x=0$

$\therefore A=1$

Put $x=-1$

$\therefore B=-1$

$$\therefore \frac{1}{x(x+1)}=\frac{1}{x}+\frac{-1}{x+1}$$

5) $1+\frac{1}{x^2-x}$ **[S-16]**

Ans. $$\frac{1}{x^2-x}=\frac{1}{(x-1)(x+1)}$$

$$\frac{1}{(x-1)(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x+1)}$$

$$\therefore \quad 1 = A(x+1)+B(x-1)$$

put $x = 1$

$$\therefore 1 = A(1+1)$$

$$\therefore \ \mathrm{A}=\frac{1}{2}$$

put $x = -1$

$$\therefore 1 = B(-1-1)$$

$$\therefore \ \mathrm{B}=-\frac{1}{2}$$

$$1+\frac{1}{x^2-x}=1+\frac{\frac{1}{2}}{x-1}+\frac{\frac{-1}{2}}{x+1}$$

6) $\frac{x}{x^2-x-2}$ **[W-18,W-14]**

Ans. $$\frac{x}{x^2-x-2}=\frac{x}{(x-2)(x+1)}$$

$$\therefore \frac{x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$$

$$\therefore x = A(x+1)+B(x-2)$$

$$\therefore \ \text{put } x=2 \ \therefore \ A={}^{2}\!/_{3}$$

$$\text{Put } x=-1 \therefore B={}^{1}\!/_{3}$$

$$\therefore \frac{x}{(x-2)(x+1)}=\frac{{}^{2}\!/_{3}}{x-2}+\frac{{}^{1}\!/_{3}}{x+1}$$

7) $\frac{2x}{x^2+x-2}$ **[W-13]**

Ans.

$$\frac{2x}{x^2+x-2}=\frac{2x}{(x-1)(x+2)}$$

$$\frac{2x}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$$

$$2x=(x+2)A+(x-1)B$$

$$put\ x=1$$

$$2=3A$$

$$A=\frac{2}{3}$$

$$put\ x=-2$$

$$-4=-3B$$

$$B=\frac{4}{3}$$

$$\frac{2x}{(x-1)(x+2)}=\frac{\frac{2}{3}}{x-1}+\frac{\frac{4}{3}}{x+2}$$

8) $\frac{x}{x^2+x-2}$ **[S-13]**

Ans.

$$\therefore\frac{x}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$$

$$x=(x-1)A+(x+2)B$$

$$\text{Put } x=-2$$

$$-2=A(-3)$$

$$A=\frac{2}{3}$$

$$\text{Put } x=1$$
$$1=3B$$
$$B=\frac{1}{3}$$
$$\frac{x}{(x+2)(x-1)}=\frac{2/3}{x-3}+\frac{1/3}{x+1}$$

9) $\frac{2x+3}{x^2-2x-3}$ **[W-19,S-13]**

Ans.
$$\therefore\frac{2x+3}{(x-3)(x+1)}=\frac{A}{x-3}+\frac{B}{x+1}$$
$$2x+3=(x+1)A+(x-3)B$$
$$\text{Put } x=-1$$
$$9=4A$$
$$A=\frac{9}{4}$$
$$\text{Put } x=3$$
$$1=-4B$$
$$B=\frac{-1}{4}$$
$$\frac{2x+3}{(x-3)(x+1)}=\frac{9/4}{x-3}+\frac{-1/4}{x+1}=\frac{1}{4}\left[\frac{9}{x-3}-\frac{1}{x+1}\right]$$

10) $\frac{1}{x^2+3x+2}$ **[W-12]**

Ans.
$$\frac{1}{x^2+3x+2}=\frac{1}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$$
$$\therefore 1=(x+2)A+(x+1)B$$
$$Put\ x=-1$$
$$\therefore 1=(-1+2)A+0$$
$$\therefore \boxed{A=1}$$

$$Put\ x=-2$$

$$\therefore 1=0+(-2+1)B$$

$$\therefore \boxed{B=-1}$$

$$\therefore \boxed{\frac{1}{x^2+3x+2}=\frac{1}{x+1}+\frac{-1}{x+2}}$$

11) $\frac{1}{1-x^2}$ **[S-14,SQP]**

Ans.

$$\frac{1}{1-x^2}=\frac{1}{(1-x)(1+x)}=\frac{A}{1-x}+\frac{B}{1+x}$$

$$\therefore \boxed{1=(1+x)A+(1-x)B}$$

$$Put\ 1-x=0 \qquad \therefore x=1$$

$$\therefore 1=(1+1)A+0$$

$$\therefore \boxed{\frac{1}{2}=A}$$

$$Put\ 1+x=0 \qquad \therefore x=-1$$

$$\therefore 1=0+(1+1)B$$

$$\therefore \boxed{\frac{1}{2}=B}$$

$$\therefore \boxed{\frac{1}{1-x^2}=\frac{\frac{1}{2}}{1-x}+\frac{\frac{1}{2}}{1+x}}$$

12) $\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}$ **[S-14]**

Ans.

$$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+3}$$

$$\therefore x^2+4x+1=(x-1)(x+1)(x+3)\left[\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+3}\right]$$

$$\therefore \boxed{x^2+4x+1=(x+1)(x+3)A+(x-1)(x+3)B+(x-1)(x+1)C}$$

$Put\ \ x=1$

$\therefore 1^2+4(1)+1=(1+1)(1+3)A+0+0$

$\therefore 6=8A$

$\therefore \boxed{\frac{3}{4}=A}$

$Put\ \ x=-1$

$\therefore (-1)^2+4(-1)+1=0+(-1-1)(-1+3)B+0$

$\therefore -2=-4B$

$\therefore \boxed{\frac{1}{2}=B}$

$Put\ \ x=-3$

$\therefore (-3)^2+4(-3)+1=0+0+(-3-1)(-3+1)C$

$\therefore -2=8C$

$\therefore \boxed{-\frac{1}{4}=C}$

$$\therefore \boxed{\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}=\frac{\frac{3}{4}}{x-1}+\frac{\frac{1}{2}}{x+1}+\frac{-\frac{1}{4}}{x+3}}$$

13) $\frac{x+3}{(x^2-1)(x+5)}$ **OR** $\frac{x+3}{(x-1)(x+1)(x+5)}$ **[W-17]**

Ans. $$\frac{x+3}{(x^2-1)(x+5)}=\frac{x+3}{(x-1)(x+1)(x+5)}$$

Let $\frac{x+3}{(x-1)(x+1)(x+5)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+5}$

$x+3=A(x+1)(x+5)+B(x-1)(x+5)+C(x-1)(x+1)$

Put $x=1$

$1+3=A(1+1)(1+5)$

$4=A(12)$

$\therefore A=\frac{1}{3}$

Put $x=-1$

$-1+3=B(-1-1)(-1+5)$

$2=B(-8)$

$\therefore B=-\frac{1}{4}$

Put $x=-5$

$-5+3=C(-5-1)(-5+1)$

$-2=C(24)$

$\therefore C=-\frac{1}{12}$

$$\therefore \frac{x+3}{(x-1)(x+1)(x+5)}=\frac{\frac{1}{3}}{x-1}+\frac{-\frac{1}{4}}{x+1}+\frac{-\frac{1}{12}}{x+5}$$

14) $\frac{3x-1}{(x-4)(x+1)(x-1)}$ **[S-19,W-18]**

Ans. $\frac{3x-1}{(x-4)(x+1)(x-1)}=\frac{A}{x-4}+\frac{B}{x+1}+\frac{C}{x-1}$

$\therefore 3x-1=A(x+1)(x-1)+B(x-4)(x-1)+C(x-4)(x+1)$

Put $x=4$

$3(4)-1=A(4+1)(4-1)$

$\therefore 11=15A$

$\therefore A=\frac{11}{15}$

Put $x=-1$

$3(-1)-1=B(-1-4)(-1-1)$

$\therefore -4=B(-5)(-2)$

$\therefore B=\frac{-2}{5}$

Put $x=1$

$3(1)-1=C(1-4)(1+1)$

$\therefore 2=C(-3)(2)$

$\therefore C=\frac{-1}{3}$

$$\therefore \frac{3x-1}{(x-4)(x+1)(x-1)}=\frac{\frac{11}{15}}{x-4}+\frac{\frac{-2}{5}}{x+1}+\frac{\frac{-1}{3}}{x-1}$$

15) $\frac{3x-1}{(x-4)(2x+1)(x-1)}$ **[S-18,W-16]**

Ans.

$$\therefore \frac{3x-1}{(x-4)(2x+1)(x-1)}=\frac{A}{x-4}+\frac{B}{2x+1}+\frac{C}{x-1}$$

$$\therefore 3x-1=A(2x+1)(x-1)+B(x-4)(x-1)+C(x-4)(2x+1)$$

Put $x=4$

$$3(4)-1=A(2(4)+1)(4-1)$$

$$11=A(9)(3)$$

$$11=A(27)$$

$$\therefore A=\frac{11}{27}$$

Put $x=\frac{-1}{2}$

$$3\left(\frac{-1}{2}\right)-1=B\left(\frac{-1}{2}-4\right)\left(\frac{-1}{2}-1\right)$$

$$\frac{-5}{2}=B\left(\frac{-9}{2}\right)\left(\frac{-3}{2}\right)$$

$$\frac{-5}{2}=B\left(\frac{27}{4}\right)$$

$$\therefore B=\frac{-10}{27}$$

Put $x=1$

$$3(1)-1=C(1-4)(2(1)+1)$$

$$2=C(-3)(3)$$

$$\therefore C=\frac{-2}{9}$$

$$\therefore \frac{3x-1}{(x-4)(2x+1)(x-1)}=\frac{\frac{11}{27}}{x-4}+\frac{\frac{-10}{27}}{2x+1}+\frac{\frac{-2}{9}}{x-1}$$

16) $\frac{x-5}{x^3+x^2-6x}$ **[S-16,SQP]**

Ans.

$$\frac{x-5}{x^3+x^2-6x}=\frac{x-5}{x(x^2+x-6)}$$

$$\frac{x-5}{x(x^2+x-6)}=\frac{x-5}{x(x-2)(x+3)}$$

$$\frac{x-5}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$x-5 = (x-2)(x+3)A + x(x+3)B + x(x-2)C$$

Put $x = 0$

$$-5 = (-2)(3)A$$

$$\therefore A = \frac{5}{6}$$

Put $x = 2$

$$-3 = (2)(5)B$$

$$\therefore B = \frac{-3}{10}$$

Put $x = -3$

$$-8 = (-3)(-5)C$$

$$\therefore C = -\frac{8}{15}$$

$$\frac{x-5}{x(x-2)(x+3)} \quad \frac{\frac{5}{6}}{x} \quad \frac{\frac{-3}{10}}{x-2} \quad \frac{-\frac{8}{15}}{x+3}$$

17) $\frac{1}{x^3+3x^2+2x}$ **[W-15]**

Ans.

$$\frac{1}{x^3+3x^2+2x} = \frac{1}{x(x+1)(x+2)}$$

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$\therefore 1=(x+1)(x+2)A+x(x+2)B+x(x+1)C$$

Put $x=0$

$$1=(1)(2)A$$

$$\therefore A=\frac{1}{2}$$

Put $x=-1$

$$1=(-1)(1)B$$

$$\therefore B=-1$$

Put $x=-2$

$$1=(-2)(-2+1)C$$

$$C=\frac{1}{2}$$

$$\therefore \frac{1}{x(x+1)(x+2)}=\frac{\frac{1}{2}}{x}+\frac{-1}{x+1}+\frac{\frac{1}{2}}{x+2}$$

18) $\frac{x^2+1}{x(x^2-1)}$ **[S-18,S-17,W-12,SQP]**

Ans.

$$\frac{x^2+1}{x(x^2-1)}=\frac{x^2+1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$$

$$\therefore x^2+1=(x+1)(x-1)A+x(x-1)B+x(x+1)C$$

Put $x=0$

$$\therefore 0+1=(0+1)(0-1)A+0+0$$

$$\therefore 1=-A$$

$$\therefore \boxed{A=-1}$$

Put $x=-1$

$$\therefore (-1)^2+1=0-1(-1-1)B+0$$

$$\therefore 2=2B$$

$$\therefore \boxed{B=1}$$

$Put\ x = 1$

$\therefore (1)^2 + 1 = 0 + 0 + 1(1+1)C$

$\therefore 2 = 2C$

$\therefore \boxed{C = 1}$

$\therefore \boxed{\frac{x^2+1}{x(x^2-1)} = \frac{-1}{x} + \frac{1}{x+1} + \frac{1}{x-1}}$

19) $\frac{x+3}{(x^2-1)(x+5)}$ **[W-17]** **OR** $\frac{x+3}{(x-1)(x+1)(x+5)}$ **[W-17]**

Ans. $\frac{x+3}{(x-1)(x+1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+5}$

$\therefore\ x + 3 = A(x+1)(x+5) + B(x-1)(x+5) + C(x-1)(x+1)$

Put $x = 1$

$4 = A(2)(6)$

$4 = 12A$

$\therefore A = \frac{1}{3}$

Put $x = -1$

$-1 + 3 = B(-2)(4)$

$2 = -8B$

$\therefore B = -\frac{1}{4}$

Put $x = -5$

$-5 + 3 = C(-6)(-4)$

$-2 = 24C$

$\therefore C = \frac{-1}{12}$

$$\frac{x+3}{(x-1)(x+1)(x+5)} = \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{12}}{x+5}$$

20) $\frac{1}{x^3-x}$ **[S-15]**

Ans. $$\frac{1}{x^3-x}=\frac{1}{x(x+1)(x-1)}$$

$$=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$$

$$\therefore \boxed{1=(x+1)(x-1)A+x(x-1)B+x(x+1)C}$$

$Put\ x=0$

$\therefore 1=(0+1)(0-1)A+0+0$

$\therefore 1=-A$

$\therefore \boxed{-1=A}$

$Put\ x=-1$

$\therefore 1=0-1(-1-1)B+0$

$\therefore 1=2B$

$\therefore \boxed{\frac{1}{2}=B}$

$Put\ x=1$

$\therefore 1=0+0+1(1+1)C$

$\therefore 1=2C$

$\therefore \boxed{\frac{1}{2}=C}$

$$\therefore \boxed{\frac{1}{x^3-x}=\frac{-1}{x}+\frac{\frac{1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}}$$

B. SIMPLE NON REPEATED LINEAR FACTORS BY SUBSTITUTION :-

Examples:-

Que. Resolve into partial fraction

1) $\frac{tan\theta}{(tan\theta+2)(tan\theta+3)}$ **[W-16]**

Ans. Let $\tan\theta = t$

$$\therefore \frac{t}{(t+2)(t+3)} = \frac{A}{t+2} + \frac{B}{t+3}$$

$$\therefore \quad t = (t+3)A + (t+2)B$$

put $t = -2$

$$-2 = (-2+3)A$$

$$\therefore A = -2$$

put $t = -3$

$$-3 = (-3+2)B$$

$$-3 = -B$$

$$\therefore B = 3$$

$$\therefore \frac{t}{(t+2)(t+3)} = \frac{-2}{t+2} + \frac{3}{t+3}$$

$$\frac{\tan\theta}{(\tan\theta+2)(\tan\theta+3)} = \frac{-2}{\tan\theta+2} + \frac{3}{\tan\theta+3}$$

2) $\frac{tan\theta+1}{(tan\theta+2)(tan\theta+3)}$ **[W-18,S-15,SQP]**

Ans. $Put \ \tan\theta = x$

$$\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)} = \frac{x+1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\therefore \boxed{x+1=(x+3)A+(x+2)B}$$

$$Put\ x=-2$$

$$\therefore -2+1=(-2+3)A+0$$

$$\therefore \boxed{A=-1}$$

$$Put\ x=-3$$

$$\therefore -3+1=0+(-3+2)B$$

$$\therefore \boxed{B=2}$$

$$\therefore \frac{x+1}{(x+2)(x+3)}=\frac{-1}{x+2}+\frac{2}{x+3}$$

$$\therefore \boxed{\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}=\frac{-1}{\tan\theta+2}+\frac{2}{\tan\theta+3}}$$

3) $\frac{sin\theta+1}{(sin\theta+2)(sin\theta+3)}$ **[W-12]**

Ans.

$$Put\ \sin\theta=x$$

$$\frac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}=\frac{x+1}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$$

$$\therefore x+1=(x+3)A+(x+2)B$$

$$Put\ x=-2$$

$$\therefore -2+1=(-2+3)A+0$$

$$\therefore \boxed{A=-1}$$

$$Put\ x=-3$$

$$\therefore -3+1=0+(-3+2)B$$

$$\therefore \boxed{B=2}$$

$$\therefore \frac{x+1}{(x+2)(x+3)}=\frac{-1}{x+2}+\frac{2}{x+3}$$

$$\therefore \boxed{\frac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}=\frac{-1}{\sin\theta+2}+\frac{2}{\sin\theta+3}}$$

4) $\frac{e^x+1}{(e^x+2)(e^x+3)}$ **[W-15]**

Ans. Put $e^x = m$

$$\therefore \frac{m+1}{(m+2)(m+3)} = \frac{A}{m+2} + \frac{B}{m+3}$$

$$\therefore m+1 = (m+3)A + (m+2)B$$

Put $m = -2$

$$\therefore -1 = A$$

Put $m = -3$

$$\therefore -2 = (-1)B$$

$$\therefore B = 2$$

$$\therefore \frac{m+1}{(m+2)(m+3)} = \frac{-1}{m+2} + \frac{2}{m+3}$$

$$\therefore \frac{e^x+1}{(e^x+2)(e^x+3)} = \frac{-1}{e^x+2} + \frac{2}{e^x+3}$$

5) $\frac{e^x}{e^{2x}+4e^x+3}$ **[W-17,S-13]**

Ans. put $e^x = t$

$$\frac{t}{t^2+4t+3} = \frac{t}{(t+3)(t+1)}$$

$$\therefore \frac{t}{(t+3)(t+1)} = \frac{A}{t+3} + \frac{B}{t+1}$$

$$\therefore t = (t+1)A + (t+3)B$$

put $t = -3$

$$-3 = -2A$$

$$\therefore A = \frac{3}{2}$$

put $t=-1$

$-1=2B$

$\therefore \text{B}=-\frac{1}{2}$

$$\therefore \frac{t}{(t+3)(t+1)}=\frac{3/2}{t+3}+\frac{-1/2}{t+1}$$

$$\therefore \frac{e^x}{(e^x+3)(e^x+1)}=\frac{3/2}{e^x+3}+\frac{-1/2}{e^x+1}$$

6) $\frac{e^x+1}{2e^{2x}+7e^x+5}$ **[W-14]**

Ans.

$$\frac{e^x+1}{2e^{2x}+7e^x+5} \qquad (Put\ e^x=y)$$

$$=\frac{y+1}{2y^2+7y+5}$$

$$=\frac{y+1}{(2y+5)(y+1)}$$

$$=\frac{1}{2y+5}$$

$$=\frac{1}{2e^x+5}$$

OR

$$\frac{e^x+1}{2e^{2x}+7e^x+5} \qquad (Put\ e^x=y)$$

$$=\frac{y+1}{2y^2+7y+5}$$

$$=\frac{y+1}{(2y+5)(y+1)}=\frac{A}{2y+5}+\frac{B}{y+1}$$

$$\therefore \boxed{y+1=(y+1)A+(2y+5)B}$$

$$Put\ 2y+5=0 \qquad \therefore y=-\frac{5}{2}$$

$$\therefore -\frac{5}{2}+1=\left(-\frac{5}{2}+1\right)A+0$$

$$\therefore -\frac{3}{2} = -\frac{3}{2}A$$

$$\therefore \boxed{1 = A}$$

$$Put\ \ y+1=0 \qquad \therefore y=-1$$

$$\therefore -1+1 = 0+(-2+5)B$$

$$\therefore 0 = 3B$$

$$\therefore \boxed{0 = B}$$

$$\therefore \frac{y+1}{2y^2+7y+5} = \frac{1}{2y+5} + \frac{0}{y+1}$$

$$\therefore \boxed{\frac{e^x+1}{2e^{2x}+7e^x+5} = \frac{1}{2e^x+5}}$$

7) $\frac{x^2}{(x^2+1)(x^2+2)}$ **[S-14]**

Ans. $Put\ \ x^2 = y$

$$\frac{x^2}{(x^2+1)(x^2+2)} = \frac{y}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$$

$$\therefore y = (y+1)(y+2)\left[\frac{A}{y+1} + \frac{B}{y+2}\right]$$

$$\therefore \boxed{y = (y+2)A + (y+1)B}$$

$$Put\ \ y = -1$$

$$\therefore -1 = (-1+2)A + 0$$

$$\therefore \boxed{-1 = A}$$

$$Put\ \ y = -2$$

$$\therefore -2 = 0 + (-2+1)B$$

$$\therefore -2 = -B$$

$$\therefore \boxed{2 = B}$$

$$\therefore \frac{y}{(y+1)(y+2)} = \frac{-1}{y+1} + \frac{2}{y+2}$$

$$\therefore \boxed{\frac{x^2}{(x^2+1)(x^2+2)} = \frac{-1}{x^2+1} + \frac{2}{x^2+2}}$$

8) $\frac{x^2}{x^4+x^2-2}$ **[W-18]**

Ans. Put $x^2 = y$

$$\therefore \frac{x^2}{\left(x^4+x^2-2\right)} = \frac{y}{y^2+y-2}$$

$$\therefore \frac{y}{y^2+y-2} = \frac{A}{y+2} + \frac{B}{y-1}$$

$$\therefore y = A(y-1) + B(y+2)$$

Put $y = -2 \quad \therefore A = {}^{2}\!/_{3}$

Put $y = 1 \quad \therefore B = {}^{1}\!/_{3}$

$$\therefore \frac{y}{y^2+y-2} = \frac{{}^{2}\!/_{3}}{y+2} + \frac{{}^{1}\!/_{3}}{y-1}$$

Re *place y by* x^2

$$\therefore \frac{x^2}{\left(x^4+x^2-2\right)} = \frac{{}^{2}\!/_{3}}{x^2+2} + \frac{{}^{1}\!/_{3}}{x^2-1}$$

9) $\frac{x^2+1}{2x^4+5x^2+2}$ **[W-14]**

Ans. $\frac{x^2+1}{2x^4+5x^2+2}$ $\quad (Put\ x^2 = y)$

$$= \frac{y+1}{2y^2+5y+2}$$

$$= \frac{y+1}{(2y+1)(y+2)} = \frac{A}{2y+1} + \frac{B}{y+2}$$

$$\therefore \boxed{y+1 = (y+2)A + (2y+1)B}$$

$Put\ 2y+1=0 \quad or \quad y = -\frac{1}{2}$

$$\therefore -\frac{1}{2}+1 = \left(-\frac{1}{2}+2\right)A + 0$$

$\therefore \frac{1}{2}=\frac{3}{2}A$

$\therefore \boxed{\frac{1}{3}=A}$

$Put \;\; y+2=0 \quad or \quad y=-2$

$\therefore -2+1=0+\left(-4+1\right)B$

$\therefore -1=-3B$

$\therefore \boxed{\frac{1}{3}=B}$

$$\therefore \frac{y+1}{2y^2+5y+2}=\frac{\frac{1}{3}}{2y+1}+\frac{\frac{1}{3}}{y+2}$$

$$\therefore \boxed{\frac{x^2+1}{2x^4+5x^2+2}=\frac{\frac{1}{3}}{2x^2+1}+\frac{\frac{1}{3}}{x^2+2}}$$

C. REPEATED LINEAR FACTORS :-

Examples:-

Que. **Resolve into partial fraction**

1) $\frac{2x+1}{x^2(x+1)}$ **[S-19,W-18,S-16,S-14]**

Ans.

$$\frac{2x+1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$$\therefore 2x+1=x^2(x+1)\left[\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}\right]$$

$$\therefore \boxed{2x+1=x(x+1)A+(x+1)B+x^2C}$$

$$Put\ x=0$$

$$\therefore 2(0)+1=0+(0+1)B+0$$

$$\therefore \boxed{1=B}$$

$$Put\ x=-1$$

$$\therefore 2(-1)+1=0+0+(-1)^2C$$

$$\therefore \boxed{-1=C}$$

$$Put\ x=1$$

$$\therefore 2(1)+1=1(1+1)A+(1+1)B+1^2C$$

$$\therefore 3=2A+2B+C$$

$$\therefore 3=2A+2(1)-1$$

$$\therefore 3=2A+1$$

$$\therefore 2=2A$$

$$\therefore \boxed{1=A}$$

$$\therefore \boxed{\frac{2x+1}{x^2(x+1)}=\frac{1}{x}+\frac{1}{x^2}+\frac{-1}{x+1}}$$

2) $\frac{2x+3}{x^2(x-1)}$ **[W-15]**

Ans. $$\frac{2x+3}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$$

$2x+3=x(x-1)A+(x-1)B+x^2C$

Put $x=0$

$3=(-1)B$

$\therefore B=-3$

Put $x=1$

$\therefore 5=C$

Put $x=-1$

$\therefore 1=(-1)(-1-1)A+(-1-1)B+(-1)^2C$

$\therefore 1=2A-2B+C$

$\therefore 1=2A+6+5$

$\therefore -10=2A$

$\therefore A=-5$

$$\therefore \frac{2x+3}{x^2(x-1)}=\frac{-5}{x}+\frac{-3}{x^2}+\frac{5}{x-1}$$

3) $\frac{3x+2}{(x+1)(x^2-1)}$ **[S-19]**

Ans. $$\frac{3x+2}{(x+1)^2(x-1)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$$

$\therefore 3x+2=A(x-1)(x+1)+B(x-1)+C(x+1)^2$

Put $x=-1$

$$\therefore -3+2=B(-1-1)$$

$$\boxed{B=\frac{1}{2}}$$

Put $x=1$

$$\therefore 3+2=C(1+1)^2$$

$$\boxed{C=\frac{5}{4}}$$

Put $x=0, B=\frac{1}{2}, C=\frac{5}{4}$

$$\therefore 2=A(0-1)(0+1)+\frac{1}{2}(0-1)+\frac{5}{4}(0+1)^2$$

$$\boxed{A=-\frac{5}{4}}$$

$$\therefore \frac{3x+2}{(x+1)^2(x-1)}=\frac{-\frac{5}{4}}{x+1}+\frac{\frac{1}{2}}{(x+1)^2}+\frac{\frac{5}{4}}{x-1}$$

4) $\frac{2x-3}{(x+1)(x^2-1)}$ **[S-18]**

Ans. Let $\frac{2x-3}{(x^2-1)(x+1)}=\frac{2x-3}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$

$$\therefore\ 2x-3=A(x+1)^2+B(x-1)(x+1)+C(x-1)$$

Put $x=1$

$$2(1)-3=A(1+1)^2$$

$$-1=A(4)$$

$$\therefore A=-\frac{1}{4}$$

Put $x=-1$

$2(-1)-3=C(-1-1)$

$-5=C(-2)$

$\therefore C=\frac{5}{2}$

Put $x=0$

$\therefore -3=A-B-C$

$\therefore -3=-\frac{1}{4}-B-\frac{5}{2}$

$\therefore B=-\frac{1}{4}-\frac{5}{2}+3$

$\therefore B=\frac{1}{4}$

$$\therefore \frac{2x-3}{(x^2-1)(x+1)}=\frac{\frac{-1}{4}}{x-1}+\frac{\frac{1}{4}}{(x+1)}+\frac{\frac{5}{2}}{(x+1)^2}$$

5) $\frac{x^2}{(x+1)(x+2)^2}$ **[S-17]**

Ans.

Let $\frac{x^2}{(x+1)(x+2)^2}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$

$x^2=(x+2)^2A+(x+1)(x+2)B+(x+1)C$

Put $x=-1$

$\therefore 1=(1)A$

$\therefore A=1$

Put $x=-2$

$\therefore 4=(-1)C$

$\therefore C=-4$

Put $x=0$

$\therefore 0=4A+2B+C$

$\therefore 0=4-2B-4$

$\therefore B=0$

$$\therefore \frac{x^2}{(x+1)(x+2)^2}=\frac{1}{x+1}+\frac{0}{x+2}+\frac{(-4)}{(x+2)^2}$$

$$\frac{x^2}{(x+1)(x+2)^2}=\frac{1}{x+1}-\frac{4}{(x+2)^2}$$

6) $\frac{x^2-2x+7}{(x+1)(x-1)^2}$ **[W-13]**

Ans.

$$\frac{x^2-2x+7}{(x+1)(x-1)^2}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$x^2-2x+7=(x-1)^2A+(x+1)(x-1)B+(x+1)C$$

$$x^2-2x+7=(x-1)^2A+(x+1)(x-1)B+(x+1)C$$

$$put\ x=-1$$

$$1+2+7=4A$$

$$10=4A$$

$$A=\frac{5}{2}$$

$$put\ x=1$$

$$1-2+7=2C$$

$$6=2C$$

$$C=3$$

$$put\ x=0$$

$$7=A-B+C$$

$$B=\frac{5}{2}+3-7$$

$$B=\frac{-3}{2}$$

$$\frac{x^2-2x+7}{(x+1)(x-1)^2}=\frac{\frac{5}{2}}{x+1}+\frac{\frac{-3}{2}}{x-1}+\frac{3}{(x-1)^2}$$

7) $\frac{9}{(x-1)(x+2)^2}$ **[S-13]**

Ans.

$$\frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$$

$$9=(x+2)^2A+(x+2)(x-1)B+(x-1)C$$

Put $x=1$

$9=9A$

$\therefore$ A=1

Put$x=-2$

$9=-3C$

$\therefore C=-3$

Put $x=0$

$9=4A-2B-C$

$9=4(1)-2B+3$

$9=7-2B$

$2B=-2$

$\therefore B=-1$

$$\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}+\frac{-1}{x+2}+\frac{-3}{(x+2)^2}$$

8) $\frac{1}{(x+2)(x+1)^2}$ **[W-12]**

Ans.

$$\frac{1}{(x+1)^2(x+2)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{(x+2)}$$

$$\therefore 1=(x+1)(x+2)A+(x+2)B+(x+1)^2C$$

$Put\ x=-1$

$\therefore 1=0+(-1+2)B+0$

$\therefore \boxed{B=1}$

$Put\ x=-2$

$\therefore 1=0+0+(-2+1)^2 C$

$\therefore \boxed{C=1}$

$Put\ x=0$

$\therefore 1=(1)(2)A+(2)B+(1)^2 C$

$\therefore 1=2A+2B+C$

$\therefore \boxed{A=-1}$

$$\therefore \boxed{\frac{1}{(x+1)^2(x+2)}=\frac{-1}{x+1}+\frac{1}{(x+1)^2}+\frac{1}{(x+2)}}$$

D. NON- REPEATED LINEAR AND IRREDUCIBLE QUADRATIC FACTORS :-

Examples:-

Que. **Resolve into partial fraction**

1) $\frac{x^2+23x}{(x+3)(x^2+1)}$ **[S-19,W-18,W-16,S-14,SQP]**

Ans.

$$\frac{x^2+23x}{(x+3)(x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{x^2+1}$$

$$\therefore x^2+23x=(x+3)(x^2+1)\left[\frac{A}{x+3}+\frac{Bx+C}{x^2+1}\right]$$

$$\therefore \boxed{x^2+23x=(x^2+1)A+(x+3)(Bx+C)}$$

$$Put\, x=-3$$

$$\therefore (-3)^2+23(-3)=((-3)^2+1)A+0$$

$$\therefore -60=10A$$

$$\therefore \boxed{-6=A}$$

$$Put\, x=0$$

$$\therefore 0^2+23(0)=(0^2+1)A+(0+3)(0+C)$$

$$\therefore 0=A+3C$$

$$\therefore 0=-6+3C$$

$$\therefore 6=3C$$

$$\therefore \boxed{2=C}$$

$$Put\, x=1$$

$$\therefore 1^2+23(1)=(1^2+1)A+(1+3)(B+C)$$

$$\therefore 24=2A+4B+4C$$

$$\therefore 24=2(-6)+4B+4(2)$$

$$\therefore 28=4B$$

$$\therefore \boxed{7=B}$$

$$\therefore \boxed{\frac{x^2+23x}{(x+3)(x^2+1)}=\frac{-6}{x+3}+\frac{7x+2}{x^2+1}}$$

2) $\frac{x^2+23x}{(x-3)(x^2+1)}$ **[W-14]**

Ans.

$$\frac{x^2+23x}{(x-3)(x^2+1)}=\frac{A}{x-3}+\frac{Bx+C}{x^2+1}$$

$$\therefore x^2+23x=(x-3)(x^2+1)\left[\frac{A}{x-3}+\frac{Bx+C}{x^2+1}\right]$$

$$\therefore \boxed{x^2+23x=(x^2+1)A+(x-3)(Bx+C)}$$

$$Put\, x=3$$

$$\therefore (3)^2+23(3)=\left((3)^2+1\right)A+0$$

$$\therefore 78=10A$$

$$\therefore \boxed{\frac{39}{5}=A}$$

$$Put\, x=0$$

$$\therefore 0^2+23(0)=(0^2+1)A+(0-3)(0+C)$$

$$\therefore 0=A-3C$$

$$\therefore 0=\frac{39}{5}-3C$$

$$\therefore 3C=\frac{39}{5}$$

$$\therefore \boxed{C=\frac{13}{5}}$$

$$Put\, x=1$$

$$\therefore 1^2+23(1)=(1^2+1)A+(1-3)(B+C)$$

$$\therefore 24=2A-2B-2C$$

$$\therefore 24=2\left(\frac{39}{5}\right)-2B-2\left(\frac{13}{5}\right)$$

$$\therefore 2B=2\left(\frac{39}{5}\right)-2\left(\frac{13}{5}\right)-24$$

$$\therefore 2B=-\frac{68}{5}$$

$$\therefore \boxed{B=-\frac{34}{5}}$$

$$\therefore \boxed{\frac{x^2+23x}{(x-3)(x^2+1)}=\frac{\frac{39}{5}}{x-3}+\frac{-\frac{34}{5}x+\frac{13}{5}}{x^2+1}}$$

3) $\frac{2x+1}{(x-1)(x^2+1)}$ **[S-18]**

Ans.

$$\frac{2x+1}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\therefore\ 2x+1=(x^2+1)A+(x-1)(Bx+C)$$

Put $x=1$

$$\therefore 2(1)+1=(1^2+1)A$$

$$\therefore 3=2A$$

$$\therefore\ A=\frac{3}{2}$$

Put $x=0$,

$$\therefore\ 2(0)+1=(0+1)A+(0-1)(B(0)+C)$$

$$\therefore 1=A-C$$

$$\therefore 1=\frac{3}{2}-C$$

$$\therefore C=\frac{1}{2}$$

Put $x=-1$,

$$\therefore\ 2(-1)+1=((-1)^2+1)A+(-1-1)(B(-1)+C)$$

$$\therefore -1=2A+2B-2C$$

$$\therefore -1=2\left(\frac{3}{2}\right)+2B-2\left(\frac{1}{2}\right)$$

$$\therefore -1=3+2B-1$$

$$\therefore B=-\frac{3}{2}$$

$$\therefore \frac{2x+1}{(x-1)(x^2+1)}=\frac{\frac{3}{2}}{x-1}+\frac{-\frac{3}{2}x+\frac{1}{2}}{x^2+1} \qquad OR$$

$$\frac{2x+1}{(x-1)(x^2+1)}=\frac{1}{2}\left[\frac{3}{x-1}+\frac{-3x+1}{x^2+1}\right]$$

4) $\frac{x^2-x+3}{(x-2)(x^2+1)}$ **[W-17]**

Ans.

$$\frac{x^2-x+3}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$$

$$\therefore x^2-x+3=(x^2+1)A+(x-2)(Bx+C)$$

Put $x=2$

$5=5A$

$A=1$

Put $x=0$

$3=A-2C$

$\therefore C=-1$

Put $x=1$

$3=2A+(-1)(B+C)$

$3=2-B+1$

$\therefore B=0$

$$\frac{x^2-x+3}{(x-2)(x^2+1)}=\frac{1}{x-2}+\frac{(0)x-1}{x^2+1}$$

$$\frac{x^2-x+3}{(x-2)(x^2+1)}=\frac{1}{x-2}-\frac{1}{x^2+1}$$

5) $\frac{x^2+36x+6}{(x-1)(x^2+2)}$ **[S-15]**

Ans.

$$\frac{x^2+36x+6}{(x-1)(x^2+2)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+2}$$

$$\therefore \boxed{x^2+36x+6=(x^2+2)A+(x-1)(Bx+C)}$$

Put $x-1=0$ *i.e.,* $x=1$

$\therefore 1^2+36(1)+6=A(1^2+2)+0$

$\therefore 43=3A$

$\therefore \boxed{\frac{43}{3}=A}$

$Put\ \ x=0$

$\therefore 0+0+6=(0+2)A+(0-1)(0+C)$

$\therefore 6=2A-C$

$\therefore 6-2A=-C$

$\therefore 6-2\left(\frac{43}{3}\right)=-C$

$\therefore -\frac{68}{3}=-C$

$\therefore \boxed{\frac{68}{3}=C}$

$Put\ \ x=2$

$\therefore 2^2+36(2)+6=(2^2+2)A+(2-1)(2B+C)$

$\therefore 82=6A+2B+C$

$\therefore 82-6A-C=2B$

$\therefore 82-6\left(\frac{43}{3}\right)-\frac{68}{3}=2B$

$\therefore -\frac{80}{3}=2B$

$\therefore \boxed{-\frac{40}{3}=B}$

$$\therefore \boxed{\frac{x^2+36x+6}{(x-1)(x^2+2)}=\frac{\frac{43}{3}}{x-1}+\frac{-\frac{40}{3}x+\frac{68}{3}}{x^2+2}}$$

6) $\frac{x^2+1}{(x+1)(x^2+4)}$ **[S-17]**

Ans.

$$\text{Let}\ \frac{x^2+1}{(x+1)(x^2+4)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$$

$\therefore x^2+1=(x^2+4)A+(x+1)(Bx+C)$

$\text{Put}\ x=-1$

$\therefore 2=5A$

$\therefore A=\frac{2}{5}$

Put $x = 0$

$$1 = 4A + (1)C$$

$$1 = 4\left(\frac{2}{5}\right) + C$$

$$\therefore C = \frac{-3}{5}$$

Put $x = 1$

$$2 = 5A + 2(B + C)$$

$$2 = 5\left(\frac{2}{5}\right) + 2B + 2\left(\frac{-3}{5}\right)$$

$$\frac{6}{5} = 2B$$

$$\therefore B = \frac{3}{5}$$

$$\therefore \frac{x^2 + 1}{(x+1)(x^2+4)} = \frac{\frac{2}{5}}{x+1} + \frac{\frac{3}{5}x - \frac{3}{5}}{x^2+4}$$

7) $\frac{3x-2}{(x+2)(x^2+4)}$ **[W-19]**

Ans. $$\frac{3x-2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

$$\therefore 3x-2 = (x^2+4)A + (x+2)(Bx+C)$$

Put $x = -2$

$$\therefore 3(-2)-2 = ((-2)^2+4)A$$

$$\therefore -8 = 8A$$

$$\therefore A = -1$$

Put $x = 0$

$$\therefore -2 = 4A + 2C$$

$$\therefore -2 = 4(-1) + 2C$$

$$\therefore 2 = 2C$$

$$\therefore C = 1$$

Put $x=1$

$$\therefore 3(1)-2=\left((1)^2+4\right)A+(1+2)\left(B(1)+C\right)$$

$$\therefore 1=5A+3B+3C$$

$$\therefore 1=5(-1)+3B+3(1)$$

$$\therefore 3=3B$$

$$\therefore B=1$$

$$\therefore \frac{3x-2}{(x+2)(x^2+4)}=\frac{-1}{x+2}+\frac{x+1}{x^2+4}$$

8) $\frac{x^2-2x+3}{x^3+x}$ **[W-17]**

Ans.

$$\frac{x^2-x+3}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$$

$$\therefore x^2-x+3=(x^2+1)A+(x-2)(Bx+C)$$

Put $x=2$

$$5=5A$$

$$A=1$$

Put $x=0$

$$3=A-2C$$

$$\therefore C=-1$$

Put $x=1$

$$3=2A+(-1)(B+C)$$

$$3=2-B+1$$

$$\therefore B=0$$

$$\frac{x^2-x+3}{(x-2)(x^2+1)}=\frac{1}{x-2}+\frac{(0)x-1}{x^2+1}$$

$$\frac{x^2-x+3}{(x-2)(x^2+1)}=\frac{1}{x-2}-\frac{1}{x^2+1}$$

9) $\frac{x-2}{x^3+1}$ **[S-18]**

Ans.

$$\frac{x-2}{x^3+1}=\frac{x-2}{(x+1)(x^2-x+1)}$$

$$\therefore \frac{x-2}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$$

$$\therefore x-2=A(x^2-x+1)+(Bx+C)(x+1)$$

Put $x=-1$

$$\therefore -3=3A$$

$$\therefore A=-1$$

Put $x=0$

$$-2=(1)A+(1)C$$

$$-2=(1)(-1)+C$$

$$\therefore C=-1$$

Put $x=1$

$$\therefore 1-2=(1)A+2(B+C)$$

$$\therefore -1=A+2B+2C$$

$$\therefore -1=-1+2B-2$$

$$\therefore -1+3=2B$$

$$\therefore 2=2B$$

$$\therefore B=1$$

$$\therefore \frac{x-2}{(x+1)(x^2-x+1)}=\frac{-1}{x+1}+\frac{x-1}{x^2-x+1}$$

10) $\frac{x^2+1}{x^3+1}$ **[W-13]**

Ans. $\frac{x^2+1}{x^3+1}=\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$

$Put\ \text{x} = \text{-1}$ $\quad (\text{-1})^2+1=A\left((\text{-1})^2-(\text{-1})+1\right) \quad \therefore 2=3A \quad \therefore A=\frac{2}{3}$

$Put\ \text{x} = 0$

1=A+C

$1=\frac{2}{3}+C$

$C=1-\frac{2}{3}$

$C=\frac{1}{3}$

$Put\ \text{x} = 1$

$(1)^2+1=A\left((1)^2-(1)+1\right)+(B(1)+C)(1+1)$

$2=\frac{2}{3}+\left(B+\frac{1}{3}\right)2$

$2=\frac{2}{3}+2B+\frac{2}{3}$

$2B=2-\frac{4}{3}$

$2B=\frac{2}{3}$

$B=\frac{1}{3}$

$$\frac{x^2+1}{x^3+1}=\frac{\frac{2}{3}}{(x+1)}+\frac{\frac{1}{3}x+\frac{1}{3}}{(x^2-x+1)}$$

11) $\frac{x}{x^3+1}$ **[W-15]**

Ans. $\frac{x}{x^3+1}=\frac{x}{(x+1)(x^2-x+1)}$

$\therefore \frac{x}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$

$\therefore x=(x^2-x+1)A+(x+1)(Bx+C)$

Put $x=-1$

$\therefore -1=3A$

$\therefore A=\dfrac{-1}{3}$

Put $x=0$

$0=(1)A+(1)C$

$0=\dfrac{-1}{3}+C$

$\therefore C=\dfrac{1}{3}$

Put $x=1$

$\therefore 1=(1)A+2(B+C)$

$\therefore 1=\dfrac{-1}{3}+2B+\dfrac{2}{3}$

$\therefore 1-\dfrac{1}{3}=2B$

$\therefore \dfrac{2}{3}=2B$

$\therefore B=\dfrac{1}{3}$

$$\therefore \frac{x}{(x+1)(x^2-x+1)}=\frac{\frac{-1}{3}}{x+1}+\frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}$$

12) $\dfrac{x}{x^3-1}$ **[W-12]**

Ans. $\dfrac{x}{x^3-1}=\dfrac{x}{(x-1)(x^2+x+1)}$

$=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+x+1}$

$\therefore x=(x^2+x+1)A+(x-1)(Bx+C)$

$Put\, x=1$

$\therefore 1=\left((-1)^2+1+1\right)A+0$

$\therefore 1=3A$

$\therefore \boxed{A=\dfrac{1}{3}}$

$Put\ x = 0$

$\therefore 0 = \left(0^2 + 0 + 1\right)A + (0-1)(0+C)$

$\therefore 0 = A - C$

$\therefore 0 = \frac{1}{3} - C$

$\therefore \boxed{C = \frac{1}{3}}$

$Put\ x = -1$

$\therefore -1 = \left(1^2 - 1 + 1\right)A + (-1-1)(-B+C)$

$\therefore -1 = A + 2B - 2C$

$\therefore -1 = \frac{1}{3} + 2B - \frac{2}{3}$

$\therefore \boxed{B = -\frac{1}{3}}$

$\therefore \boxed{\frac{x}{x^3-1} = \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}x + \frac{1}{3}}{x^2+x+1}}$

13) $\frac{x+2}{(x-1)(x^2+x+1)}$ **[W-18]**

Ans.

Let $\frac{x+2}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

$\therefore x + 2 = A\left(x^2 + x + 1\right) + (Bx + C)(x-1)$

Put $x = 1$

$\therefore 3 = 3A$

$\therefore A = 1$

Put $x = 0$

$2 = (1)A + (-1)C$

$2 = (1)(1) - C$

$\therefore C = -1$

Put $x = -1$

$\therefore -1 + 2 = (1)A + (-2)(-B + C)$

$\therefore 1 = A + 2B - 2C$

$\therefore 1 = 1 + 2B - 2(-1)$

$\therefore 1 - 3 = 2B$

$\therefore -2 = 2B$

$\therefore B = -1$

$$\therefore \frac{x+2}{(x-1)(x^2+x+1)} = \frac{1}{x-1} + \frac{-x-1}{x^2+x+1}$$

$$\therefore \frac{x+2}{(x-1)(x^2+x+1)} = \frac{1}{x-1} - \frac{x+1}{x^2+x+1}$$

E. IMPROPER FRACTION :-

Examples:-

Que. **Resolve into partial fraction**

1) $\frac{x^4}{x^3+1}$ **[S-19,S-17]**

Ans.

$$\begin{array}{r|l} & x \\ x^3+1 & x^4 \\ & x^4 + x \\ & - \quad - \\ & \text{----------} \\ & -x \end{array}$$

$$\therefore \frac{x^4}{x^3+1} = x - \frac{x}{x^3+1}$$

$$\therefore \frac{x}{x^3+1} = \frac{x}{(x+1)(x^2-x+1)}$$

$$\frac{x}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$\therefore x = (x^2-x+1)A + (x+1)(Bx+C)$$

$\text{Put } x = -1$

$-1 = 3A$

$A = -\frac{1}{3}$

$\text{Put } x = 0$

$0 = A + C$

$\therefore C = \frac{1}{3}$

$\text{Put } x = 1$

$1 = A + 2(B+C)$

$1 = -\frac{1}{3} + 2B + 2\left(\frac{1}{3}\right)$

$\therefore B = \frac{1}{3}$

$$\therefore \frac{x}{(x+1)(x^2-x+1)} = \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1}$$

$$\therefore \frac{x^4}{x^3+1} = x - \left(\frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x+\frac{1}{3}}{x^2-x+1} \right)$$

2) $\frac{x^4}{x^2-1}$ **[S-18]**

Ans.

$$\begin{array}{r|l} & \quad x^2+1 \\ x^2-1 & \quad x^4 \\ & \quad x^4-x^2 \\ & \; - \quad + \\ \hline & \qquad x^2 \\ & \qquad x^2-1 \\ & \quad - \quad + \\ \hline & \qquad\quad 1 \end{array}$$

$$\therefore \frac{x^4}{x^2-1} = (x^2+1) + \frac{1}{x^2-1}$$

$$\text{Let} \frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\therefore \quad 1 = A(x-1) + B(x+1)$$

put $x = -1$

$$\therefore \quad 1 = A(-1-1)$$

$$\therefore A = -\frac{1}{2}$$

put $x = 1$

$$\therefore \quad 1 = B(1+1)$$

$$\therefore B = \frac{1}{2}$$

$$\frac{1}{x^2-1} = \frac{\frac{-1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1} \right)$$

$$\therefore \frac{x^4}{x^2-1} = (x^2+1) + \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1} \right)$$

3) $\frac{x^3}{x^2-1}$ **[S-16]**

Ans.

$$\begin{array}{r|l} & x \\ \hline x^2-1 & x^3 \\ & x^3 - x \\ & - \quad + \\ \hline & x \end{array}$$

$$\therefore \frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$$

$$\therefore \frac{x}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$\therefore x = (x+1)A + (x-1)B$$

Put $x = 1$

$1 = 2A \qquad A = \frac{1}{2}$

Put $x = -1$

$-1 = -2B \qquad B = \frac{1}{2}$

$$\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1} = x + \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}$$

4) $\frac{x^3+x}{x^2-9}$ **[W-17,W-14]**

Ans.

$$\frac{x^3+x}{x^2-9} = x + \frac{10x}{x^2-9}$$

$$\therefore \frac{10x}{x^2-9} = \frac{10x}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$\therefore \boxed{10x = (x+3)A + (x-3)B}$$

$Put \quad x-3=0 \quad i.e., \ x=3$

$$\therefore 30 = 6A + 0$$

$$\therefore \boxed{5 = A}$$

$$Put\ \ x+3=0\quad i.e.,\ x=-3$$

$$\therefore -30=0-6B$$

$$\therefore \boxed{5=B}$$

$$\therefore \frac{10x}{x^2-9}=\frac{5}{x-3}+\frac{5}{x+3}$$

$$\therefore \boxed{\frac{x^3+x}{x^2-9}=x+\frac{5}{x-3}+\frac{5}{x+3}}$$

5) $\frac{x^3+x}{x^2-4}$ **[W-16]**

Ans.

$$\begin{array}{r} x \\ x^2-4\overline{)x^3+x} \\ x^3-4x \\ -\quad + \\ \hline 5x \end{array}$$

$$\therefore \frac{x^3+x}{x^2-4}=x+\frac{5x}{(x-2)(x+2)}$$

$$\therefore \frac{5x}{(x-2)(x+2)}=\frac{A}{x-2}+\frac{B}{x+2}$$

$$\therefore 5x=(x+2)A+(x-2)B$$

Put $x=2$ $\quad\therefore 5(2)=(2+2)A$

$$A=\frac{10}{4}$$

$$A=\frac{5}{2}$$

Put $x=-2$ $\quad\therefore 5(-2)=(-2-2)B$

$$B=\frac{-10}{-4}$$

$$B=\frac{5}{2}$$

$$\therefore \frac{5x}{(x-2)(x+2)}=\frac{\frac{5}{2}}{x-2}+\frac{\frac{5}{2}}{x+2}$$

6) $\frac{x^3+2}{x^2-1}$ **[S-13]**

Ans.

$$\begin{array}{r|l} & x \\ x^2-1 & x^3+2 \\ & x^3-x \\ & -\quad + \\ \hline & x+2 \end{array}$$

$$\therefore \frac{x^3+2}{x^2-1}=x+\frac{x+2}{x^2-1}$$

Consider, $\frac{x+2}{x^2-1}=\frac{x+2}{(x+1)(x-1)}$

$$\therefore \frac{x+2}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$$

$$x+2=(x-1)A+(x+1)B$$

Put $x=-1$

$1=-2A$

$\therefore A=-\frac{1}{2}$

Put $x=1$

$3=2B$

$\therefore B=\frac{3}{2}$

$$\therefore \frac{x+2}{(x+1)(x-1)}=\frac{-1/2}{x+1}+\frac{3/2}{x-1}$$

$$\therefore \frac{x^3+2}{x^2-1}=x+\frac{-1/2}{x+1}+\frac{3/2}{x-1}$$

7) $\frac{x^3+1}{x^2+2x}$ **[W-15]**

Ans.

$$\begin{array}{r} x-2 \\ x^2+2x\overline{\smash{)}x^3+1} \\ x^3+2x^2 \\ -\quad\;\; - \\ \hline -2x^2+1 \\ -2x^2-4x \\ +\qquad\;\; + \\ \hline 4x+1 \end{array}$$

$$\therefore \frac{x^3+1}{x^2+2x}=(x-2)+\frac{4x+1}{x^2+2x}$$

$$\therefore \frac{4x+1}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2}$$

$$\therefore 4x+1=(x+2)A+xB$$

Put $x=0$

$$1=2A$$

$$A=\frac{1}{2}$$

Put $x=-2$

$$-7=-2B$$

$$B=\frac{7}{2}$$

$$\therefore \frac{4x+1}{x(x+2)}=\frac{\frac{1}{2}}{x}+\frac{\frac{7}{2}}{x+2}$$

$$\frac{x^3+1}{x^2+2x}=(x-2)+\frac{\frac{1}{2}}{x}+\frac{\frac{7}{2}}{x+2}$$

8) $\frac{x^3+1}{x^2+6x}$ **[S-15]**

Ans.

$$\frac{x^3+1}{x^2+6x}=x-6+\frac{36x+1}{x^2+6x}$$

$$\therefore \frac{36x+1}{x^2+6x}=\frac{36x+1}{x(x+6)}=\frac{A}{x}+\frac{B}{x+6}$$

$$\therefore \boxed{36x+1=(x+6)A+xB}$$

$$Put\ x=0$$

$$\therefore 0+1=(0+6)A+0$$

$$\therefore 1=6A$$

$$\therefore \boxed{\frac{1}{6}=A}$$

$$Put\ x+6=0 \quad i.e.,\ x=-6$$

$$\therefore 36(-6)+1=0-6B$$

$$\therefore -215=-6B$$

$$\therefore \boxed{\frac{215}{6}=B}$$

$$\therefore \frac{x^3+1}{x^2+6x}=\frac{\frac{1}{6}}{x}+\frac{\frac{215}{6}}{x+2}$$

$$\therefore \boxed{\frac{x^3+1}{x^2+6x}=x-6+\frac{\frac{1}{6}}{x}+\frac{\frac{215}{6}}{x+2}}$$

TUTORIAL NO.5

PARTIAL FRACTION

Que. **Resolve into partial fraction**

1) $\frac{3x-1}{(x-4)(2x+1)(x-1)}$ **[S-18,W-16]**

2) $\frac{x^2+1}{x(x^2-1)}$ **[S-18,S-17,W-12,SQP]**

3) $\frac{tan\theta+1}{(tan\theta+2)(tan\theta+3)}$ **[W-18,S-15,SQP]**

4) $\frac{e^x+1}{2e^{2x}+7e^x+5}$ **[W-14]**

5) $\frac{2x+1}{x^2(x+1)}$ **[S-19,W-18,S-16,S-14]**

6) $\frac{x^2}{(x+1)(x+2)^2}$ **[S-17]**

7) $\frac{x^2+23x}{(x+3)(x^2+1)}$ **[S-19,W-18,W-16,S-14,SQP]**

8) $\frac{x-2}{x^3+1}$ **[S-18]**

9) $\frac{x^4}{x^3+1}$ **[S-19,S-17]**

10) $\frac{x^3+1}{x^2+6x}$ **[S-15]**

TRIGONOMETRIC RATIOS OF ALLIED , COMPOUND , MULTIPLE & SUB-MULTIPLE ANGLES.

Position in Question Paper **Total Marks-14**

Q.1. c) 2-Marks.

Q.3. a) 4-Marks.

Q.4. c) 4-Marks.

Q.4. e) 4-Marks.

TRIGONOMETRIC RATIOS OF ALLIED & COMPOUND ANGLES

Allied Angles :- [S-18,S-13]

If the sum or difference of two angles is either 0 or is an integral multiple of $\frac{\pi}{2}$ i.e.90^0 **Forms of Allied Angles**:- $\theta, \frac{\pi}{2}+\theta, \frac{\pi}{2}-\theta, \frac{\pi}{2}+\theta, \frac{3\pi}{2}+\theta$ ...

Compound Angles:- [W-16,S-15,SQP]

If the sum or difference of two or more angles are generally called compound angles & respective angles are called **Constituent angles**.

Forms of Compound Angles:-

$A+B,\ A-B,\ A+B+C,\ A-B+C,\ A+B-C,\ A-B-C.$

Examples:-

Que. **Write the formulae for**

1) $sin(A+B)$ **[S-14]**

Ans. $sin(A+B) = sinA.cosB + cosA.sinB$

2) $sin(A-B)$

Ans. $sin(A-B) = sinA.cosB - cosA.sinB$

3) $cos(A+B)$

Ans. $cos(A+B) = cosA.cosB - sinA.sinB$

4) $cos(A-B)$ **[S-14]**

Ans. $cos(A-B) = cosA.cosB + sinA.sinB$

5) $tan(A+B)$

Ans. $$tan(A+B) = \frac{tanA + tanB}{1 - tanA.tanB}$$

6) $tan(A-B)$

Ans. $$tan(A-B) = \frac{tanA - tanB}{1 + tanA.tanB}$$

Que. **Without using calculator find the value of**

1) $sin\ 15^0$ **[W-18]**

Ans.

$$\sin\left(15^0\right)$$
$$=\sin\left(45^0-30^0\right)$$
$$=\sin 45^0\cos 30^0-\cos 45^0\sin 30^0$$
$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$=\frac{\sqrt{3}-1}{2\sqrt{2}}\quad \text{or}\quad 0.2588$$

2) $cos\ 15^0$

Ans. $cos15^0 = cos(45^0 - 30^0)$

$$= cos45^0cos30^0 + sin45^0sin30^0$$
$$=\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}.\frac{1}{2}$$
$$=\frac{\sqrt{3}+1}{\sqrt{2}.2}$$

3) $sin\ 75^0$ **[S-19,W-15]**

Ans.

$$\sin 75^0=\sin\left(45^0+30^0\right)$$
$$=\sin 45^0\cos 30^0+\cos 45^0\sin 30^0$$
$$=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\frac{1}{2}$$
$$=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

4) $cos\ 75^0$ **[S-17]**

Ans. $\cos(75)^0 = \cos(30^0 + 45^0)$

$$=\cos 30^0.\cos 45^0 - \sin 30^0.\sin 45^0$$

$$=\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} - \frac{1}{2}.\frac{1}{\sqrt{2}}$$

$$=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

5) $tan\ 75^0$ **[W-12]**

Ans. $\tan(75^\circ) = \tan(30^\circ + 45^\circ)$

$$=\frac{\tan(30^\circ)+\tan(45^\circ)}{1-\tan(30^\circ)\tan(45^\circ)}$$

$$=\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}\cdot 1}$$

$$=\frac{1+\sqrt{3}}{\sqrt{3}-1}$$

6) $sin\ 105^0$ **[W-17]**

Ans. $\sin(105^0)$

$$=\sin(60^0+45^0)$$

$$=\sin 60^0 \cos 45^0 + \cos 60^0 \sin 45^0$$

$$=\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} + \frac{1}{2}\frac{1}{\sqrt{2}}$$

$$=\frac{\sqrt{3}+1}{2\sqrt{2}} \quad \text{OR} \quad 0.9659$$

7) $\cos 105^0$ **[S-19]**

Ans. $\cos(105^0) = \cos(60^0 + 45^0)$

$$= \cos 60^0 \cos 45^0 - \sin 60^0 \sin 45^0$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}} \quad \text{or} \quad -0.2588$$

8) $\tan 105^0$

Ans. $\tan 105^0 = \tan(60^0 + 45^0)$

$$= \frac{\tan 60^0 + \tan 45^0}{1 - \tan 60^0 . \tan 45^0}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}.1}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

9) $\sin^2 120^0$

Ans. $\sin^2 120^0 = (\sin 120^0)^2$

$$= [\sin(60^0 + 60^0)]^2$$

$$= [\sin 60^0 . \cos 60^0 + \cos 60^0 . \sin 60^0]^2$$

$$= [\frac{\sqrt{3}}{2} . \frac{1}{2} + \frac{1}{2} . \frac{\sqrt{3}}{2}]^2$$

$$= [\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}]^2$$

$$= [\frac{2\sqrt{3}}{4}]^2$$

$$= [\frac{\sqrt{3}}{2}]^2$$

$$= \frac{3}{4}$$

10) $sin(-330^0)$ **[S-14,SQP]**

Ans.

$$\begin{aligned}\sin(-330^\circ) &= -\sin 330^\circ\\ &= -\sin(360^\circ - 30^\circ)\\ &= -\sin(-30^\circ)\\ &= \sin(30^\circ)\\ &= \frac{1}{2} \quad or \quad 0.5\end{aligned}$$

11) $sin(-765^0)$ **[W-19,W-16,SQP]**

Ans.

$$\begin{aligned}\sin(-765^0) &= -\sin 765^0\\ &= -\sin(8\times 90^0 + 45^0) \quad \text{or} \quad -\sin\left(8\times\frac{\pi}{2}+45^0\right)\\ &= -\sin 45^0\\ &= -\frac{1}{\sqrt{2}} \quad \text{or} \quad -0.7071\end{aligned}$$

12) $cos(-390^0)$

Ans.

$$\begin{aligned}cos(-390^0) &= cos(390^0)\\ &= cos(360^0 + 30^0)\\ &= cos30^0\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

13) $cos(3660^0)$ **[W-15]**

Ans.

$$\begin{aligned}\cos(3660) &= \cos(3600+60)\\ &= \cos(40\times 90+60)\end{aligned}$$

$$=\cos(60)$$

$$=\frac{1}{2}$$

14) $sec(3660^0)$ **[S-18]**

Ans.

$$\sec(3660^0)=\sec(40\times 90^0+60^0)$$

$$=\sec 60^0$$

$$=2$$

15) $cosec\frac{5\pi}{6}$

Ans. $cosec\frac{5\pi}{6}=cosec150$

$$=cosec(180-30)$$

$$=cosec(180-30)$$

$$=cosec30$$

$$=2$$

16) $tan\frac{\pi}{4}$

Ans. $tan\frac{\pi}{4}=\tan(45)=1$

17) $sec\frac{3\pi}{2}$

Ans. $sec\frac{3\pi}{2}=\sec(270)=\infty$

18) $cos\frac{7\pi}{2}$

Ans. $cos\frac{7\pi}{2} = \cos(630) = 0$

19) $sin\frac{\pi}{12}$ **[W-17]**

Ans.

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(15^0\right) = \sin\left(45^0 - 30^0\right)$$

$$= \sin 45^0.\cos 30^0 - \cos 45^0.\sin 30^0$$

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

20) $cot\frac{19\pi}{6}$

Ans. $cot\frac{19\pi}{6} = \cot(570)$

$$= \cot(540 + 30)$$

$$= \cot(30)$$

$$= \sqrt{3}$$

Que. **Without using calculator find the value of**

1) $sin(22^0).cos(38^0) + cos(22^0).sin(38^0)$

Ans. $sin(22^0).cos(38^0) + cos(22^0).sin(38^0)$

$= sin(22 + 38)$

$= sin(60)$

$= \frac{\sqrt{3}}{2}$

2) $sin(\frac{5\pi}{12}).cos(\frac{\pi}{12}) + cos(\frac{5\pi}{12}).sin(\frac{\pi}{12})$

Ans. $sin(\frac{5\pi}{12}).cos(\frac{\pi}{12}) + cos(\frac{5\pi}{12}).sin(\frac{\pi}{12})$

$= sin(\frac{5\pi}{12} + \frac{\pi}{12})$

$= sin(\frac{6\pi}{12})$

$= sin(\frac{\pi}{2})$

$= 1$

3) $sin^2 60 + tan^2 45 - cosec^2 30$ **[S-16]**

Ans. $sin^2 60 + tan^2 45 - cosec^2 30$

$= (sin60)^2 + (tan45)^2 - (cosec30)^2$

$= (\frac{\sqrt{3}}{2})^2 + (1)^2 - (2)^2$

$= \frac{3}{4} + 1 - 4$

$= \frac{-9}{4}$

4) $tan(585^0).cot(-495^0) - cot(405^0).tan(-495^0)$ **[W-16]**

Ans.

$$\begin{aligned}\tan(585^\circ) &= \tan(6\times 90^0+45^\circ)\\ &= \tan 45^\circ\\ &=1\end{aligned}$$

$$\begin{aligned}\cot(-495^\circ) &= -\cot(495^\circ)\\ &= -\cot(5\times 90^0+45^\circ)\\ &= \tan 45^0\\ &=1\end{aligned}$$

$$\begin{aligned}\cot(405^\circ) &= \cot(4\times 90^0+45^\circ)\\ &= \cot 45^\circ\\ &=1\end{aligned}$$

$$\begin{aligned}\tan(-495^\circ) &= -\tan(495^\circ)\\ &= -\tan(5\times 90^0+45^\circ)\\ &= \cot 45^\circ\\ &=1\end{aligned}$$

$$\begin{aligned}&\tan(585^\circ).\ \cot(-495^\circ)-\cot(405^\circ).\ \tan(-495^\circ)\\ &=(1)(1)-(1)(1)\\ &=0\end{aligned}$$

5) $sin(150^0) + cos(300^0) - tan(315^0) + \sec^2(360^0)$ **[S-15]**

Ans.

$$\begin{aligned}\sin(150^\circ) &= \sin(90^\circ+60^\circ)\\ &= \cos 60^\circ\\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\cos(300^\circ) &= \cos(270^\circ+30^\circ)\\ &= \cos(3\times 90^\circ+30^\circ)\\ &= \sin 30^\circ\\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\tan(315^\circ) &= \tan(270^\circ+45^\circ)\\ &= \tan(3\times90^\circ+45^\circ)\\ &= -\cot 45^\circ\\ &= -1\end{aligned}$$

$$\sec^2(360^\circ)=\sec^2(0^\circ)=1$$

$$\therefore \sin(150^\circ)-\tan(315^\circ)+\cos(300^\circ)+\sec^2(360^\circ)$$

$$=\frac{1}{2}+1+\frac{1}{2}+1$$

$$=3$$

6) $sin(150^0)+cos(300^0)-tan(315^0)+\sec^2(3660^0)$ **[W-17,S-17]**

Ans.

$$\sin(150^0)=\sin(2\times90-30)=\sin 30=\frac{1}{2}$$

$$\cos(300^0)=\cos(4\times90-60)=\cos 60=\frac{1}{2}$$

$$\tan(315^0)=\tan(4\times90-45)=-\tan 45=-1$$

$$\sec^2(3660^0)=\left[\sec(3660^0)\right]^2=\left[\sec(40\times90+60)\right]^2=\left[\sec(60)\right]^2=[2]^2=4$$

$$\therefore \sin(150^0)+\cos(300^0)-\tan(315^0)+\sec^2(3660^0)$$

$$=\frac{1}{2}+\frac{1}{2}-(-1)+4$$

$$=6$$

7) $sin(510^0).cos(570^0)\ +\ sin(-330^0).cos(-390^0)$ **[S-18]**

Ans.

$$\begin{aligned}\sin(-330^0) &= -\sin(330^0)\\ &= -\sin(4\times90^0-30^0)=-(-\sin 30^0)=\frac{1}{2}\end{aligned}$$

$$\cos 570^0 = \cos\left(6\times 90^0 + 30^0\right)$$

$$\cos 570^0 = -\cos 30^0 = -\frac{\sqrt{3}}{2}$$

$$\sin\left(-330^0\right) = -\sin\left(330^0\right)$$

$$= -\sin\left(4\times 90^0 - 30^0\right) = -\left(-\sin 30^0\right) = \frac{1}{2}$$

$$\cos\left(-390^0\right) = \cos 390^0$$

$$= \cos\left(4\times 90^0 + 30^0\right) = \cos 30^0 = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos 570^0 \sin 510^0 + \sin\left(-330^0\right)\cos\left(-390^0\right)$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= 0$$

8) $sin(\,420).cos(390)\ +\ cos(-300).sin(330)$ **[W-18]**

Ans.

$$\sin 420^0 \cdot \cos 390^0 + \cos 300^0 \cdot \sin 330^0$$

$$= \sin(360^0 + 60^0)\cdot\cos(360^0 + 30^0) + \cos(360^0 - 60^0)\sin(360^0 - 30^0)$$

$$= \sin 60^0 \cdot \cos 30^0 + \cos 60^0 \cdot (-\sin 30^0)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{1}{2}$$

9) $\frac{4}{3tan^2 30} + 3sin^2 120 - cosec^2 30 - \frac{3}{4cot^2 120} + cos^2 270$ **[W-14]**

Ans.

$$\tan^2 30^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

$$\sin 120^\circ = \sin\left(90^\circ + 30^\circ\right) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \sin^2 120^\circ = \frac{3}{4}$$

$$\cos ec 30^\circ = 2$$

$$\therefore \cos ec^2 30^\circ = 4$$

$$\begin{aligned}\cot 120^\circ &= \cot\left(90^\circ + 30^\circ\right)\\ &= -\tan 30^\circ\\ &= -\frac{1}{\sqrt{3}}\end{aligned}$$

$$\therefore \cot^2 120^\circ = \frac{1}{3}$$

$$\begin{aligned}\cos 270^\circ &= \cos\left(3\times 90^\circ + 0\right)\\ &= \sin 0\\ &= 0\end{aligned}$$

$$\therefore \cos^2 270^\circ = 0$$

But given that

$$\frac{4}{3\tan^2 30^\circ} + 3\sin^2 120^\circ - \cos ec^2 30^\circ - \frac{3}{4\cot^2 120^\circ} + \cos^2 270^\circ$$

$$= \frac{4}{3\left(\frac{1}{3}\right)} + 3\left(\frac{3}{4}\right) - 4 - \frac{3}{4\left(\frac{1}{3}\right)} + 0$$

$$= \frac{9}{2} \quad or \quad 4.5$$

1) Prove that: $cos(\ 510).cos(330)\ +\ sin(390).cos(120) = -1$ **[W-12]**

Ans.

$$\begin{aligned}\cos(510^\circ) &= \cos(6\times 90^\circ - 30^\circ)\\ &= -\cos 30^\circ\\ &= -\frac{\sqrt{3}}{2} \qquad or \quad -0.866\end{aligned}$$

$$\begin{aligned}\cos(330^\circ) &= \cos(4\times 90^\circ - 30^\circ)\\ &= \cos 30^\circ\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\sin(390^\circ) &= \sin(4\times 90^\circ + 30^\circ)\\ &= \sin 30^\circ\\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\cos(120^\circ) &= \cos(90^\circ + 30^\circ)\\ &= -\sin 30^\circ\\ &= -\frac{1}{2}\end{aligned}$$

$$\cos(510^\circ)\cos(330^\circ)+\sin(390^\circ)\cos(120^\circ)$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$= -1$$

2) Prove that: $sin\ (420).cos(390)\ + cos(-300).sin(-330)\ = 1$

[S-19,S-16]

Ans.

$$\begin{aligned}\sin 420^0 &= \sin(90^0\times 4 + 60^0)\\ &= \sin 60^0 = \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\cos 390^0 &= \cos(90^0\times 4 + 30^0)\\ &= \cos 30^0 = \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\cos(-300^0) &= \cos(300^0)\\ &= \cos(90^0\times 3 + 30^0)\\ &= \sin 30^0 = \frac{1}{2}\end{aligned}$$

$$\sin\left(-330^0\right)=-\sin\left(330^0\right)$$
$$=-\sin\left(90^0\times3+60^0\right)$$
$$=-\left(-\cos 60^0\right)=\frac{1}{2}$$
$$\sin420^0\cos 390^0+\cos\left(-300^0\right)\sin\left(-330^0\right)$$
$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$
$$=1$$

3) Evaluate: $\frac{sec^2(135^0)}{\cos(-240^0)-2\sin(930^0)}$ **[W-15]**

Ans.
$$\frac{\sec^2 135^0}{\cos\left(-240^0\right)-2\sin\left(930^0\right)}=\frac{\sec^2 135^0}{\cos\left(240^0\right)-2\sin\left(930^0\right)}$$
$$=\frac{\sec^2\left(90^0+45^0\right)}{\cos\left(2\times90^0+60^0\right)-2\sin\left(10\times90^0+30^0\right)}$$
$$=\frac{\cos ec^2\left(45^0\right)}{-\cos\left(60^0\right)+2\sin\left(30^0\right)}$$
$$=\frac{2}{-\frac{1}{2}+2\frac{1}{2}}=\frac{2}{-\frac{1}{2}+1}$$
$$=4$$

4) Simplify: $\frac{\cos^2(180-\theta)}{\sin(-\theta)}+\frac{\cos^2(270+\theta)}{\sin(180+\theta)}$ **[W-19,SQP]**

Ans.
$$\cos^2\left(180^0-\theta\right)=\left(-\cos\theta\right)^2=\cos^2\theta$$
$$\cos^2\left(270^0+\theta\right)=\sin^2\theta$$
$$\sin\left(-\theta\right)=-\sin\theta$$
$$\sin\left(180+\theta\right)=-\sin\theta$$

$$\therefore \frac{\cos^2(180^0-\theta)}{\sin(-\theta)}+\frac{\cos^2(270^0+\theta)}{\sin(180+\theta)}$$

$$=\frac{\cos^2\theta}{-\sin\theta}+\frac{\sin^2\theta}{-\sin\theta}$$

$$=\frac{\cos^2\theta+\sin^2\theta}{-\sin\theta}$$

$$=\frac{1}{-\sin\theta}$$

$$=-\cos ec\theta$$

5) Evaluate: $\frac{\tan66^0+\tan69^0}{1-\tan66^0.\tan69^0}$ **[W-14,SQP]**

Ans.

$$\frac{\tan 66°+\tan 69°}{1-\tan 66°\tan 69°}=\tan(66°+69°)$$

$$=\tan 135°$$

$$=\tan(90°+45°) \quad OR \quad \tan(180°-45°)$$

$$=-\cot 45° \quad OR \quad -\tan(45°)$$

$$=-1$$

6) Evaluate $\frac{\tan32^0+\tan88^0}{1-\tan32^0.\tan88^0}$ **[S-16]**

Ans. $\frac{\tan32^0+\tan88^0}{1-\tan32^0.\tan88^0}$

$$=\tan(32^0+88^0)$$

$$=\tan120^0$$

$$=\tan(90^0+30^0)$$

$$=-\cot30^0$$

$$=-\sqrt{3}$$

7) Evaluate: $\frac{\tan 85^0 - \tan 10^0}{1+\tan 85^0 . \tan 10^0}$ **[S-13]**

Ans.

$$\frac{\tan 85^0 - \tan 40^0}{1+\tan 85^0 . \tan 40^0}$$

$$= \tan\left(85^0 - 40^0\right)$$

$$= \tan 45^0$$

$$= 1$$

8) Prove that: $cosec^2\theta - cos^2\theta . cosec^2\theta = 1$ **[W-13]**

Ans.

$$L.H.S. = \cos ec^2\theta - \cos^2\theta \cos ec^2\theta$$

$$= \cos ec^2\theta\left(1-\cos^2\theta\right)$$

$$= \cos ec^2\theta \sin^2\theta$$

$$= \frac{1}{\sin^2\theta}\sin^2\theta$$

$$=1=R.H.S.$$

9) Prove that: $\frac{1}{1-COSA} + \frac{1}{1+COSA} = 2cosec^2A$ **[S-17]**

Ans.

$$\frac{1}{1-\cos A} + \frac{1}{1+\cos A}$$

$$= \frac{(1+\cos A)+(1-\cos A)}{(1-\cos A)(1+\cos A)}$$

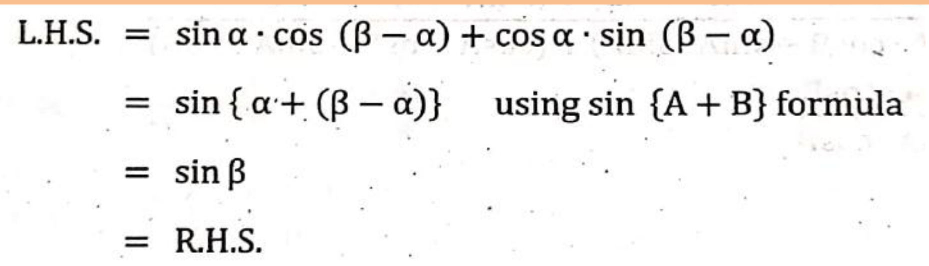
$$= \frac{1+\cos A+1-\cos A}{1^2-\cos^2 A}$$

$$= \frac{2}{\sin^2 A}$$

$$= 2\operatorname{cosec}^2 A$$

10) Show that: $sin\alpha\ .\ cos(\beta-\alpha)+\ cos\alpha\ .\ sin(\beta-\alpha)=sin\beta$

Ans.



$$\begin{aligned} \text{L.H.S.} &= \sin\alpha\cdot\cos\ (\beta-\alpha)+\cos\alpha\cdot\sin\ (\beta-\alpha) \\ &= \sin\{\alpha+(\beta-\alpha)\} \quad \text{using } \sin\ \{A+B\} \text{ formula} \\ &= \sin\beta \\ &= \text{R.H.S.} \end{aligned}$$

11) Prove that:

$$sin\left(\frac{\pi}{4}+A\right).cos\left(\frac{\pi}{4}+B\right)-cos\left(\frac{\pi}{4}+A\right).sin\left(\frac{\pi}{4}+B\right)\ =\ sin(A-B)$$

Ans.

$$sin\left(\frac{\pi}{4}+A\right).cos\left(\frac{\pi}{4}+B\right)-cos\left(\frac{\pi}{4}+A\right).sin\left(\frac{\pi}{4}+B\right)\ =\ sin[\left(\frac{\pi}{4}+A\right)-\left(\frac{\pi}{4}+B\right)]$$

$$sin\left(\frac{\pi}{4}+A\right).cos\left(\frac{\pi}{4}+B\right)-cos\left(\frac{\pi}{4}+A\right).sin\left(\frac{\pi}{4}+B\right)\ =\ sin[\frac{\pi}{4}+A-\frac{\pi}{4}-B]$$

$$sin\left(\frac{\pi}{4}+A\right).cos\left(\frac{\pi}{4}+B\right)-cos\left(\frac{\pi}{4}+A\right).sin\left(\frac{\pi}{4}+B\right)\ =\ sin[A-B]$$

12) Prove that: $sin\left(\theta+\frac{\pi}{6}\right)-sin\left(\theta-\frac{\pi}{6}\right)\ =\ cos\theta$ **[W-15,W-12]**

Ans.

$$\therefore \sin\left(\theta+\frac{\pi}{6}\right)-\sin\left(\theta-\frac{\pi}{6}\right)$$

$$=\left(\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right)-\left(\sin\theta\cos\frac{\pi}{6}-\cos\theta\sin\frac{\pi}{6}\right)$$

$$=2\cos\theta\sin\frac{\pi}{6}$$

$$=2\cos\theta\cdot\frac{1}{2}$$

$$=\cos\theta$$

13) Prove that: $sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$

Ans.

$$sin\left(\frac{\pi}{2}+\theta\right) = sin\frac{\pi}{2}.cos\theta + cos\frac{\pi}{2}.sin\theta$$

$$sin\left(\frac{\pi}{2}+\theta\right) = 1.cos\theta + 0.sin\theta$$

$$sin\left(\frac{\pi}{2}+\theta\right) = cos\theta + 0$$

$$sin\left(\frac{\pi}{2}+\theta\right) = cos\theta$$

14) Prove that: $tan(\frac{\pi}{4} - \theta) = \frac{1-tan\theta}{1+tan\theta}$ **[W-18,W-14]**

OR

Prove that: $tan(45 - \theta) = \frac{cos\theta - sin\theta}{cos\theta + sin\theta}$

OR

Prove that: $tan(\frac{\pi}{4} - \theta) = \frac{cos\theta - sin\theta}{cos\theta + sin\theta}$

Ans.

$$tan(\frac{\pi}{4} - \theta) = \frac{tan\frac{\pi}{4} - tan\theta}{1 + tan\frac{\pi}{4}.tan\theta}$$

$$= \frac{1-tan\theta}{1+1.tan\theta}$$

$$= \frac{1-tan\theta}{1+tan\theta}$$

$$= \frac{1-\frac{sin\theta}{cos\theta}}{1+\frac{sin\theta}{cos\theta}}$$

$$= \frac{\frac{cos\theta - sin\theta}{cos\theta}}{\frac{cos\theta+sin\theta}{cos\theta}}$$

$$= \frac{cos\theta - sin\theta}{cos\theta + sin\theta}$$

15) Prove that: $tan(\frac{\pi}{4} - A) = \frac{cosA+sinA}{cosA-sinA}$

Ans.

$$\tan\left(\frac{\pi}{4}+A\right)$$

$$= \frac{\tan\frac{\pi}{4}+\tan A}{1-\tan\frac{\pi}{4}\tan A}$$

$$= \frac{1+\tan A}{1-\tan A}$$

$$= \frac{1+\frac{\sin A}{\cos A}}{1-\frac{\sin A}{\cos A}}$$

$$= \frac{\cos A+\sin A}{\cos A-\sin A}$$

16) Prove that: $tan\left(\frac{\pi}{4}+A\right).tan\left(\frac{\pi}{4}-A\right) = 1$

Ans.

$$\text{L.H.S.} = \tan\left(\frac{\pi}{4}+\text{A}\right)\tan\left(\frac{\pi}{4}-\text{A}\right)$$

$$= \left(\frac{\tan\frac{\pi}{4}+\tan \text{A}}{1-\tan\frac{\pi}{4}\cdot\tan \text{A}}\right)\left(\frac{\tan\frac{\pi}{4}-\tan \text{A}}{1+\tan\frac{\pi}{4}\cdot\tan \text{A}}\right)$$

$$= \left(\frac{1+\tan \text{A}}{1-1\cdot\tan \text{A}}\right)\left(\frac{1-\tan \text{A}}{1+1\cdot\tan \text{A}}\right) \qquad \boxed{\tan\frac{\pi}{4}=1}$$

$$= \left(\frac{1+\tan \text{A}}{1-\tan \text{A}}\right)\times\left(\frac{1-\tan \text{A}}{1+\tan \text{A}}\right) = 1$$

$$= \text{R.H.S.}$$

17) If $tanA = \frac{1}{2}$, $tanB = \frac{1}{3}$ find $tan(A+B)$ **[W-18,S-16]**

Ans.

$$\tan\left(A+B\right)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$
$$=1$$

18) If $tan\,x = \frac{5}{6}$ & $tan\,y = \frac{1}{11}$, prove that $x + y = \frac{\pi}{4}$ **[W-14]**

Ans.

$$\tan x=\frac{5}{6},\ \tan y=\frac{1}{11}$$
$$\therefore x=\tan^{-1}\left(\frac{5}{6}\right),\ y=\tan^{-1}\left(\frac{1}{11}\right)$$
$$\therefore x+y=\tan^{-1}\left(\frac{5}{6}\right)+\tan^{-1}\left(\frac{1}{11}\right)$$
$$=\tan^{-1}\left(\frac{\frac{5}{6}+\frac{1}{11}}{1-\left(\frac{5}{6}\right)\left(\frac{1}{11}\right)}\right)$$
$$=\tan^{-1}\left(1\right)$$
$$=\frac{\pi}{4}$$

19) If $A + B = \frac{\pi}{4}$ Show that $(1+tanA)(1+tanB) = 2$ **[S-19,S-16]**

Ans. $A = \frac{\pi}{4} - B$ Taking tan on both side

$$(1+tanA)(1+tanB) = \left[1+ta\,n\left(\frac{\pi}{4}-B\right)\right].[1+tanB]$$

$$= \left[1 + \frac{\tan\frac{\pi}{4} - tanB}{1 + tan\frac{\pi}{4}.tanB}\right].[1 + tanB]$$

$$= \left[1 + \frac{1 - tanB}{1 + 1.tanB}\right].[1 + tanB]$$

$$= \left[1 + \frac{1 - tanB}{1 + tanB}\right].[1 + tanB]$$

$$= \left[\frac{1 + tanB + 1 - tanB}{1 + tanB}\right].[1 + tanB]$$

$$= 1 + 1$$

$$= 2$$

20) If $tan(A + B) = 3$, $tan(A - B) = 5$ find $tan2A$.

Ans. $2A = A + A$

$2A = A + A + B - B$

$2A = (A + B) + (A - B)$

Taking tan on both side

$tan(2A) = tan[(A + B) + (A - B)]$

$$tan(2A) = \frac{tan[(A + B) + tan(A - B)]}{1 - tan[(A + B).tan(A - B)]}$$

$$tan(2A) = \frac{3 + 5}{1 - 3.5}$$

$$tan(2A) = \frac{8}{-14}$$

$$tan(2A) = \frac{-4}{7}$$

21) Given that $tan(x + y) = \frac{3}{4}$, $tan(x - y) = \frac{8}{15}$ prove that $tan\ 2x = \frac{77}{36}$

[S-18,W-17]

Ans. $$2x = x + y + x - y$$

$$\tan 2x = \tan(x + y + x - y)$$

$$= \frac{\tan(x+y) + \tan(x-y)}{1 - \tan(x+y)\tan(x-y)}$$

$$= \frac{\frac{3}{4} + \frac{8}{15}}{1 - \frac{3}{4}\frac{8}{15}}$$

$$= \frac{77}{36}$$

22) Given that $tan(A+B) = \frac{3}{4}$, $tan(A-B) = \frac{77}{36}$ find

i) $tan\,2A$ **[S-16]** ***ii***) $tan\,2B$ **[S-19]**

Ans. ***i***)

$$Let\ A + B = x,\ A - B = y$$

$$x + y = A + B + A - B = 2A$$

$$\therefore \tan(x + y) = \tan 2A$$

$$\therefore \tan 2A = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$= \frac{\frac{3}{4} + \frac{77}{36}}{1 - \frac{3}{4} \times \frac{77}{36}}$$

$$= -\frac{416}{87} = -4.7816$$

ii)

$$\tan 2B = \tan\left[(A+B)-(A-B)\right]$$
$$= \frac{\tan(A+B)-\tan(A-B)}{1+\tan(A+B)\tan(A-B)}$$
$$= \frac{\frac{3}{4}-\frac{77}{36}}{1+\frac{3}{4}\times\frac{77}{36}}$$
$$= \frac{-8}{15}$$

23) Given that $tan(x+y)=\frac{1}{2}$, $tan(x-y)=\frac{1}{3}$ find

i) $tan2x$ **[W-16]** ***ii***) $tan2y$ **[W-16]**

Ans.

$$(i)\tan 2x = \tan\left[(x+y)+(x-y)\right]$$
$$= \frac{\tan(x+y)+\tan(x-y)}{1-\tan(x+y)\tan(x-y)}$$
$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$$
$$= 1$$

$$(ii)\tan 2y = \tan\left[(x+y)-(x-y)\right]$$
$$= \frac{\tan(x+y)-\tan(x-y)}{1+\tan(x+y)\tan(x-y)}$$

$$= \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2} \times \frac{1}{3}}$$

$$= \frac{1}{7}$$

24) In any ΔABC show that $tanA + tanB + tanC = tanA.tanB.tanC$

[S-19,S-18,W-14]

Ans.

$$We\ \ have,\ \ A+B+C=180^{\circ}\ \ or\ \ \pi$$

$$\therefore A+B=180^{\circ}-C$$

$$\therefore \tan(A+B)=\tan(180^{\circ}-C)$$

$$\therefore \frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$$\therefore \tan A+\tan B=-\tan C[1-\tan A\tan B]$$

$$\therefore \tan A+\tan B=-\tan C+\tan A\tan B\tan C$$

$$\therefore \tan A+\tan B+\tan C=\tan A\tan B\tan C$$

25) Show that: $tan70^0 - tan50^0 - tan20^0 = tan70^0.tan50^0.tan20^0$

[S-19,S-18]

Ans.

$$\text{consider}\ \ \tan 70^0=\tan\left(50^0+20^0\right)$$

$$\tan 70^0=\frac{\tan 50^0+\tan 20^0}{1-\tan 50^0.\tan 20^0}$$

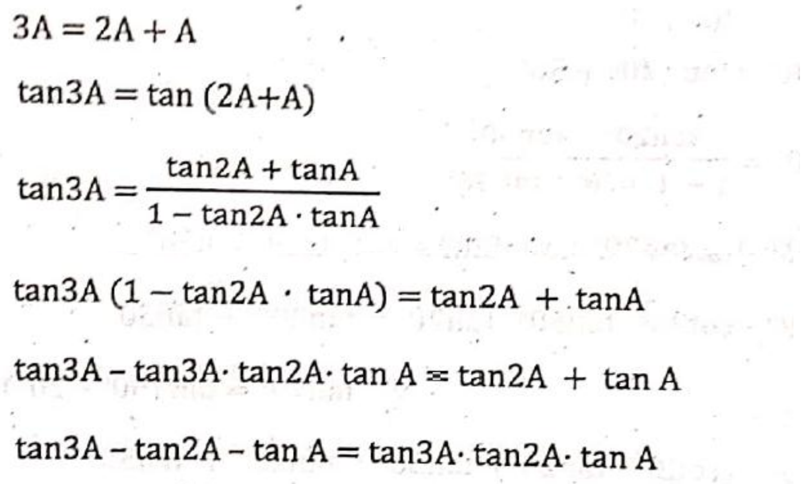
$$\tan 70^0\left(1-\tan 50^0.\tan 20^0\right)=\tan 50^0+\tan 20^0$$

$$\tan 70^0-\tan 70^0.\tan 50^0.\tan 20^0=\tan 50^0+\tan 20^0$$

$$\tan 70^0-\tan 50^0-\tan 20^0=\tan 70^0.\tan 50^0.\tan 20^0$$

26) Show that: $tan3A - tan2A - tanA = tan3A.tan2A.tanA$

Ans.



$3A = 2A + A$

$\tan 3A = \tan(2A+A)$

$\tan 3A = \dfrac{\tan 2A + \tan A}{1 - \tan 2A \cdot \tan A}$

$\tan 3A\,(1 - \tan 2A \cdot \tan A) = \tan 2A + \tan A$

$\tan 3A - \tan 3A \cdot \tan 2A \cdot \tan A = \tan 2A + \tan A$

$\tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$

27) Prove that: $cos(A+B).cos(A-B) = cos^2A - sin^2B$

Ans.

$cos(A+B).cos(A-B) = [cosA.cosB - sinA.sinB].[cosA.cosB + sinA.sinB]$

$cos(A+B).cos(A-B) = [cosA.cosB]^2 - [sinA.sinB]^2$

$cos(A+B).cos(A-B) = cos^2A.cos^2B - sin^2A.sin^2B$

$cos(A+B).cos(A-B) = cos^2A.(1 - sin^2B) - (1 - cos^2A).sin^2B$

$cos(A+B).cos(A-B) = cos^2A.-cos^2Asin^2B - (sin^2B - sin^2Bcos^2A)$

$cos(A+B).cos(A-B) = cos^2A.-cos^2Asin^2B - sin^2B + sin^2Bcos^2A$

$cos(A+B).cos(A-B) = cos^2A - sin^2B$

28) Prove that: $sin(A+B).Sin(A-B) = sin^2A - sin^2B$ **[W-17,S-14]**

$= \cos^2B - \cos^2A$ **[S-18]**

Ans.

$\sin(A+B)\sin(A-B)$

$=(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)$

$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$

$=\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B$

$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$

$=\sin^2 A-\sin^2 B$

$$\begin{aligned}
\text{LHS} &= \sin(A+B).\sin(A-B)\\
&= (\sin A.\cos B + \cos A.\sin B)(\sin A.\cos B - \cos A.\sin B)\\
&= (\sin A.\cos B)^2 - (\cos A.\sin B)^2\\
&= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B\\
\text{LHS} &= (1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)\\
&= \cos^2 B - \cos^2 B.\cos^2 A - \cos^2 A + \cos^2 B.\cos^2 A\\
&= \cos^2 B - \cos^2 A\\
&= \text{RHS}
\end{aligned}$$

29) If $sin\theta = \frac{15}{17}$ where θ lies in II^{nd} quadrant find the value of $tan\theta$ **[W-13]**

Ans.

$$\begin{aligned}
\cos\theta &= \sqrt{1-\sin^2\theta}\\
&= \sqrt{1-\left(\frac{15}{17}\right)^2}\\
&= \sqrt{\frac{64}{289}}\\
\cos\theta &= \frac{-8}{17} \qquad \because \theta \text{ is in II quadrant}\\
\therefore \tan\theta &= \frac{\sin\theta}{\cos\theta}\\
&= \frac{15/17}{-8/17}\\
&= \frac{15}{-8}
\end{aligned}$$

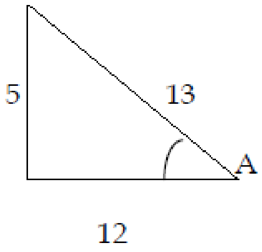
30) If A & B both are obtuse angles & $sin\text{A} = \frac{5}{13}$, $cos\text{B} = \frac{-4}{5}$. Evaluate

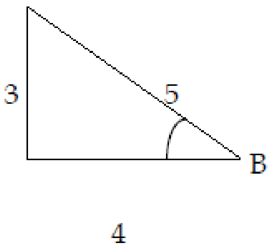
$\boldsymbol{i})$ $sin(A+B)$ **[S-18,S-13]** $\boldsymbol{ii})$ $cos(A+B)$

$\boldsymbol{iii})$ $tan(A+B)$ $\boldsymbol{iv})$ Quadrant of angle A + B **[S-15]**

Ans. *i*)

Given $\sin A = \frac{5}{13}$ $\cos B = -\frac{4}{5}$





$\therefore \cos A = \frac{12}{13}$ $\sin B = \frac{3}{5}$

$\therefore A$ and B are obtuse

$\cos A = \frac{-12}{13}$, $\sin B = \frac{3}{5}$

$$\therefore \sin(A+B) = \sin A.\cos B + \cos A.\sin B$$

$$= \frac{5}{13}.\frac{-4}{5} + \frac{-12}{13}.\frac{3}{5}$$

$$= \frac{-56}{65}$$

ii)

$$cos(A+B) = cosA.cosB - sinA.sinB$$

$$cos(A+B) = \frac{12}{13}.\frac{-4}{5} - \frac{5}{13}.\frac{3}{5}$$

$$cos(A+B) = \frac{-48}{65} - \frac{15}{65}$$

$$cos(A+B) = \frac{-63}{65}$$

$iii)$

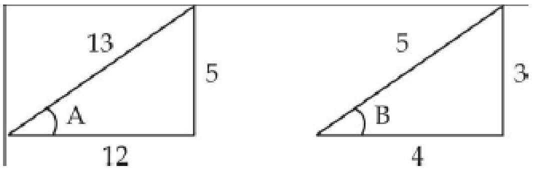
$$tan(A+B)=\frac{sin(A+B)}{cos(A+B)}$$

$$tan(A+B)=\frac{\frac{-56}{63}}{\frac{-63}{65}}$$

$$tan(A+B)=\frac{56}{63}$$

$iv)$

$\sin A=\frac{5}{13}$ $\cos B=-\frac{4}{5}$



As A and B are abtuse angles, $\cos A$ is negative and $\sin B$ is positive.

$\therefore \cos A=-\frac{12}{13}$

$\sin B=\frac{3}{5}$

$\therefore \cos(A+B)=\cos A\cos B-\sin A\sin B$

$$=-\frac{12}{13}\cdot\left(-\frac{4}{5}\right)-\frac{5}{13}\cdot\frac{3}{5} \qquad ---(*)$$

$$=\frac{33}{65} \qquad ---(*)$$

As A and B are abtuse angles, $180°<A+B<360°$.

In III quadrant, cos is -ve.

$\therefore A+B$ is in IV quadrant.

31) If A & B both are obtuse angles & $sin A = \frac{12}{13}$, $cos B = \frac{-4}{5}$. Evaluate

i) $sin(A+B)$ **[SQP]** ***ii***) $cos(A+B)$ **[W-19]**

Ans. ***i***)

$$\sin A = \frac{12}{13},\ \cos B = \frac{-4}{5}$$

$$\cos^2 A = 1 - \sin^2 A$$
$$= 1 - \left(\frac{12}{13}\right)^2$$
$$= 1 - \frac{144}{169} = \frac{25}{169}$$
$$\cos A = \pm\frac{5}{13}$$
$$\therefore \cos A = -\frac{5}{13} \quad (\angle A \text{ is obtuse angle})$$
$$\sin^2 B = 1 - \cos^2 B$$
$$= 1 - \left(-\frac{4}{5}\right)^2$$
$$\sin^2 B = 1 - \frac{16}{25} = \frac{9}{25}$$
$$\sin B = \pm\frac{3}{5}$$
$$\therefore \sin B = \frac{3}{5} \quad (\angle B \text{ is obtuse angle})$$

$$sin(A+B) = sinA.cosB + cosA.sinB$$
$$sin(A+B) = \frac{12}{13}.\frac{-4}{5} + \frac{-5}{13}.\frac{3}{5}$$
$$sin(A+B) = \frac{-48}{65} + \frac{-15}{65}$$
$$sin(A+B) = \frac{-63}{65}$$

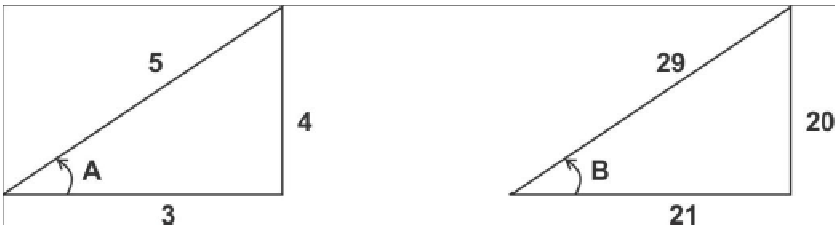
$ii)$

$$\therefore \cos(A+B) = \cos A.\cos B - \sin A.\sin B$$
$$= \left(-\frac{5}{13}\right)\times\left(-\frac{4}{5}\right)-\left(\frac{12}{13}\right)\times\left(\frac{3}{5}\right)$$
$$= -\frac{16}{65}$$

32) If $cosA = \frac{-3}{5}$ & $sinB = \frac{20}{29}$ where A & B are in III^{rd} & II^{nd}quadrant respectively find $tan(A+B)$ **[S-15]**

Ans.

$$\cos A = -\frac{3}{5} \qquad \sin B = \frac{20}{29}$$



As A is the third quadrant, tan A is positive and B is in the second quadrant, tan B is negative.

$$\therefore \tan A = \frac{4}{3} \quad and \quad \tan B = -\frac{20}{21}$$

$$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{4}{3}-\frac{20}{21}}{1-\left(\frac{4}{3}\right)\left(-\frac{20}{21}\right)} \qquad ---(*)$$

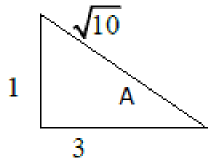
$$= \frac{24}{143} \quad or \quad 0.168 \qquad ---(**)$$

33) If $tanA = \frac{1}{3}$, $0 < A < \frac{\pi}{2}$, & $tanB = \frac{1}{4}$, $\pi < B < \frac{3\pi}{2}$ find $sin(A+B)$

[W-16]

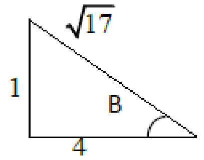
Ans.

$\tan A = \frac{1}{3}$,



$0 < A < \frac{\pi}{2}$ (A lies in first quadrant) $\sin A = \frac{1}{\sqrt{10}}$, $\cos A = \frac{3}{\sqrt{10}}$

$\tan B = \frac{1}{4}$,



$\pi < B < \frac{3\pi}{2}$ (B lies in third quadrant) $\sin B = \frac{-1}{\sqrt{17}}$, $\cos B = \frac{-4}{\sqrt{17}}$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$= \left(\frac{1}{\sqrt{10}}\right)\left(\frac{-4}{\sqrt{17}}\right) + \left(\frac{3}{\sqrt{10}}\right)\left(\frac{-1}{\sqrt{17}}\right)$$

$$= -\frac{-7}{\sqrt{170}} \quad \text{or} \quad -0.5369$$

34) If $sin\alpha = \frac{-5}{13}$, $cos\beta = \frac{-7}{25}$ and α & β lies in third quadrant find $sin(\alpha - \beta)$ **[W-13]**

Ans. $\therefore \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ ------------------ (1)

$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$= \sqrt{1-\left(\frac{-5}{13}\right)^2}$$

$$= \sqrt{1-\frac{25}{169}}$$

$$= \sqrt{\frac{169-25}{169}}$$

$\cos\alpha = \frac{-12}{13}$ $\because \alpha$ lies in III quadrant

$$\sin\beta = \sqrt{1-\cos^2\beta}$$
$$= \sqrt{1-\left(\frac{-7}{25}\right)^2}$$
$$= \sqrt{1-\frac{49}{625}}$$
$$= \sqrt{\frac{625-49}{625}}$$
$$\sin\beta = \frac{-24}{25} \qquad \because \beta \text{ lies in III quadrant}$$
$$\therefore \sin(\alpha-\beta) = \left(\frac{-5}{13}\right)\left(\frac{-7}{25}\right)-\left(\frac{-12}{13}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{-53}{325}$$

35) If $sin\alpha = \frac{12}{13}$, $cos\beta = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$ find $cos(\alpha + \beta)$

[W-17]

Ans.

$$\because \cos^2\alpha = 1-\sin^2\alpha$$
$$= 1-\frac{144}{169} = \frac{25}{169}$$
$$\therefore \cos\alpha = \pm\frac{5}{13}$$
$$\therefore \cos\alpha = -\frac{5}{13} \quad \text{as } \frac{\pi}{2} < \alpha < \pi$$
$$\cos\beta = \frac{3}{5}$$
$$\therefore \sin^2\beta = 1-\cos^2\beta$$
$$= 1-\frac{9}{25} = \frac{16}{25}$$
$$\therefore \sin\beta = \pm\frac{4}{5}$$
$$\therefore \sin\beta = \frac{4}{5} \quad \text{as } 0 < \beta < \frac{\pi}{2}$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(-\frac{5}{13}\cdot\frac{3}{5}\right)-\left(\frac{12}{13}\cdot\frac{4}{5}\right)$$
$$= \frac{-63}{65}$$

TRIGONOMETRIC RATIOS OF MULTIPLE & SUB-MULTIPLE ANGLES

Multiple Angles:- If A is an angle then the angles 2A,3A,4A… are called multiple angles.

Sub-multiple Angles:- If A is an angle then the angles A/2 , A/3 , A/4 … are called sub-multiple angles.

Examples:-

1) If $sinA = 0.4$ find $sin3A$ **[W-14]**

Ans.

$$\begin{aligned}\sin 3A &= 3\sin A - 4\sin^3 A\\ &= 3(0.4) - 4(0.4)^3\\ &= 0.944\end{aligned}$$

2) If $sinA = \frac{1}{2}$ find $sin3A$ **[W-17,S-17,S-14]**

Ans.

$$\begin{aligned}\sin 3A &= 3\sin A - 4\sin^3 A\\ &= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3\\ &= 1\end{aligned}$$

3) If $cos\alpha = 0.4$ find $cos3\alpha$ **[W-13]**

Ans.

$$\begin{aligned}\cos 3A &= 4\cos^3 A - 3\cos A\\ &= 4(0.4)^3 - 3(0.4)\\ &= 0.256 - 1.2\\ &= -0.944\end{aligned}$$

4) If $tan\frac{A}{2} = \frac{1}{\sqrt{3}}$ find

$i)$ $sinA$ **[W-17,S-17,W-16]** $ii)$ $cosA$ **[SQP]**

Ans. $i)$

$$\tan\left(\frac{A}{2}\right) = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{A}{2} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\frac{A}{2} = 30^0$$

$$\therefore A = 60^0$$

$$\therefore \sin A = \sin\left(60^0\right) = \frac{\sqrt{3}}{2}$$

$ii)$

$$cosA = cos60^0 = \frac{1}{2}$$

5) If $tan\frac{A}{2} = \frac{2}{3}$ find $2sinA + 3cosA$ **[S-18]**

Ans. $2\sin\theta + 3\cos\theta$

$$= 2\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) + 3\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)$$

$$= 2\left(\frac{2\times\frac{2}{3}}{1+\left(\frac{2}{3}\right)^2}\right) + 3\left(\frac{1-\left(\frac{2}{3}\right)^2}{1+\left(\frac{2}{3}\right)^2}\right)$$

$$= 3$$

6) If $A = 30^0$ Verify that

$i)$ $sin2A = 2sinA.cosA$ **[W-17]** $ii)$ $cos2A = \frac{1-tan^2A}{1+tan^2A}$ **[W-17]**

Ans.

$$i) L.H.S. = \sin 2A$$
$$= \sin 2(30^0)$$
$$= \sin 60^0$$
$$= \frac{\sqrt{3}}{2}$$
$$R.H.S. = 2\sin A\cos A$$
$$= 2\sin 30^0 \cos 30^0$$
$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$
$$\therefore \sin 2A = 2\sin A\cos A$$

$$ii) L.H.S. = \cos 2A = \cos 2\left(30^0\right)$$
$$= \cos 60^0$$
$$= \frac{1}{2}$$
$$R.H.S. = \frac{1-\tan^2 A}{1+\tan^2 A}$$
$$= \frac{1-\tan^2 30^0}{1+\tan^2 30^0}$$
$$= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \frac{1}{2}$$
$$\therefore \cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

7) Prove that: $8cos^3\frac{\pi}{9} - 6cos\frac{\pi}{9} = 1$

Ans. $L.H.S. = 8cos^3\frac{\pi}{9} - 6cos\frac{\pi}{9}$

$= 2[4cos^3\frac{\pi}{9} - 3cos\frac{\pi}{9}]$

$= 2[cos3\frac{\pi}{9}]$

$= 2[cos\frac{\pi}{3}]$

$= 2[\frac{1}{2}]$

$= 1$

$= R.H.S.$

8) Prove that: $1 + \ tan\ \theta\ tan\ 2\theta\ = \ sec2\theta$ **[W-19,W-18,S-15]**

Ans.

$$1 + \tan A \cdot \tan 2A = 1 + \frac{\sin A}{\cos A} \times \frac{\sin 2A}{\cos 2A}$$

$$= \frac{\cos A \cos 2A + \sin A \sin 2A}{\cos A \cos 2A}$$

$$= \frac{\cos(A - 2A)}{\cos A \cos 2A}$$

$$= \frac{\cos(-A)}{\cos A \cos 2A}$$

$$= \frac{\cos A}{\cos A \cos 2A}$$

$$= \sec 2A$$

9) Prove that: $\frac{1-tan2\theta tan\theta}{1+tan2\theta tan\theta} = \frac{cos3\theta}{cos\theta}$ **[S-17,W-13]**

Ans.

$$L.H.S. = \frac{1 - \tan 2\theta \tan \theta}{1 + \tan 2\theta \tan \theta}$$

$$=\frac{1-\frac{\sin 2\theta}{\cos 2\theta}\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin 2\theta}{\cos 2\theta}\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}$$

$$=\frac{\cos(2\theta+\theta)}{\cos(2\theta-\theta)}$$

$$=\frac{\cos 3\theta}{\cos\theta}=R.H.S.$$

10) Prove that: $\frac{cotA-cot2A}{cotA+cot2A}=\frac{sinA}{sin3A}$

Ans. $L.H.S.=\frac{cotA-cot2A}{cotA+cot2A}$

$$=\frac{\frac{cosA}{sinA}-\frac{cos2A}{sin2A}}{\frac{cosA}{sinA}+\frac{cos2A}{sin2A}}$$

$$=\frac{\frac{cosA.sin2A-cos2A.sinA}{sinA.sin2A}}{\frac{cosA.sin2A+cos2A.sinA}{sinA.sin2A}}$$

$$=\frac{\sin A}{\sin 3A}$$

$$=R.H.S.$$

11) Prove that: $\frac{sin2\theta}{1+cos2\theta}=tan\theta$

Ans. $L.H.S.=\frac{sin2\theta}{1+cos2\theta}$

$$=\frac{2sin\theta cos\theta}{2cos^2\theta}$$

$$=\frac{sin\theta}{cos\theta}$$

$$=tan\theta$$

$$=R.H.S.$$

12) Prove that: $\frac{sin\theta}{1+cos\theta} = tan\frac{\theta}{2}$

Ans. $L.H.S. = \frac{sin\theta}{1+cos\theta}$

$$= \frac{2sin\frac{\theta}{2}cos\frac{\theta}{2}}{2cos^2\frac{\theta}{2}}$$

$$= \frac{sin\frac{\theta}{2}}{cos\frac{\theta}{2}}$$

$$= tan\frac{\theta}{2}$$

$$= R.H.S.$$

13) Prove that: $\frac{sin4\theta+sin2\theta}{1+cos2\theta+cos4\theta} = tan2\theta$ **[W-18,S-18,S-15,S-13,W-12]**

Ans. $$\frac{\sin 4\theta+\sin 2\theta}{1+\cos 2\theta+\cos 4\theta} = \frac{\sin 2(2\theta)+\sin 2\theta}{1+\cos 2(2\theta)+\cos 2\theta}$$

$$= \frac{2\sin 2\theta\cos 2\theta+\sin 2\theta}{2\cos^2 2\theta+\cos 2\theta}$$

$$= \frac{\sin 2\theta(2\cos 2\theta+1)}{\cos 2\theta(2\cos 2\theta+1)}$$

$$= \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \tan 2\theta$$

14) Prove that: $\frac{sin2\theta}{sin\theta} - \frac{cos2\theta}{cos\theta} = sec\theta$ **[W-17]**

Ans. Consider $\frac{\sin 2\theta}{\sin\theta}-\frac{\cos 2\theta}{\cos\theta}$

$$=\frac{\sin 2\theta.\cos\theta-\cos 2\theta.\sin\theta}{\sin\theta.\cos\theta}$$

$$=\frac{\sin(2\theta-\theta)}{\sin\theta.\cos\theta}$$

$$=\frac{\sin(\theta)}{\sin\theta.\cos\theta}$$

$$=\frac{1}{\cos\theta}$$

$$=\sec\theta$$

15) Prove that: $\frac{sin3\theta}{sin\theta}-\frac{cos3\theta}{cos\theta}=2$

Ans.

$$\frac{sin3\theta}{sin\theta}-\frac{cos3\theta}{cos\theta}=\frac{sin3\theta.cos\theta-cos3\theta.sin\theta}{sin\theta.cos\theta}$$

$$=\frac{sin(3\theta-\theta)}{sin\theta.cos\theta}$$

$$=\frac{sin(2\theta)}{sin\theta.cos\theta}$$

$$=\frac{2si\,n\,\theta.cos\theta}{sin\theta.cos\theta}$$

$$=2$$

16) Prove that: $\frac{sin9\theta}{sin3\theta}-\frac{cos9\theta}{cos3\theta}=2$

Ans.

$$\frac{sin9\theta}{sin3\theta}-\frac{cos9\theta}{cos3\theta}=\frac{sin9\theta.cos3\theta-cos9\theta.sin3\theta}{sin3\theta.cos3\theta}$$

$$=\frac{sin(9\theta-3\theta)}{sin3\theta.cos3\theta}$$

$$=\frac{sin(6\theta)}{sin3\theta.cos3\theta}$$

$$=\frac{si\,n\,2(3\theta)}{sin3\theta.cos3\theta}$$

$$=\frac{sin(3\theta)\,cos(3\theta)}{sin3\theta.cos3\theta}$$

$$=2$$

17) Prove that: $\frac{cos3\theta}{cos\theta}+\frac{sin3\theta}{sin\theta}=4cos2\theta$ **[S-19,W-14,SQP]**

Ans.

$$\frac{\cos 3\theta}{\cos\theta}+\frac{\sin 3\theta}{\sin\theta}=\frac{\sin\theta\cos 3\theta+\cos\theta\sin 3\theta}{\cos\theta\sin\theta}$$

$$=\frac{\sin(\theta+3\theta)}{\cos\theta\sin\theta}$$

$$=\frac{\sin 4\theta}{\cos\theta\sin\theta}$$

$$=\frac{\sin 2(2\theta)}{\cos\theta\sin\theta}$$

$$=\frac{2\sin 2\theta\cos 2\theta}{\cos\theta\sin\theta}$$

$$=\frac{2\cdot 2\sin\theta\cos\theta\cdot\cos 2\theta}{\cos\theta\sin\theta}$$

$$=4\cos 2\theta$$

18) Prove that: $\frac{1+sin2A+cos2A}{1+sin2A-cos2A}=cotA$ **[SQP]**

Ans.

$$\frac{1+sin2A+cos2A}{1+sin2A-cos2A}=\frac{1+cos2A+sin2A}{1-cos2A+sin2A}$$

$$=\frac{2cos^2A+sin2A}{2sin^2A+sin2A}$$

$$=\frac{2cos^2A+2sinA.cosA}{2sin^2A+2sinA.cosA}$$

$$=\frac{2cosA(cosA+sinA)}{2sinA(sinA+cosA)}$$

$$=\frac{cosA}{sinA}$$

$$=cotA$$

19) Prove that: $\frac{1+sin\theta-cos\theta}{1+sin\theta+cos\theta}=tan\frac{\theta}{2}$ **[S-19]**

Ans.

$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$$

$$=\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}$$

$$=\frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}$$

$$=\frac{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}$$

$$=\tan\left(\frac{\theta}{2}\right)$$

20) Prove that: $sinA.\,sin(60^0-A)\,.\,sin(60^0+A)=\frac{sin3A}{4}$ **[W-18,W-12]**

Ans.

$$\sin A\sin(60-A)\sin(60+A)=\sin A\left(\sin^2 60-\sin^2 A\right)$$

$$=\sin A\left(\frac{3}{4}-\sin^2 A\right)$$

$$=\frac{1}{4}\sin A\left[3-4\sin^2 A\right]$$

$$=\frac{1}{4}\left[3\sin A-4\sin^3 A\right]$$

$$=\frac{1}{4}\sin 3A$$

21) Prove that: $cosA.\,cos(60^0-A)\,.\,\cos(60^0+A)=\frac{cos3A}{4}$

Ans. $L.H.S.=cosA.\,cos(60^0-A)\,.\,\cos(60^0+A)$

$$=cosA.\,[\cos 60^0.\,cosA+sin60^0.\,sinA]\,.\,[\cos60^0.\,cosA-sin60^0.\,sinA]$$

$$=cosA.\,[\frac{1}{2}.\,cosA+\frac{\sqrt{3}}{2}.\,sinA]\,.\,[\frac{1}{2}.\,cosA-\frac{\sqrt{3}}{2}.\,sinA]$$

$$= cosA.\{[\frac{1}{2}.cosA]^2 - [\frac{\sqrt{3}}{2}.sinA]^2\}$$

$$= cosA.[\frac{1}{4}.cos^2A - \frac{3}{4}sin^2A]$$

$$= cosA.\frac{1}{4}[cos^2A - 3sin^2A]$$

$$= cosA.\frac{1}{4}[cos^2A - 3(1 - cos^2A)]$$

$$= cosA.\frac{1}{4}[cos^2A - 3 + 3cos^2A]$$

$$= cosA.\frac{1}{4}[4cos^2A - 3]$$

$$= \frac{1}{4}[4cos^3A - 3cosA]$$

$$= \frac{1}{4}cos3A$$

$$= R.H.S.$$

22) Prove that: $tanA.tan(60^0 - A).tan(60^0 + A) = tan3A$

[W-19,S-17,W-15]

Ans. $\tan A \tan\left(60^0 - A\right)\tan\left(60^0 + A\right)$

$$= \tan A\left(\frac{\tan 60^0 - \tan A}{1 + \tan 60^0 \tan A}\right)\left(\frac{\tan 60^0 + \tan A}{1 - \tan 60^0 \tan A}\right)$$

$$= \tan A\left(\frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}\right)\left(\frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A}\right)$$

$$= \tan A\left(\frac{3 - \tan^2 A}{1 - 3\tan^2 A}\right)$$

$$=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$$
$$=\tan 3A$$

23) Prove that: $\sqrt{2+\sqrt{2+2cos4\theta}} = 2\cos\theta$ **[S-14]**

Ans.

$$\sqrt{2+\sqrt{2+2\cos 4\theta}} =\sqrt{2+\sqrt{2\left(1+\cos 4\theta\right)}}$$
$$=\sqrt{2+\sqrt{2\left(2\cos^2 2\theta\right)}}$$
$$=\sqrt{2+\sqrt{4\cos^2 2\theta}}$$
$$=\sqrt{2+2\cos 2\theta}$$
$$=\sqrt{2\left(1+\cos 2\theta\right)}$$
$$=\sqrt{2\left(2\cos^2\theta\right)}$$
$$=\sqrt{4\cos^2\theta}$$
$$=2\cos\theta$$

24) Prove that: $\sqrt{2+\sqrt{2+\sqrt{2+2cos8\theta}}} = 2\cos\theta$ **[W-18]**

Ans.

$$LHS=\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8A}}}$$
$$=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8A)}}}$$
$$=\sqrt{2+\sqrt{2+\sqrt{2\cdot 2\cos^2 4A}}}$$
$$=\sqrt{2+\sqrt{2+2\cos 4A}}$$
$$=\sqrt{2+\sqrt{2(1+\cos 4A)}}$$

$$= \sqrt{2+\sqrt{2\cdot 2\cos^2 2A}}$$
$$= \sqrt{2+2\cdot\cos 2A}$$
$$= \sqrt{2(1+\cos 2A)}$$
$$= \sqrt{2\cdot 2\cos^2 A}$$
$$= 2\cos A$$
$$= R.H.S$$

25) Prove that: $\frac{sec4\theta-1}{sec2\theta-1}=\frac{tan4\theta}{tan\theta}$ **[W-15,W-13]**

Ans.

$$\frac{\sec 4\theta-1}{\sec 2\theta-1}$$
$$= \frac{\frac{1}{\cos 4\theta}-1}{\frac{1}{\cos 2\theta}-1}$$
$$= \frac{\cos 2\theta(1-\cos 4\theta)}{\cos 4\theta(1-\cos 2\theta)}$$
$$= \frac{2\cos 2\theta\sin^2 2\theta}{2\cos 4\theta\sin^2\theta}$$
$$= \frac{\sin 4\theta\sin 2\theta}{2\cos 4\theta\sin^2\theta}$$
$$= \frac{2\tan 4\theta\sin\theta\cos\theta}{2\sin^2\theta}$$
$$= \tan 4\theta\cot\theta$$
$$= \frac{\tan 4\theta}{\tan\theta}$$

26) Prove that: $\frac{sec8\theta-1}{sec4\theta-1}=\frac{tan8\theta}{tan2\theta}$

OR

Prove that: $\frac{sec8A-1}{sec4A-1}=\frac{cot2A}{cot8A}$ **[W-17]**

Ans.

$$L.H.S.=\frac{\sec 8A-1}{\sec 4A-1}=\frac{\frac{1}{\cos 8A}-1}{\frac{1}{\cos 4A}-1}$$

$$=\frac{(1-\cos 8A)\cos 4A}{(1-\cos 4A)\cos 8A}$$

$$=\frac{2\sin^2 4A\cos 4A}{2\sin^2 2A\cos 8A}$$

$$=\frac{\sin 4A\sin 4A\cos 4A}{\sin 2A\sin 2A\cos 8A}$$

$$=\frac{2\sin 2A\cos 2A\sin 4A\cos 4A}{\sin 2A\sin 2A\cos 8A}$$

$$=\frac{\cos 2A\sin 8A}{\sin 2A\cos 8A}$$

$$=\cot 2A\times\tan 8A$$

$$=\frac{\cot 2A}{\cot 8A}$$

TUTORIAL NO.6

TRIGONOMETRIC RATIOS OF ALLIED , COMPOUND , MULTIPLE & SUB-MULTIPLE ANGLES.

1) Without using calculator find the value of $sin(150^0)+cos(300^0)-tan(315^0)+\sec^2(3660^0)$ **[W-17,S-17]**

2) Prove that $sin\left(\theta+\frac{\pi}{6}\right)-sin\left(\theta-\frac{\pi}{6}\right)=\cos\theta$ **[W-15,W-12]**

3) If $tanA=\frac{1}{2}$, $tanB=\frac{1}{3}$ find $tan(A+B)$ **[W-18,S-16]**

4) If $A+B=\frac{\pi}{4}$ Show that $(1+tanA)(1+tanB)=2$ **[S-19,S-16]**

5) Given that $tan(A+B)=\frac{3}{4}$, $tan(A-B)=\frac{77}{36}$ find

i) $tan\ 2A$ **[S-19,S-16]** ***ii***) $tan\ 2B$ **[S-19]**

6) In any ΔABC show that $tanA+tanB+tanC=tanA.tanB.tanC$

[S-19,S-18,W-14]

7) Prove that: $\frac{1-tan2\theta tan\theta}{1+tan2\theta tan\theta}=\frac{cos3\theta}{cos\theta}$ **[S-17,W-13]**

8) Prove that: $\frac{sin4\theta+sin2\theta}{1+cos2\theta+cos4\theta}=tan2\theta$ **[W-18,S-18,S-15,S-13,W-12]**

9) Prove that: $\frac{cos3\theta}{cos\theta}+\frac{sin3\theta}{sin\theta}=4cos2\theta$ **[S-19,W-14,SQP]**

10) Prove that: $sinA.sin(60^0-A).sin(60^0+A)=\frac{sin3A}{4}$ **[W-18,W-12]**

FACTORIZATION & DE-FACTORIZATION FORMULAE

Position in Question Paper **Total Marks-12**

Q.3. b) 4-Marks.

Q.3. c) 4-Marks.

Q.4. d) 4-Marks.

De- factorization: - To express product as sum or difference.

Factorization: - To express sum or difference as product.

Examples:-

Que. **Express as product:**

1) $\cos 4\theta + \cos 8\theta$ **[S-17]**

Ans.

$$\cos 4\theta + \cos 8\theta$$

$$= 2\cos\left(\frac{4\theta + 8\theta}{2}\right).\cos\left(\frac{4\theta - 8\theta}{2}\right)$$

$$= 2\cos(6\theta).\cos(-2\theta)$$

$$= 2\cos(6\theta).\cos(2\theta)$$

2) $\sin 5\theta + \sin 7\theta$

Ans.

$$sin\ 5\theta + sin\ 7\theta = 2\,sin\left(\frac{5\theta + 7\theta}{2}\right).cos(\frac{5\theta - 7\theta}{2})$$

$$sin\ 5\theta + sin\ 7\theta = 2\,sin\left(\frac{12\theta}{2}\right).cos(\frac{-2\theta}{2})$$

$$sin\ 5\theta + sin\ 7\theta = 2\,sin(6\theta).cos(-\theta)$$

$$sin\ 5\theta + sin\ 7\theta = 2\,sin(6\theta).cos(\theta)$$

3) $sin\ 7\theta + sin\ 3\theta$

Ans.

$$sin\ 7\theta + sin3\theta = 2\,sin\left(\frac{7\theta + 3\theta}{2}\right).cos(\frac{7\theta - 3\theta}{2})$$

$$sin\ 7\theta + sin3\theta = 2\,sin\left(\frac{10\theta}{2}\right).cos(\frac{4\theta}{2})$$

$$sin\ 7\theta + sin3\theta = 2\,sin(5\theta).cos(2\theta)$$

4) $sin99 - sin81$

Ans.

$$sin99 + sin81 = 2\,cos\left(\frac{99 + 81}{2}\right).sin(\frac{99 - 81}{2})$$

$$sin99 + sin81 = 2\,cos\left(\frac{180}{2}\right).sin(\frac{18}{2})$$

$$sin99 + sin81 = 2cos90.sin18$$

$$sin99 + sin81 = 2.0.sin18$$

$$sin99 + sin81 = 0$$

5) $cos\frac{5\pi}{11} + cos\frac{7\pi}{11}$

Ans.

$$cos\frac{5\pi}{11} + cos\frac{7\pi}{11} = 2\,cos\left(\frac{5\pi + 7\pi}{2}\right).cos(\frac{5\pi - 7\pi}{2})$$

$$cos\frac{5\pi}{11} + cos\frac{7\pi}{11} = 2\,cos\left(\frac{12\pi}{2}\right).cos(\frac{-2\pi}{2})$$

$$cos\frac{5\pi}{11} + cos\frac{7\pi}{11} = 2\,cos(6\pi)\,.\,cos(-\pi)$$

$$cos\frac{5\pi}{11} + cos\frac{7\pi}{11} = 2\,cos(6\pi)\,.\,cos(\pi)$$

6) $cos\frac{3\pi}{7} + cos\frac{2\pi}{7}$

Ans.

$$cos\frac{3\pi}{7} + cos\frac{2\pi}{7} = 2\,cos\left(\frac{\frac{3\pi}{7}+\frac{2\pi}{7}}{2}\right).cos(\frac{\frac{3\pi}{7}-\frac{2\pi}{7}}{2})$$

$$cos\frac{3\pi}{7} + cos\frac{2\pi}{7} = 2\,cos\left(\frac{5\pi}{14}\right).cos(\frac{\pi}{14})$$

7) $cos\frac{7\pi}{13} + cos\frac{6\pi}{13}$

Ans.

$$cos\frac{7\pi}{13} + cos\frac{6\pi}{13} = 2\,cos\left(\frac{\frac{7\pi}{13}+\frac{6\pi}{13}}{2}\right).cos(\frac{\frac{7\pi}{13}-\frac{6\pi}{13}}{2})$$

$$cos\frac{7\pi}{13} + cos\frac{6\pi}{13} = 2\,cos\left(\frac{13\pi}{26}\right).cos(\frac{\pi}{26})$$

$$cos\frac{7\pi}{13} + cos\frac{6\pi}{13} = 2\,cos\left(\frac{\pi}{2}\right).cos(\frac{\pi}{26})$$

$$cos\frac{7\pi}{13} + cos\frac{6\pi}{13} = 0.\,cos(\frac{\pi}{26})$$

$$cos\frac{7\pi}{13} + cos\frac{6\pi}{13} = 0$$

Que. **Express as sum or difference:**

1) $2sin\,4\theta.cos\,2\theta$

Ans. $2sin\,4\theta.cos\,2\theta = sin(4\theta + 2\theta) + sin(4\theta - 2\theta)$

$2sin\,4\theta.cos\,2\theta = sin(6\theta) + sin(2\theta)$

2) $2sin\,15.cos15$

Ans. $2sin\,15.cos15 = sin(15 + 15).cos(15 - 15)$

$2sin\,15.cos15 = sin(30).cos(0)$

$2sin\,15.cos15 = \frac{1}{2}.1$

$2sin\,15.cos15 = \frac{1}{2}$

3) $2cos\,75.cos15$ **[S-18,W-17,S-14]**

Ans. $2\cos 75^{o}\cos 15^{o} = \cos(75^{o}+15^{o}) + \cos(75^{o}-15^{o})$

$= \cos(90^{o}) + \cos(60^{o})$

$= 0 + \frac{1}{2}$

$= \frac{1}{2} \quad or \quad 0.5$

4) $4cos30.sin\,20$

Ans. $4cos30.sin\ 20 = 2.2cos30.sin20$

$4cos30.sin\ 20 = 2.[sin(30+20) - sin(30-10)]$

$4cos30.sin\ 20 = 2.[sin50 - sin20]$

$4cos30.sin\ 20 = 2.sin50 - 2sin20$

5) $4sin\ 50.sin\ 20$

Ans. $4sin\ 50.sin\ 20 = 2.2sin50.sin20$

$4sin\ 50.sin\ 20 = 2.[2cos(50-20) - cos(50+20)]$

$4sin\ 50.sin\ 20 = 2.[2cos(30) - cos(70)]$

$4sin\ 50.sin\ 20 = 2.[2.\frac{1}{2} - cos(70)]$

$4sin\ 50.sin\ 20 = 2.[1 - cos(70)]$

$4sin\ 50.sin\ 20 = 2 - 2.cos70$

6) $cos\frac{3\pi}{9}.cos\frac{7\pi}{9}$

Ans. $cos\frac{3\pi}{9}.cos\frac{7\pi}{9} = cos\left(\frac{3\pi}{9} + \frac{7\pi}{9}\right) + cos(\frac{3\pi}{9} - \frac{7\pi}{9})$

$cos\frac{3\pi}{9}.cos\frac{7\pi}{9} = cos\left(\frac{10\pi}{9}\right) + cos(\frac{-4\pi}{9})$

Que. **Find A and B**

1) $2sin\ 40.cos\ 10 = \ sin\ A + sin\ B$ **[SQP]**

Ans. $2sin\ 40.cos\ 10 = \ sin\ A + sin\ B$

$sin(40 + 10) + sin(40 - 10) = \ sin\ A + sin\ B$

$sin(50) + sin(30) = \ sin\ A + sin\ B$

$\therefore A = 50 \quad , \quad B = 30$

2) $2\ cos\ 70\ sin\ 50 = sinA - sinB$ **[W-18,W-12]**

Ans. $2\cos 70^{o} \sin 50^{o} = \sin A - \sin B$

$\therefore \sin\left(70^{o} + 50^{o}\right) - \sin\left(70^{o} - 50^{o}\right) = \sin A - \sin B$

$\therefore \sin\left(120^{o}\right) - \sin\left(20^{o}\right) = \sin A - \sin B$

$\therefore A = 120^{o}$

$B = 20^{o}$

3) $2\ cos\ 60.cos10 = cosA + cosB$ **[S-13]**

Ans. $\cos 60^{0}.\cos 10^{0} = \cos A + \cos B$

$\cos\left(60^{0} + 10^{0}\right) + \cos\left(60^{0} - 10^{0}\right) = \cos A + \cos B$

$\cos 70^{0} + \cos 50^{0} = \cos A + \cos B$

$\therefore A = 70^{0}$ and $B = 50^{0}$

4) $sin80 + sin50 = 2sinA.cosB$ **[S-18]**

Ans. $sin80 + sin50 = 2sinA.cosB$

$$2\,sin\left(\frac{80+50}{2}\right).sin\left(\frac{80-50}{2}\right) = 2sinA.cosB$$

$$2\,sin\left(\frac{130}{2}\right).sin\left(\frac{30}{2}\right) = 2sinA.cosB$$

$$2\,sin(65).sin(15) = 2sinA.cosB$$

$$A = 65 \quad , \quad B = 15$$

5) $sin40 - cos70 = \sqrt{3}\,cosA$

Ans. $sin40 - cos70 = \sqrt{3}\,cosA$

$$sin40 - cos(90 - 20) = \sqrt{3}\,cosA$$

$$sin40 - sin\,20 = \sqrt{3}\,cosA$$

$$2\,cos\left(\frac{40+20}{2}\right).sin(\frac{40-20}{2}) = \sqrt{3}\,cosA$$

$$2\,cos\left(\frac{60}{2}\right).sin(\frac{20}{2}) = \sqrt{3}\,cosA$$

$$2\,cos(30).sin(10) = \sqrt{3}\,cosA$$

$$2\frac{\sqrt{3}}{2}.sin(10) = \sqrt{3}\,cosA$$

$$\sqrt{3}.sin(10) = \sqrt{3}\,cosA$$

$$\sqrt{3}.sin(90 - 80) = \sqrt{3}\,cosA$$

$$\sqrt{3}.cos80 = \sqrt{3}\,cosA$$

$$A = 80$$

1)	Find the exact value of $sin(52.5)^0.cos(7.5)^0$

Ans.

$$sin(52.5)^0.cos(7.5)^0 = \frac{2}{2}[sin(52.5)^0.cos(7.5)^0]$$

$$sin(52.5)^0.cos(7.5)^0 = \frac{1}{2}[2.sin(52.5)^0.cos(7.5)^0]$$

$$sin(52.5)^0.cos(7.5)^0 = \frac{1}{2}[sin(52.5+7.5)+sin(52.5-7.5)]$$

$$sin(52.5)^0.cos(7.5)^0 = \frac{1}{2}[sin(60)+sin(45)]$$

$$sin(52.5)^0.cos(7.5)^0 = \frac{1}{2}[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}]$$

$$sin(52.5)^0.cos(7.5)^0 = \frac{\sqrt{3}}{4}+\frac{1}{2\sqrt{2}}$$

2)	Show that $cos59^0 + sin59^0 = \sqrt{2}.cos14^0$ **[W-17]**

Ans.

$$\text{L.H.S.} = \cos 59^0 + \sin 59^0$$

$$= \cos 59^0 + \cos 31^0$$

$$= 2\cos\left(\frac{59^0+31^0}{2}\right)\cdot\cos\left(\frac{59^0-31^0}{2}\right)$$

$$= 2\cos 45^0\cdot\cos 14^0$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)\cdot\cos 14^0$$

$$= \sqrt{2}\cos 14^0$$

$$= R.H.S.$$

3) Prove that: $\frac{sinA-sin3A}{sin^2A-cos^2A} = 2sinA$ **[W-12]**

Ans.

$$\frac{\sin A-\sin 3A}{\sin^2 A-\cos^2 A} = \frac{2\cos 2A\sin(-A)}{\sin^2 A-\cos^2 A}$$

$$= \frac{-2\cos 2A\sin A}{-\left(\cos^2 A-\sin^2 A\right)}$$

$$= \frac{-2\cos 2A\sin A}{-\cos 2A}$$

$$= 2\sin A$$

4) Prove that: $\frac{sin3A-sinA}{cos3A+cosA} = tanA$ **[S-18]**

Ans.

$$\text{LHS} = \frac{\sin 3A-\sin A}{\cos 3A+\cos A}$$

$$= \frac{2.\cos\left(\frac{3A+A}{2}\right).\sin\left(\frac{3A-A}{2}\right)}{2.\cos\left(\frac{3A+A}{2}\right).\cos\left(\frac{3A-A}{2}\right)}$$

$$= \frac{2\cos 2A.\sin A}{2\cos 2A.\cos A}$$

$= \tan A$

=RHS

5) Prove that: $\frac{cos3A+cosA}{sin3A-sinA} = cotA$

Ans.

$$\frac{cos3A + cosA}{sin3A - sinA} = \frac{2\,cos\left(\frac{3A+A}{2}\right).cos(\frac{3A-A}{2})}{2\,cos\left(\frac{3A+A}{2}\right).sin(\frac{3A-A}{2})}$$

$$\frac{cos3A + cosA}{sin3A - sinA} = \frac{2\,cos\left(\frac{4A}{2}\right).cos(\frac{2A}{2})}{2\,cos\left(\frac{4A}{2}\right).sin(\frac{2A}{2})}$$

$$\frac{cos3A + cosA}{sin3A - sinA} = \frac{2\,cos(2A).cos(A)}{2\,cos(2A).sin(A)}$$

$$\frac{cos3A + cosA}{sin3A - sinA} = \frac{cos(A)}{sin(A)}$$

$$\frac{cos3A + cosA}{sin3A - sinA} = cotA$$

6) Prove that: $\frac{sin8A+sin2A}{cos8A\ +cos2A} = tan5A$

Ans.

$$\frac{sin8A + sin2A}{cos8A\ + cos2A} = \frac{2\,sin\left(\frac{8A+2A}{2}\right).cos(\frac{8A-2A}{2})}{2\,cos\left(\frac{8A+2A}{2}\right).cos(\frac{8A-2A}{2})}$$

$$\frac{sin8A + sin2A}{cos8A\ + cos2A} = \frac{2\,sin\left(\frac{10A}{2}\right).cos(\frac{6A}{2})}{2\,cos\left(\frac{10A}{2}\right).cos(\frac{6A}{2})}$$

$$\frac{sin8A + sin2A}{cos8A\ + cos2A} = \frac{2\,sin(5A).cos(3A)}{2\,cos(5A).cos(3A)}$$

$$\frac{sin8A + sin2A}{cos8A\ + cos2A} = tan5A$$

7) Prove that: $\frac{sin7x+sinx}{cos5x\ -cos3x} = sin2x - cos2x.cotx$ **[S-14,W-13,SQP]**

Ans.

$$\frac{\sin 7x+\sin x}{\cos 5x-\cos 3x}=\frac{2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)}{-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin(4x)\cos(3x)}{-2\sin(4x)\sin(x)}$$

$$=\frac{\cos(3x)}{-\sin x}$$

$$=\frac{\cos(x+2x)}{-\sin x}$$

$$=\frac{\cos x\cos 2x-\sin x\sin 2x}{-\sin x}$$

$$=\frac{\cos x\cos 2x}{-\sin x}-\frac{\sin x\sin 2x}{-\sin x}$$

$$=-\cot x\cos 2x+\sin 2x$$

$$OR\ \ \sin 2x-\cot x\cos 2x$$

8) Prove that: $\frac{cos3A-cos7A}{sin9A+sinA} = cos2Atan4A - sin2A$

Ans.

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{-2\,sin\left(\frac{3A+7A}{2}\right).sin(\frac{3A-7A}{2})}{2\,sin\left(\frac{9A+A}{2}\right).cos(\frac{9A-A}{2})}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{-2\,sin\left(\frac{10A}{2}\right).sin(\frac{-4A}{2})}{2\,sin\left(\frac{10A}{2}\right).cos(\frac{8A}{2})}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{-sin(\frac{-4A}{2})}{cos(\frac{8A}{2})}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{-sin(-2A)}{cos(4A)}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{-- sin(2A)}{cos(4A)}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{sin(2A)}{cos(4A)}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{sin(4A - 2A)}{cos\,4A}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{sin4A.cos2A - cos4A.sin2A}{cos\,4A}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = \frac{sin4A.cos2A}{cos\,4A} - \frac{cos4A.sin2A}{cos\,4A}$$

$$\frac{cos3A - cos7A}{sin9A + sinA} = tan4A.cos2A - sin2A$$

9) Prove that: $\frac{sin8\theta - sin5\theta}{cos7\theta + cos6\theta} = sin\theta + cos\theta.tan\frac{\theta}{2}$ **[S-15,S-13]**

Ans.

$$\frac{sin8x - sin5x}{cos7x + cos6x} = sinx + cosx.tan\frac{x}{2}$$

$$= \frac{2cos\frac{8x+5x}{2}.sin\frac{8x-5x}{2}}{2cos\frac{7x+6x}{2}.cos\frac{7x-6x}{2}}$$

$$\frac{2cos\frac{13x}{2}.sin\frac{3x}{2}}{2cos\frac{13x}{2}.cos\frac{x}{2}}$$

$$= \frac{sin\frac{3x}{2}}{cos\frac{x}{2}}$$

$$=\frac{sin\left(x+\frac{x}{2}\right)}{cos\frac{x}{2}}$$

$$=\frac{sinxcos\frac{x}{2}+cosxsin\frac{x}{2}}{cos\frac{x}{2}}$$

$$=sinx+cosx.tan\frac{x}{2}$$

10) Prove that: $\frac{sin4A+sin5A+sin6A}{cos4A+cos5A+cos6A}=tan5A$

[W-19,S-19,W-18, W-17,S-17,S-16, S-15,W-14,SQP]

Ans.

$$\frac{\sin 4A+\sin 5A+\sin 6A}{\cos 4A+\cos 5A+\cos 6A}=\frac{\sin 4A+\sin 6A+\sin 5A}{\cos 4A+\cos 6A+\cos 5A}$$

$$=\frac{2\sin 5A\cos(-A)+\sin 5A}{2\cos 5A\cos(-A)+\cos 5A}$$

$$=\frac{\sin 5A[2\cos(-A)+1]}{\cos 5A[2\cos(-A)+1]}$$

$$=\tan 5A$$

11) Prove that: $\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=tan3A$

Ans. $\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=\frac{sinA+sin5A+sin3A}{cosA+cos5A+cos3A}$

$$\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=\frac{2\,sin\left(\frac{A+5A}{2}\right).cos(\frac{A-5A}{2})+sin3A}{2\,cos\left(\frac{A+5A}{2}\right).cos(\frac{A-5A}{2})+cos3A}$$

$$\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=\frac{2\,sin\left(\frac{6A}{2}\right).cos(\frac{-4A}{2})+sin3A}{2\,cos\left(\frac{6A}{2}\right).cos(\frac{-4A}{2})\,+cos3A}$$

$$\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=\frac{2\,sin(3A).cos(-2A)+sin3A}{2\,cos(3A).cos(-2A)\,+cos3A}$$

$$\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=\frac{sin(3A)[2cos(-2A)+1]}{cos(3A)[2cos(-2A)\,+1]}$$

$$\frac{sinA+sin3A+sin5A}{cosA+cos3A+cos5A}=tan3A$$

12) Prove that: $\frac{sinx+sin5x+sin9x}{cosx+cos5x+cos9x}=tan5x$

Ans.

$$\frac{sinx+sin5x+sin9x}{cosx+cos5x+cos9x}=\frac{sinx+sin9x+sin5x}{cosx+cos9x+cos5x}$$

$$\frac{sinx+sin9x+sin5x}{cosx+cos9x+cos5x}=\frac{2\,sin\left(\frac{x+9x}{2}\right).cos(\frac{x-9x}{2})+sin5x}{2\,cos\left(\frac{x+9x}{2}\right).cos(\frac{x-9x}{2})\,+cos5x}$$

$$\frac{sinx+sin5x+sin9x}{cosx+cos5x+cos9x}=\frac{2\,sin\left(\frac{10x}{2}\right).cos(\frac{-8x}{2})+sin5x}{2\,cos\left(\frac{10x}{2}\right).cos(\frac{-8x}{2})\,+cos9x}$$

$$\frac{sinx+sin5x+sin9x}{cosx+cos5x+cos9x}=\frac{2\,sin(5x).cos(-4x)+sin5x}{2\,cos(5x).cos(-4x)\,+cos5x}$$

$$\frac{sinx+sin5x+sin9x}{cosx+cos5x+cos9x}=\frac{sin(5x)[2cos(-4x+1]}{cos(5x)[2cos(-4x+1]}$$

$$\frac{sinx+sin5x+sin9x}{cosx+cos5x+cos9x}=tan5x$$

13) Prove that: $\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{sin8A}{sin11A}$

Ans.

$$\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{sin5A+sin11A\,+2sin8A}{sin8A+sin14A+2sin11A}$$

$$\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{2\,sin\left(\frac{5A+11A}{2}\right).cos(\frac{5A-11A}{2})\,+2sin8A}{2\,sin\left(\frac{8A+14A}{2}\right).cos(\frac{8A-14A}{2})+2sin11A}$$

$$\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{2\,sin\left(\frac{16A}{2}\right).cos(\frac{-6A}{2})\ +2sin8A}{2\,sin\left(\frac{22A}{2}\right).cos(\frac{-6A}{2})+2sin11A}$$

$$\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{2\,sin(8A).cos(-3A)\ +2sin8A}{2\,sin(11A).cos(-3A)+2sin11A}$$

$$\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{2\,sin(8A)[cos(-3A)\ +1]}{2\,sin(11A)[cos(-3A)+1]}$$

$$\frac{sin5A+2sin8A+sin11A}{sin8A+2sin11A+sin14A}=\frac{sin(8A)}{sin(11A)}$$

14) Prove that: $\frac{\text{sinA+2sin2A+sin3A}}{\text{cosA+2cos2A+cos3A}}=\text{tan2A}$ **[S-14]**

Ans.

$$\frac{\sin A+2\sin 2A+\sin 3A}{\cos A+2\cos 2A+\cos 3A}=\frac{\sin A+\sin 3A+2\sin 2A}{\cos A+\cos 3A+2\cos 2A}$$

$$=\frac{2\sin 2A\cos(-A)+2\sin 2A}{2\cos 2A\cos(-A)+2\cos 2A}$$

$$=\frac{\sin 2A\left[2\cos(-A)+2\right]}{\cos 2A\left[2\cos(-A)+2\right]}$$

$$=\frac{\sin 2A\left[2\cos(-A)+2\right]}{\cos 2A\left[2\cos(-A)+2\right]}$$

$$=\frac{\sin 2A}{\cos 2A}$$

$$=\tan 2A$$

15) Prove that: $\frac{cos3A+\ 2\ cos5A\ +\ cos7A}{cosA\ +\ \ 2cos3A\ +\ cos5A}=\ cos2A-sin2A.tan3A$ **[W-14]**

Ans.

$$\frac{\cos 3A+2\cos 5A+\cos 7A}{\cos A+2\cos 3A+\cos 5A}=\frac{\cos 3A+\cos 7A+2\cos 5A}{\cos A+\cos 5A+2\cos 3A}$$

$$=\frac{2\cos 5A\cos(-2A)+2\cos 5A}{2\cos 3A\cos(-2A)+2\cos 3A}$$

$$=\frac{\cos 5A[2\cos(-2A)+2]}{\cos 3A[2\cos(-2A)+2]}$$

$$=\frac{\cos 5A}{\cos 3A}$$

$$=\frac{\cos(2A+3A)}{\cos 3A}$$

$$=\frac{\cos 2A\cos 3A-\sin 2A\sin 3A}{\cos 3A}$$

$$=\cos 2A-\sin 2A\tan 3A$$

16) Prove that: $\frac{cos2A+2\,cos4A+cos6A}{cosA+2cos3A+cos5A}=cosA-tan3A.sinA$

[S-19,S-18,W-13]

Ans.

$$\text{LHS}=\frac{\cos 2A+2\cos 4A+\cos 6A}{\cos A+2\cos 3A+\cos 5A}$$

$$=\frac{\cos 2A+\cos 6A+2\cos 4A}{\cos A+\cos 5A+2\cos 3A}$$

$$=\frac{2.\cos\left(\frac{2A+6A}{2}\right).\cos\left(\frac{2A-6A}{2}\right)+2\cos 4A}{2.\cos\left(\frac{A+5A}{2}\right).\cos\left(\frac{A-5A}{2}\right)+2\cos 3A}$$

$$=\frac{2\cos 4A.\cos(-2A)+2\cos 4A}{2\cos 3A.\cos(-2A)+2\cos 3A}$$

$$=\frac{2\cos 4A\left[\cos(-2A)+1\right]}{2\cos 3A\left[\cos(-2A)+1\right]}$$

$$=\frac{\cos(3A+A)}{\cos 3A}$$

$$=\frac{\cos 3A.\cos A}{\cos 3A}-\frac{\sin 3A.\sin A}{\cos 3A}$$

$$=\cos A-\tan 3A.\sin A=\text{ RHS}$$

17) Prove that: $\frac{sinA+sin2A+sin3A+sin4A}{cosA+cos2A+cos3A+cos4A}=tan(\frac{5A}{2})$ **[S-19,W-16,W-12]**

Ans.

$$\frac{\sin A+\sin 2A+\sin 3A+\sin 4A}{\cos A+\cos 2A+\cos 3A+\cos 4A}$$

$$=\frac{(\sin A+\sin 4A)+(\sin 2A+\sin 3A)}{(\cos A+\cos 4A)+(\cos 2A+\cos 3A)}$$

$$=\frac{2\sin\left(\frac{5A}{2}\right)\cos\left(\frac{-3A}{2}\right)+2\sin\left(\frac{5A}{2}\right)\cos\left(\frac{-A}{2}\right)}{2\cos\left(\frac{5A}{2}\right)\cos\left(\frac{-3A}{2}\right)+2\cos\left(\frac{5A}{2}\right)\cos\left(\frac{-A}{2}\right)}$$

$$=\frac{2\sin\left(\frac{5A}{2}\right)\left[\cos\left(\frac{-3A}{2}\right)+\cos\left(\frac{-A}{2}\right)\right]}{2\cos\left(\frac{5A}{2}\right)\left[\cos\left(\frac{-3A}{2}\right)+\cos\left(\frac{-A}{2}\right)\right]}$$

$$=\frac{\sin\left(\frac{5A}{2}\right)}{\cos\left(\frac{5A}{2}\right)}$$

$$=\tan\left(\frac{5A}{2}\right)$$

18) Prove that: $\frac{sinx-sin5x+sin9x-sin13x}{cosx-cos5x-cos9x+cos13x}=cot4x$ **[S-18,W-15]**

Ans.

$$LHS=\frac{\sin x-\sin 5x+\sin 9x-\sin 13x}{\cos x-\cos 5x-\cos 9x+\cos 13x}$$

$$\text{LHS}=\frac{(\sin x+\sin 9x)-(\sin 5x+\sin 13x)}{(\cos x-\cos 9x)-(\cos 5x-\cos 13x)}$$

$$=\frac{2.\sin\left(\frac{x+9x}{2}\right).\cos\left(\frac{x-9x}{2}\right)-2.\sin\left(\frac{5x+13x}{2}\right).\cos\left(\frac{5x-13x}{2}\right)}{2.\sin\left(\frac{x+9x}{2}\right).\sin\left(\frac{9x-x}{2}\right)-2.\sin\left(\frac{5x+13x}{2}\right).\sin\left(\frac{13x-5x}{2}\right)}$$

$$=\frac{\sin 5x.\cos(-4x)-\sin 9x.\cos(-4x)}{\sin 5x.\sin 4x-\sin 9x.\sin 4x}$$

$$=\frac{\cos(-4x)[\sin 5x-\sin 9x]}{\sin 4\text{x}[\sin 5x-\sin 9x]}$$

$$=\frac{\cos 4x}{\sin 4x}$$

$$=\cot 4x=\text{RHS}$$

19) Prove that: $\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=tan8x$

Ans.

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{2sin11x.sinx+2sin7x.sin3x}{2cos11x.sinx+2cos7x.sin3x}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{[cos(11x-x)-cos(11x+x)]+[cos(7x-3x)-cos(7x+3x)]}{[sin(11x+x)-cos(11x-x)]+[sin(7x+3x)-sin(7x-3x)]}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{[cos(10x)-cos(12x)]+[cos(4x)-cos(10x)]}{[sin(12x)-cos(10x)]+[sin(10x)-sin(4x)]}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{cos(10x)-cos(12x)+cos(4x)-cos(10x)}{sin(12x)-sin(10x)+sin(10x)-sin(4x)}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{-cos(12x)+cos(4x)}{sin(12x)-sin(4x)}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{cos(4x)-cos(12x)}{sin(12x)-sin(4x)}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{-2\,sin\left(\frac{4x+12x}{2}\right).sin(\frac{4x-12x}{2})}{2\,cos\left(\frac{12x+4x}{2}\right).sin(\frac{12x-4x}{2})}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{-2\,sin\left(\frac{16x}{2}\right).sin(\frac{-8x}{2})}{2\,cos\left(\frac{16x}{2}\right).sin(\frac{8x}{2})}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{-2\,sin(8x).sin(-4x)}{2\,cos(8x).sin(4x)}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{--2\,sin(8x).sin(4x)}{2\,cos(8x).sin(4x)}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=\frac{sin(8x)}{cos(8x)}$$

$$\frac{sin11x.sinx+sin7x.sin3x}{cos11x.sinx+cos7x.sin3x}=tan8x$$

20) Prove that: $\frac{cos3A.sin9A-sinA.cos5A}{cosA.cos5A-sin3A.sin9A}=tan8A$ **[W-17]**

Ans.

$$L.H.S.=\frac{\cos 3A\sin 9A-\sin A\cos 5A}{\cos A\cos 5A-\sin 3A\sin 9A}$$

$$=\frac{2\cos 3A\sin 9A-2\sin A\cos 5A}{2\cos A\cos 5A-2\sin 3A\sin 9A}$$

$$=\frac{\left[\sin(3A+9A)-\sin(3A-9A)\right]-\left[\sin(A+5A)+\sin(A-5A)\right]}{\left[\cos(A+5A)+\cos(A-5A)\right]-\left[\cos(3A-9A)-\cos(3A+9A)\right]}$$

$$=\frac{\left[\sin(12A)-\sin(-6A)\right]-\left[\sin(6A)+\sin(-4A)\right]}{\left[\cos(6A)+\cos(-4A)\right]-\left[\cos(-6A)-\cos(12A)\right]}$$

$$=\frac{\left[\sin(12A)+\sin(6A)\right]-\left[\sin(6A)-\sin(4A)\right]}{\left[\cos(6A)+\cos(4A)\right]-\left[\cos(6A)-\cos(12A)\right]}$$

$$=\frac{\sin(12A)+\sin(6A)-\sin(6A)+\sin(4A)}{\cos(6A)+\cos(4A)-\cos(6A)+\cos(12A)}$$

$$=\frac{\sin(12A)+\sin(4A)}{\cos(4A)+\cos(12A)}$$

$$=\frac{2\sin\left(\frac{12A+4A}{2}\right).\cos\left(\frac{12A-4A}{2}\right)}{2\cos\left(\frac{4A+12A}{2}\right).\cos\left(\frac{4A-12A}{2}\right)}$$

$$=\frac{2\sin(8A).\cos(4A)}{2\cos(8A).\cos(-4A)}$$

$$=\frac{\sin(8A).\cos(4A)}{\cos(8A).\cos(4A)}$$

$$=\tan 8A = R.H.S.$$

21) Prove that: $\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=tan2\theta$

Ans.

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{2sin8\theta.cos\theta-2cos3\theta.sin6\theta}{2cos2\theta.cos\theta-2sin3\theta.sin4\theta}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{[sin(8\theta+\theta)+sin(8\theta-\theta)]-[sin(3\theta+6\theta)-sin(3\theta-6\theta)]}{[cos(2\theta+\theta)+cos(2\theta-\theta)]-[cos(3\theta-4\theta)-cos(3\theta+4\theta)]}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{[sin(9\theta)+sin(7\theta)]-[sin(9\theta)-sin(-3\theta)]}{[cos(3\theta)+cos(\theta)]-[cos(-\theta)-cos(7\theta)]}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{[sin(9\theta)+sin(7\theta)]-[sin(9\theta)--sin(3\theta)]}{[cos(3\theta)+cos(\theta)]-[cos(\theta)-cos(7\theta)]}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{[sin(9\theta)+sin(7\theta)]-[sin(9\theta)+sin(3\theta)]}{[cos(3\theta)+cos(\theta)]-[cos(\theta)-cos(7\theta)]}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{sin(9\theta)+sin(7\theta)-sin(9\theta)-sin(3\theta)}{cos(3\theta)+cos(\theta)-cos(\theta)+cos(7\theta)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{sin(7\theta)-sin(3\theta)}{cos(3\theta)+cos(7\theta)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{2\,cos\left(\frac{7\theta+3\theta}{2}\right).sin\left(\frac{7\theta-3\theta}{2}\right)}{2\,cos\left(\frac{3\theta+7\theta}{2}\right).cos\left(\frac{3\theta-7\theta}{2}\right)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{2\,cos\left(\frac{10\theta}{2}\right).sin\left(\frac{4\theta}{2}\right)}{2\,cos\left(\frac{10\theta}{2}\right).cos\left(\frac{-4\theta}{2}\right)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{2\,cos(5\theta).sin(2\theta)}{2\,cos(5\theta).cos(-2\theta)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{sin(2\theta)}{cos(-2\theta)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=\frac{sin(2\theta)}{cos(2\theta)}$$

$$\frac{sin8\theta.cos\theta-cos3\theta.sin6\theta}{cos2\theta.cos\theta-sin3\theta.sin4\theta}=tan2\theta$$

22) Prove that: $\frac{cotA+tanB}{tanA+cotB}=cotA.tanB$

Ans.

$$\frac{cotA\ +\ tanB}{tanA\ +\ cotB}=\frac{\frac{cosA}{sinA}+\frac{sinB}{cosB}}{\frac{sinA}{cosA}+\frac{cosB}{sinB}}$$

$$\frac{cotA\ +\ tanB}{tanA\ +\ cotB}=\frac{\frac{cosA.cosB+sinA.sinB}{sinA.cosB}}{\frac{sinA.sinB+cosB.cosA}{cosA.sinB}}$$

$$\frac{cotA\ +\ tanB}{tanA\ +\ cotB}=\frac{\frac{1}{sinA.cosB}}{\frac{1}{cosA.sinB}}$$

$$\frac{cotA\ +\ tanB}{tanA\ +\ cotB}=\frac{cosA.sinB}{sinA.cosB}$$

$$\frac{cotA\ +\ tanB}{tanA\ +\ cotB}=cotA.tanB$$

23) Prove that: $\frac{cos21^0-sin21^0}{cos21^0+sin21^0}=cot66^0=tan24^0$

Ans.

$$\frac{cos21^0-sin21^0}{cos21^0+sin21^0}=\frac{cos21-sin21}{cos21+sin21}$$

$$=\frac{cos21-sin(90-69)}{cos21+sin(90-69)}$$

$$=\frac{cos21-cos69}{cos21+cos69}$$

$$=\frac{-2\,sin\left(\frac{21+69}{2}\right).sin(\frac{21-69}{2})}{2\,cos\left(\frac{21+69}{2}\right).cos(\frac{21-69}{2})}$$

$$=\frac{-2\,sin\left(\frac{90}{2}\right).sin(\frac{-48}{2})}{2\,cos\left(\frac{90}{2}\right).cos(\frac{-48}{2})}$$

$$= \frac{-2\,sin(45).sin(-24)}{2\,cos(45).cos(-24)}$$

$$= \frac{--2\,sin(45).sin(24)}{2\,cos(45).cos(24)}$$

$$= \frac{2\frac{1}{\sqrt{2}}.sin(24)}{2\frac{1}{\sqrt{2}}.cos(24)}$$

$$= tan(24)$$

$$= cot(66)$$

$$= R.H.S.$$

24) Prove that: $\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = tan10^0 = cot80^0$

Ans. $$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{cos(90-55) - sin35}{cos(90-55) + sin35}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{sin55 - sin35}{sin55 + sin35}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{2\,cos\left(\frac{55+35}{2}\right).sin(\frac{55-35}{2})}{2\,sin\left(\frac{55+35}{2}\right).cos(\frac{55-35}{2})}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{2\,cos\left(\frac{90}{2}\right).sin(\frac{20}{2})}{2\,sin\left(\frac{90}{2}\right).cos(\frac{20}{2})}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{2\,cos(45).sin(10)}{2\,sin(45).cos(10)}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{2\frac{1}{\sqrt{2}}.sin(10)}{2\frac{1}{\sqrt{2}}.cos(10)}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = \frac{sin(10)}{cos(10)}$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = tan10$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = tan(90-80)$$

$$\frac{cos35^0 - sin35^0}{cos35^0 + sin35^0} = cot80$$

25) Prove that: $\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = tan56^0 = cot34^0$

Ans.

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{cos(90-79) + sin11}{cos(90-79) - sin11}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{sin79 + sin11}{sin79 - sin11}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{2\,sin\left(\frac{79+11}{2}\right).cos(\frac{79-11}{2})}{2\,cos\left(\frac{79+11}{2}\right).sin(\frac{79-11}{2})}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{2\,sin\left(\frac{90}{2}\right).cos(\frac{68}{2})}{2\,cos\left(\frac{90}{2}\right).sin(\frac{68}{2})}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{2\,sin(45).cos(34)}{2\,cos(45).sin(34)}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{2\frac{1}{\sqrt{2}}.cos(34)}{2\frac{1}{\sqrt{2}}.sin(34)}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = \frac{cos(34)}{sin(34)}$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = cot34$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = cot(90 - 56)$$

$$\frac{cos11^0 + sin11^0}{cos11^0 - sin11^0} = tan56$$

26) Prove that: $sin50 - sin70 + sin10 = 0$ **[S-17]**

Ans.

$$\sin 50^0 - \sin 70^0 + \sin 10^0$$

$$= 2\cos\left(\frac{50^0+70^0}{2}\right)\sin\left(\frac{50^0-70^0}{2}\right)+\sin 10^0$$

$$= 2\cos 60^0 \sin\left(-10^0\right)+\sin 10^0$$

$$= 2\left(\frac{1}{2}\right)\sin\left(-10^0\right)+\sin 10^0$$

$$= -\sin 10^0 + \sin 10^0$$

$$= 0$$

27) Prove that: $sin20.sin40.sin60.sin80 = \frac{3}{16}$

[S-19,W-17,S-17,S-16, W-15,S-14,W-13,S-13,SQP]

Ans.

$$L.H.S. = \sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0$$

$$= \sin 20^0 \sin 40^0 \frac{\sqrt{3}}{2} \sin 80^0$$

$$= \frac{\sqrt{3}}{4}[2\sin 20^0 \sin 40^0]\sin 80^0$$

$$= \frac{\sqrt{3}}{4}\left(\cos 20^0 - \cos 60^0\right)\sin 80^0$$

$$= \frac{\sqrt{3}}{4}\left(\cos 20^0 \sin 80^0 - \cos 60^0 \sin 80^0\right)$$

$$= \frac{\sqrt{3}}{8}\left(2\cos 20^0 \sin 80^0 - 2\frac{1}{2}\sin 80^0\right)$$

$$= \frac{\sqrt{3}}{8}\left(\sin 100^0 + \sin 60^0 - \sin 80^0\right)$$

$$= \frac{\sqrt{3}}{8}\left(\sin\left(2\times 90-80\right)+\frac{\sqrt{3}}{2}-\sin 80^0\right)$$

$$=\frac{\sqrt{3}}{8}\left(\sin 80^0+\frac{\sqrt{3}}{2}-\sin 80^0\right)$$

$$=\frac{3}{16}=R.H.S.$$

28) Prove that: $cos20.cos40.cos60.cos80=\frac{1}{16}$

[W-19,S-19,W-18,S-18,W-17,W-14]

Ans.

$$\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ=\cos 20^\circ\cos 40^\circ\left(\frac{1}{2}\right)\cos 80^\circ$$

$$=\frac{1}{2}\cdot\frac{1}{2}\left(2\cos 20^\circ\cos 40^\circ\right)\cos 80^\circ$$

$$=\frac{1}{4}\left(\cos 60^\circ+\cos 20^\circ\right)\cos 80^\circ$$

$$=\frac{1}{4}\left(\frac{1}{2}+\cos 20^\circ\right)\cos 80^\circ$$

$$=\frac{1}{4}\left(\frac{1}{2}\cos 80^\circ+\cos 80^\circ\cos 20^\circ\right)$$

$$=\frac{1}{4}\left(\frac{1}{2}\cos 80^\circ+\frac{1}{2}\cdot 2\cos 80^\circ\cos 20^\circ\right)$$

$$=\frac{1}{4}\cdot\frac{1}{2}\left[\cos 80^\circ+\left(\cos 100^\circ+\cos 60^\circ\right)\right]$$

$$=\frac{1}{8}\left[\cos 80^\circ+\cos 100^\circ+\frac{1}{2}\right]$$

$$=\frac{1}{8}\left[2\cos 90^\circ\cos\left(-10^\circ\right)+\frac{1}{2}\right]$$

$$=\frac{1}{8}\left[0+\frac{1}{2}\right]$$

$$=\frac{1}{16}$$

29) Prove that: $sin10.sin30.sin50.sin70=\frac{1}{16}$ **[W-16]**

Ans.

$$\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0$$

$$= \sin 10^0 \frac{1}{2} \sin 50^0 \sin 70^0$$

$$= \frac{1}{4}\left[2\sin 10^0 \sin 50^0\right]\sin 70^0$$

$$= \frac{1}{4}\left[\cos\left(-40^0\right) - \cos 60^0\right]\sin 70^0$$

$$= \frac{1}{4}\left[\cos 40^0 - \frac{1}{2}\right]\sin 70^0$$

$$= \frac{1}{4}\left[\cos 40^0 \sin 70^0 - \frac{1}{2}\sin 70^0\right]$$

$$= \frac{1}{4}\left[\frac{1}{2} 2\cos 40^0 \sin 70^0 - \frac{1}{2}\sin 70^0\right]$$

$$= \frac{1}{8}\left[\sin 110^0 - \sin\left(-30\right) - \sin 70^0\right]$$

$$= \frac{1}{8}\left[\sin\left(2\times 90^0 - 70\right) + \frac{1}{2} - \sin 70^0\right]$$

$$= \frac{1}{8}\left[\sin 70^0 + \frac{1}{2} - \sin 70^0\right]$$

$$= \frac{1}{16}$$

30) Prove that: $cos(15).cos(30)cos(60).cos(75) = \frac{\sqrt{3}}{16}$ **[SQP]**

Ans.

$$L.H.S. = cos(15).cos(30)cos(60).cos(75)$$

$$L.H.S. = cos(15).\frac{\sqrt{3}}{2}\frac{1}{2}.cos(75)$$

$$L.H.S. = \frac{\sqrt{3}}{4}.cos(15).cos(75)$$

$$L.H.S. = \frac{\sqrt{3}}{4}.\frac{2}{2}.cos(15).cos(75)$$

$$L.H.S. = \frac{\sqrt{3}}{8}.2.cos(15).cos(75)$$

$$L.H.S. = \frac{\sqrt{3}}{8}.[cos(15+75) + cos(15-75)]$$

$$L.H.S. = \frac{\sqrt{3}}{8}.[cos(90) + cos(-60)]$$

$$L.H.S. = \frac{\sqrt{3}}{8}.[0 + cos(60)]$$

$$L.H.S. = \frac{\sqrt{3}}{8}.[cos(60)]$$

$$L.H.S. = \frac{\sqrt{3}}{8}.[\frac{1}{2}]$$

$$L.H.S. = \frac{\sqrt{3}}{16}$$

TUTORIAL NO.7

FACTORIZATION & DE-FACTORIZATION FORMULAE

1) Prove that: $\frac{sin8\theta - sin5\theta}{cos7\theta + cos6\theta} = sin\theta + cos\theta . tan\frac{\theta}{2}$ **[S-15,S-13]**

2) Prove that: $\frac{sin7x + sinx}{cos5x - cos3x} = sin2x - cos2x. cotx$ **[S-14,W-13,SQP]**

3) Prove that: $\frac{sin4A + sin5A + sin6A}{cos4A + cos5A + cos6A} = tan5A$

[W-19,S-19,W-18, W-17,S-17,S-16, S-15,W-14,SQP]

4) Prove that: $\frac{cos2A + 2\,cos4A + cos6A}{cosA + 2cos3A + cos5A} = cosA - tan3A. sinA$**[S-19,S-18,W-13]**

5) Prove that: $\frac{sinA + sin2A + sin3A + sin4A}{cosA + cos2A + cos3A + cos4A} = tan(\frac{5A}{2})$ **[S-19,W-16,W-12]**

6) Prove that: $\frac{sinx - sin5x + sin9x - sin13x}{cosx - cos5x - cos9x + cos13x} = cot4x$ **[S-18,W-15]**

7) Prove that: $\frac{cos3A.sin9A - sinA.cos5A}{cosA.cos5A - sin3A.sin9A} = tan8A$ **[W-17]**

8) Prove that: $sin20. sin40. sin60. sin80 = \frac{3}{16}$

[S-19,W-17,S-17,S-16, W-15,S-14,W-13,S-13,SQP]

9) Prove that: $cos20. cos40. cos60. cos80 = \frac{1}{16}$

[W-19,S-19,W-18,S-18,W-17,W-14]

10) Prove that: $sin10. sin30. sin50. sin70 = \frac{1}{16}$ **[W-16]**

INVERSE TRIGONOMETRIC RATIOS

Position in Question Paper **Total Marks-04**

Q.3. d) 4-Marks.

Examples:-

Que. **Find the principal value of**

1) $sin^{-1}(\frac{1}{\sqrt{2}})$

Ans. $sin^{-1}(\frac{1}{\sqrt{2}}) = \frac{\pi}{4}$

2) $cos^{-1}(\frac{-1}{\sqrt{2}})$

Ans. $cos^{-1}(\frac{-1}{\sqrt{2}}) = \pi - cos^{-1}(\frac{1}{\sqrt{2}})$

$cos^{-1}(\frac{-1}{\sqrt{2}}) = \pi - \frac{\pi}{4}$

$cos^{-1}(\frac{-1}{\sqrt{2}}) = \frac{3\pi}{4}$

3) $cos^{-1}(\frac{-1}{2})$ **[W-17]**

Ans. $cos^{-1}(\frac{-1}{2}) = \pi - cos^{-1}(\frac{1}{2})$

$cos^{-1}(\frac{-1}{2}) = \pi - cos^{-1}(\frac{\pi}{6})$

4) $sin^{-1}(\frac{-1}{2})$

Ans. $sin^{-1}(\frac{-1}{2}) = -sin^{-1}(\frac{1}{2})$

$sin^{-1}(\frac{-1}{2}) = -\frac{\pi}{6}$

5) $sin^{-1}(\frac{-1}{\sqrt{2}})$

Ans. $sin^{-1}(\frac{-1}{\sqrt{2}}) = -sin^{-1}(\frac{1}{\sqrt{2}})$

$sin^{-1}(\frac{-1}{\sqrt{2}}) = -sin^{-1}(\frac{\pi}{4})$

6) $tan^{-1}(\frac{-1}{\sqrt{3}})$

Ans. $tan^{-1}(\frac{-1}{\sqrt{3}}) = -tan^{-1}(\frac{1}{\sqrt{3}})$

$tan^{-1}(\frac{-1}{\sqrt{3}}) = -\frac{\pi}{6}$

7) $tan^{-1}(-1)$

Ans. $tan^{-1}(-1) = -tan^{-1}(1)$

$tan^{-1}(-1) = -\frac{\pi}{4}$

8) $tan^{-1}(\sqrt{3})$

Ans. $tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

9) $cot^{-1}(-\sqrt{3})$

Ans. $cot^{-1}(\,-\sqrt{3}\,) = -cot^{-1}(\,\sqrt{3}\,)$

$cot^{-1}(\,-\sqrt{3}\,) = -tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$cot^{-1}(\,-\sqrt{3}\,) = -tan^{-1}\left(\frac{\pi}{6}\right)$

10) $sec^{-1}(\,-2\,)$

Ans. $sec^{-1}(\,-2\,) = \pi - sec^{-1}(\,2\,)$

$sec^{-1}(\,-2\,) = \pi - cos^{-1}(\frac{1}{2})$

$sec^{-1}(\,-2\,) = \pi - \frac{\pi}{3}$

$sec^{-1}(\,-2\,) = \frac{2\pi}{3}$

11) $sin[cos^{-1}(\,\frac{-1}{2}\,)]$

Ans. $sin[cos^{-1}(\,\frac{-1}{2}\,)] = sin[\pi - cos^{-1}(\,\frac{1}{2}\,)]$

$sin[cos^{-1}(\,\frac{-1}{2}\,)] = sin[\pi - \frac{\pi}{3}]$

$sin[cos^{-1}(\,\frac{-1}{2}\,)] = sin[\frac{2\pi}{3}]$

$sin[cos^{-1}(\,\frac{-1}{2}\,)] = sin[120]$

$sin[cos^{-1}(\,\frac{-1}{2}\,)] = sin[180 - 60]$

$sin[cos^{-1}(\,\frac{-1}{2}\,)] = sin[60]$

$sin[cos^{-1}(\,\frac{-1}{2}\,)] = \frac{\sqrt{3}}{2}$

12) $sec[cos^{-1}(\,\frac{\sqrt{3}}{2}\,)]$ **[S-19,W-16]**

Ans.

$$\sec\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$=\sec 30^0 \quad \text{or} \quad \sec\frac{\pi}{6}$$

$$=\frac{2}{\sqrt{3}} \quad \text{or } 1.1547$$

13) $sin[\frac{\pi}{2} - cos^{-1}(\frac{1}{2})]$

Ans.

$$sin[\frac{\pi}{2} - cos^{-1}(\frac{1}{2})] = sin[\frac{\pi}{2} - \frac{\pi}{3}]$$

$$sin[\frac{\pi}{2} - cos^{-1}(\frac{1}{2})] = sin[\frac{\pi}{6}]$$

$$sin[\frac{\pi}{2} - cos^{-1}(\frac{1}{2})] = \frac{1}{2}$$

14) $sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})]$

Ans.

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = sin[\frac{\pi}{2} - -sin^{-1}(\frac{1}{2})]$$

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = sin[\frac{\pi}{2} + \frac{\pi}{6}]$$

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = sin[\frac{2\pi}{3}]$$

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = sin[120]$$

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = sin[180 - 60]$$

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = sin[60]$$

$$sin[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = \frac{\sqrt{3}}{2}$$

15) $cos[\frac{\pi}{2} - sin^{-1}(\frac{1}{2})]$ **[S-15]**

Ans.

$$\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$$

$$=\cos\left[\frac{\pi}{3}\right]$$

$$=\frac{1}{2} \quad or \quad 0.5$$

16) $cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})]$

Ans.

$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = cos[\frac{\pi}{2} - -sin^{-1}(\frac{1}{2})]$$

$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = cos[\frac{\pi}{2} + \frac{\pi}{6}]$$

$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = cos[\frac{2\pi}{3}]$$

$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = cos[120]$$

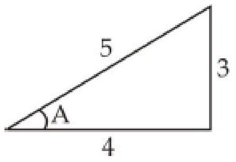
$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = cos[180 - 60]$$

$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = -cos[60]$$

$$cos[\frac{\pi}{2} - sin^{-1}(\frac{-1}{2})] = -\frac{1}{2}$$

17) $cos[tan^{-1}(\frac{3}{4})]$

Ans.



$$tan^{-1}\left(\frac{3}{4}\right) = A$$

$$\therefore cos[tan^{-1}(\frac{3}{4})] = cosA$$

$$\therefore cos[tan^{-1}(\frac{3}{4})] = \frac{4}{5}$$

18) $tan[2\,tan^{-1}(\frac{1}{5})]$ **[S-17]**

Ans.

$$tan[2\,tan^{-1}(\frac{1}{5})] = tan\{tan^{-1}[\frac{2.\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}]\}$$

$$tan[2\,tan^{-1}(\frac{1}{5})] = tan\{tan^{-1}[\frac{\frac{2}{5}}{1-\frac{1}{25}}]\}$$

$$tan[2\,tan^{-1}(\frac{1}{5})] = tan\{tan^{-1}[\frac{\frac{2}{5}}{\frac{24}{25}}]\}$$

$$tan[2\,tan^{-1}(\frac{1}{5})] = tan\{tan^{-1}[\frac{5}{12}]\}$$

$$tan[2\,tan^{-1}(\frac{1}{5})] = \frac{5}{12}$$

19) $sin[\,2\,tan^{-1}(\frac{4}{3})\,]$

Ans. $tan^{-1}\left(\frac{4}{3}\right) = \theta$ $\therefore \frac{4}{3} = tan\theta$ $\therefore sin\theta = \frac{4}{5}$ $\therefore cos\theta = \frac{3}{5}$

$$sin[\,2\,tan^{-1}(\frac{4}{3})\,] = sin[2\theta]$$

$$sin[\,2\,tan^{-1}(\frac{4}{3})\,] = 2sin\theta.cos\theta$$

$$sin[\,2\,tan^{-1}(\frac{4}{3})\,] = 2.\frac{4}{5}.\frac{3}{5}$$

$$sin[\,2\,tan^{-1}(\frac{4}{3})\,] = \frac{24}{25}$$

20) $sin[2\ sin^{-1}(\frac{4}{5})\ ]$

Ans. $sin^{-1}\left(\frac{4}{5}\right) = \theta \qquad \therefore \left(\frac{4}{5}\right) = sin\theta \qquad \therefore\ cos\theta = \left(\frac{3}{5}\right)$

$sin[2\ sin^{-1}(\frac{4}{5})\ ] = sin[2\theta]$

$sin[2\ sin^{-1}(\frac{4}{5})\ ] = 2.sin\theta.cos\theta$

$sin[2\ sin^{-1}(\frac{4}{5})\ ] = 2.\frac{4}{5}.\frac{3}{5}$

$sin[2\ sin^{-1}(\frac{4}{5})\ ] = \frac{24}{25}$

21) $cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{1}{2}\right)$

Ans. $cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right) - sin^{-1}\left(\frac{1}{2}\right)$

$cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{6} - \frac{\pi}{3}$

$cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

22) $cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{-1}{2}\right)$

Ans. $cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{-1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right) - -sin^{-1}\left(\frac{1}{2}\right)$

$cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{-1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{1}{2}\right)$

$cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{-1}{2}\right) = \pi - \frac{\pi}{6} + \frac{\pi}{3}$

$cos^{-1}\left(\frac{-1}{2}\right) - sin^{-1}\left(\frac{-1}{2}\right) = \frac{7\pi}{6}$

23) $cos[sin^{-1}(\frac{3}{5}) + sin^{-1}(\frac{5}{13})]$

Ans. $sin^{-1}(\frac{3}{5}) = A \qquad \therefore \frac{3}{5} = sinA \qquad \therefore cosA = \frac{4}{5}$

$sin^{-1}(\frac{5}{13}) = B \qquad \therefore \frac{5}{13} = sinB \qquad \therefore cosB = \frac{12}{13}$

$$cos[sin^{-1}(\frac{3}{5}) + sin^{-1}(\frac{5}{13})] = cos[A + B]$$

$$cos[sin^{-1}(\frac{3}{5}) + sin^{-1}(\frac{5}{13})] = cosA.cosB - sinA.sinB$$

$$cos[sin^{-1}(\frac{3}{5}) + sin^{-1}(\frac{5}{13})] = \frac{4}{5}.\frac{12}{13} - \frac{3}{5}.\frac{5}{13}$$

$$cos[sin^{-1}(\frac{3}{5}) + sin^{-1}(\frac{5}{13})] = \frac{48}{65} - \frac{15}{65}$$

$$cos[sin^{-1}(\frac{3}{5}) + sin^{-1}(\frac{5}{13})] = \frac{33}{65}$$

24) $sin^{-1}(\frac{1}{2}) + cos^{-1}(-\frac{1}{2}) - tan^{-1}(\infty)$ **[W-18]**

Ans.

$$\sin^{-1}\left(\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)-\tan^{-1}(\infty)$$

$$=\sin^{-1}\left(\frac{1}{2}\right)+\pi-\cos^{-1}\left(\frac{1}{2}\right)-\tan^{-1}(\infty)$$

$$=\frac{\pi}{6}+\pi-\frac{\pi}{3}-\frac{\pi}{2}$$

$$=\frac{\pi}{3}$$

1) Using principal value find the value of x if

$tan^{-1}(1) + tan^{-1}(x) = 0$ **[S-17,W-13]**

Ans. $\tan^{-1}(1)+\tan^{-1}(x)=0$

$$\frac{\pi}{4}+\tan^{-1}(x)=0$$

$$\tan^{-1}(x)=-\frac{\pi}{4}$$

$$x=\tan\left(-\frac{\pi}{4}\right)$$

$$x=-\tan\left(\frac{\pi}{4}\right)$$

$$\therefore x=-1$$

2) Prove that: $2tan^{-1}(x) = tan^{-1}\left(\frac{2x}{1-x^2}\right)$ **[S-15,W-12]**

Ans.

$$2\tan^{-1}x=\tan^{-1}x+\tan^{-1}x$$

$$=\tan^{-1}\left(\frac{x+x}{1-x.x}\right)$$

$$=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

3) Prove that: $sin^{-1}(x) = cot^{-1}(\frac{\sqrt{1-x^2}}{x})$ **[W-15]**

Ans.

Put $x=\sin\theta$

$$=\cot^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$$

$$=\cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$=\cot^{-1}(\cot\theta)$$

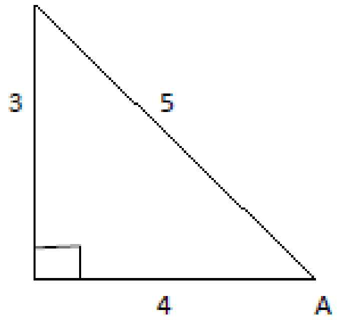
$$=\theta$$

$$=\sin^{-1}x$$

4) Prove that: $\cos[sin^{-1}(\frac{3}{5})] = \frac{4}{5}$ **[W-15]**

Ans. Put $\sin^{-1}\left(\frac{3}{5}\right) = \theta$

$\therefore \sin\theta = \frac{3}{5}$



$\therefore \cos\theta = \frac{4}{5}$

$\therefore \cos\left[\sin^{-1}\left(\frac{3}{5}\right)\right] = \frac{4}{5}$

5) Prove that: $sin^{-1}(\frac{-1}{\sqrt{2}}) + 2\,cos^{-1}(\frac{-1}{\sqrt{2}}) + 3\,sin^{-1}(-1) = -\frac{\pi}{4}$ **[W-15]**

Ans. $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 3\sin^{-1}(-1)$

$= -\sin^{-1}\frac{1}{\sqrt{2}} + 2\left(\pi - \cos^{-1}\frac{1}{\sqrt{2}}\right) - 3\sin^{-1}1$

$= -\frac{\pi}{4} + 2\left(\pi - \frac{\pi}{4}\right) - 3\frac{\pi}{2}$

$= -\frac{\pi}{4}$

6) Verify that: $sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = tan^{-1}(\infty)$

Ans.

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = \frac{\pi}{6} + \frac{\pi}{3}$$

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = \frac{3\pi + 6\pi}{18}$$

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = \frac{9\pi}{18}$$

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = \frac{\pi}{2}$$

$$tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\therefore sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = tan^{-1}(\infty)$$

7) Verify that: $sin^{-1}(\frac{-1}{2}) + cos^{-1}(-\frac{1}{2}) = tan^{-1}(\infty)$

Ans.

$$sin^{-1}(\frac{-1}{2}) + cos^{-1}(\frac{-1}{2}) = -sin^{-1}(\frac{1}{2}) + [\pi - cos^{-1}(\frac{1}{2})]$$

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = -\frac{\pi}{6} + [\pi - \frac{\pi}{3}]$$

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = -\frac{\pi}{6} + \pi - \frac{\pi}{3}$$

$$sin^{-1}(\frac{1}{2}) + cos^{-1}(\frac{1}{2}) = \frac{\pi}{2}$$

$$tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\therefore sin^{-1}(\frac{-1}{2}) + cos^{-1}(\frac{-1}{2}) = tan^{-1}(\infty)$$

8) Prove that: $\cos^{-1}(\frac{4}{5}) + \cos^{-1}(\frac{12}{13}) = \cos^{-1}(\frac{33}{65})$

[S-19,W-18,S-18,W-17,W-16,S-16,W-15,W-14,W-13,SQP]

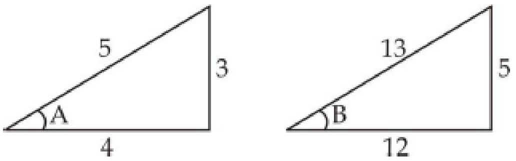
OR

Prove that: $sin^{-1}(\frac{3}{5}) + cos^{-1}(\frac{12}{13}) = cos^{-1}(\frac{33}{65}) = sin^{-1}(\frac{56}{65})$ **[W-12]**

Ans.

$$A=\sin^{-1}\left(\frac{3}{5}\right) \qquad B=\cos^{-1}\left(\frac{12}{13}\right)$$

$$\therefore \sin A=\frac{3}{5} \qquad \cos B=\frac{12}{13}$$



$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$=\frac{3}{5}\times\frac{12}{13}+\frac{4}{5}\times\frac{5}{13}$$

$$=\frac{36}{65}+\frac{20}{65}$$

$$=\frac{36+20}{65}$$

$$=\frac{56}{65}$$

$$\therefore A+B=\sin^{-1}\left(\frac{56}{65}\right)$$

$$\therefore \sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

$$\therefore sin^{-1}\left(\frac{56}{65}\right)=cos^{-1}(\frac{33}{65})$$

9) Prove that: $cos^{-1}(\frac{4}{5}) - cos^{-1}(\frac{12}{13}) = cos^{-1}(\frac{63}{65}) = sin^{-1}(\frac{16}{65})$ **[S-17]**

OR

Prove that: $cos^{-1}(\frac{4}{5}) - sin^{-1}(\frac{5}{13}) = cos^{-1}(\frac{63}{65})$

Ans. $cos^{-1}\left(\frac{4}{5}\right)=A \qquad \therefore \left(\frac{4}{5}\right)=cosA \qquad \therefore sinA=\frac{3}{5}$

$cos^{-1}(\frac{12}{13}) = B$ $\therefore \frac{12}{13} = cosB$ $\therefore\ sinB = \frac{5}{13}$

$$cos(A + B) = cosA.cosB - sinA.sinB$$

$$cos(A + B) = \frac{4}{5}.\frac{12}{13} - \frac{3}{5}.\frac{5}{13}$$

$$cos(A + B) = \frac{48}{65} - \frac{15}{65}$$

$$cos(A + B) = \frac{33}{65}$$

$$A + B = cos^{-1}(\frac{33}{65})$$

$$cos^{-1}\left(\frac{4}{5}\right) + cos^{-1}(\frac{12}{13}) = cos^{-1}(\frac{33}{65})$$

$$cos^{-1}(\frac{63}{65}) = sin^{-1}(\frac{16}{65})$$

10) Prove that: $sin^{-1}(\frac{3}{5}) - cos^{-1}(\frac{5}{13}) = cos^{-1}(\frac{56}{65})$ **[S-13]**

Ans. $sin^{-1}\left(\frac{3}{5}\right) = A$ $\therefore \left(\frac{3}{5}\right) = sinA$ $\therefore cosA = \frac{4}{5}$

$cos^{-1}(\frac{5}{13}) = B$ $\therefore \left(\frac{5}{13}\right) = cosB$ $\therefore sinB = \frac{12}{13}$

$$cos(A - B) = cosA.cosB + sinA.sinB$$

$$cos(A - B) = \frac{4}{5}.\frac{5}{13} + \frac{3}{5}.\frac{12}{13}$$

$$cos(A - B) = \frac{20}{65} + \frac{36}{65}$$

$$cos(A - B) = \frac{56}{65}$$

$$A - B = cos^{-1}(\frac{56}{65})$$

$$sin^{-1}\left(\frac{3}{5}\right) - cos^{-1}(\frac{5}{13}) = cos^{-1}(\frac{56}{65})$$

11) Prove that: $cos^{-1}(\frac{4}{5})$ – $sin^{-1}(\frac{8}{17})$ $= cos^{-1}(\frac{84}{85})$ **[S-19]**

OR

Prove that: $sin^{-1}(\frac{3}{5})$ – $sin^{-1}(\frac{8}{17})$ $= cos^{-1}(\frac{84}{85}) = sin^{-1}(\frac{13}{85})$ **[W-17]**

Ans. $cos^{-1}\left(\frac{4}{5}\right) = A$ $\therefore \left(\frac{4}{5}\right) = cosA$ $\therefore sinA = \frac{3}{5}$

$sin^{-1}(\frac{8}{17}) = B$ $\therefore \left(\frac{8}{17}\right) = sinB$ $\therefore cosB = \frac{15}{17}$

$$cos(A - B) = cosA.cosB + sinA.sinB$$

$$cos(A - B) = \frac{4}{5}.\frac{15}{17} + \frac{3}{5}.\frac{8}{17}$$

$$cos(A - B) = \frac{60}{85} + \frac{24}{85}$$

$$cos(A - B) = \frac{84}{85}$$

$$A - B = cos^{-1}(\frac{84}{85})$$

$$cos^{-1}\left(\frac{4}{5}\right) - sin^{-1}(\frac{8}{17}) = cos^{-1}(\frac{84}{85})$$

12) Prove that: $sin^{-1}(\frac{4}{5}) + sin^{-1}(\frac{8}{17}) = sin^{-1}(\frac{84}{85})$ **[S-13]**

Ans. $sin^{-1}(\frac{4}{5}) = A$ $\therefore (\frac{4}{5}) = sinA$ $\therefore cosA = \frac{3}{5}$

$sin^{-1}\left(\frac{8}{17}\right) = B$ $\therefore \left(\frac{8}{17}\right) = sinB$ $\therefore cosB = \frac{15}{17}$

$$sin(A + B) = sinA.cosB + cosA.sinB$$

$$sin(A+B)=\frac{4}{5}.\frac{15}{17}+\frac{3}{5}.\frac{8}{17}$$

$$sin(A+B)=\frac{60}{85}+\frac{24}{85}$$

$$sin(A+B)=\frac{84}{85}$$

$$A+B=sin^{-1}(\frac{84}{85})$$

$$sin^{-1}(\frac{4}{5})+sin^{-1}\left(\frac{8}{17}\right)=sin^{-1}(\frac{84}{85})$$

13) Prove that: $sec^{-1}(\frac{5}{4})\ +tan^{-1}(\frac{3}{5})\ =\ tan^{-1}(\frac{27}{11})$

OR

Prove that: $cos^{-1}(\frac{4}{5})\ +tan^{-1}(\frac{3}{5})\ =\ tan^{-1}(\frac{27}{11})$

[W-17,W-15,S-15, S-14,W-13,W-12]

Ans. $sec^{-1}(\frac{5}{4})=cos^{-1}(\frac{4}{5})$

$cos^{-1}(\frac{4}{5})=A \qquad \therefore (\frac{4}{5})=cosA \qquad \therefore tanA=\frac{3}{4}$

$$tan^{-1}(\frac{3}{5})=B$$

$$\therefore tan(A+B)=\frac{tanA+tanB}{1-tanA.tanB}$$

$$\therefore tan(A+B)=\frac{(\frac{3}{4})+(\frac{3}{5})}{1-(\frac{3}{4}).(\frac{3}{5})}$$

$$tan(A+B)=(\frac{27}{11})$$

$$cos^{-1}(\frac{4}{5})+tan^{-1}(\frac{3}{5})=tan^{-1}(\frac{27}{11})$$

14) Prove that: $tan^{-1}(\frac{1}{11})\ +cot^{-1}(\frac{6}{5})\ =\ sec^{-1}(\sqrt{2})\ =cos^{-1}(\frac{1}{\sqrt{2}})$ **[W-17]**

OR

Prove that: $tan^{-1}(\frac{1}{11}) + tan^{-1}(\frac{5}{6}) = sec^{-1}(\sqrt{2}) = cos^{-1}(\frac{1}{\sqrt{2}})$

Ans.

$$\text{Let } \cot^{-1}\left(\frac{6}{5}\right)=\tan^{-1}\left(\frac{5}{6}\right)$$

$$L.H.S.=\cot^{-1}\left(\frac{6}{5}\right)+\tan^{-1}\left(\frac{1}{11}\right)$$

$$=\tan^{-1}\left(\frac{5}{6}\right)+\tan^{-1}\left(\frac{1}{11}\right)$$

$$=\tan^{-1}\left(\frac{\frac{5}{6}+\frac{1}{11}}{1-\frac{5}{6}\frac{1}{11}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{55+6}{66}}{\frac{66-5}{66}}\right)$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}=R.H.S.$$

$$\therefore R.H.S=\sec^{-1}\left(\sqrt{2}\right)=\frac{\pi}{4}$$

15) Prove that: $tan^{-1}(\frac{1}{7}) + tan^{-1}(\frac{1}{13}) = tan^{-1}(\frac{2}{9}) = cot^{-1}(\frac{9}{2})$

[W-18,S-18,S-14,S-13,W-12,SQP]

Ans.

$$\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\cdot\frac{1}{13}}\right)$$

$$=\tan^{-1}\left(\frac{20}{90}\right)$$

$$=\tan^{-1}\left(\frac{2}{9}\right)$$

$$tan^{-1}\left(\frac{2}{9}\right)=cot^{-1}(\frac{9}{2})$$

16) Prove that: $tan^{-1}(\frac{1}{4}) + tan^{-1}(\frac{2}{9}) = tan^{-1}(\frac{1}{2}) = cot^{-1}(2)$

[S-19,S-18]

Ans.

$$\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$$

$$=\tan^{-1}\left[\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right]$$

$$=\tan^{-1}\left(\frac{1}{2}\right)$$

$$=\cot^{-1}2$$

$$\therefore \tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\cot^{-1}2$$

17) Prove that: $tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

[W-19,S-19,S-18,W-16,S-16,SQP]

Ans.

$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

18) Prove that: $tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{8}) = \frac{\pi}{4}$ **[S-16]**

Ans.

$$tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{8}) = tan^{-1}(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\cdot\frac{1}{5}}) + tan^{-1}(\frac{1}{8})$$

$$tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{8}) = tan^{-1}(\frac{7}{9}) + tan^{-1}(\frac{1}{8})$$

$$tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{8}) = tan^{-1}(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\cdot\frac{1}{8}})$$

$$tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{8}) = tan^{-1}(1)$$

$$tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{8}) = \frac{\pi}{4}$$

19) Prove that: $tan^{-1}(\frac{3}{4}) + tan^{-1}(\frac{3}{5}) - tan^{-1}(\frac{8}{19}) = \frac{\pi}{4}$

Ans.

$$tan^{-1}(\frac{3}{4}) + tan^{-1}(\frac{3}{5}) - tan^{-1}\left(\frac{8}{19}\right) = tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right) - tan^{-1}(\frac{8}{19})$$

$$tan^{-1}(\frac{3}{4}) + tan^{-1}(\frac{3}{5}) - tan^{-1}\left(\frac{8}{19}\right) = tan^{-1}\left(\frac{27}{11}\right) - tan^{-1}(\frac{8}{19})$$

$$tan^{-1}(\frac{3}{4}) + tan^{-1}(\frac{3}{5}) - tan^{-1}\left(\frac{8}{19}\right) = tan^{-1}(\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\cdot\frac{8}{19}})$$

$$tan^{-1}(\frac{3}{4}) + tan^{-1}(\frac{3}{5}) - tan^{-1}\left(\frac{8}{19}\right) = tan^{-1}(1)$$

$$tan^{-1}(\frac{3}{4}) + tan^{-1}(\frac{3}{5}) - tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$$

20) Prove that: $tan^{-1}(\frac{1}{5}) + tan^{-1}(\frac{1}{7}) + tan^{-1}(\frac{1}{3}) + tan^{-1}(\frac{1}{8}) = \frac{\pi}{4}$

[W-16]

Ans.

$$\therefore \tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left[\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right]+\tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{12}{35}}{1-\frac{1}{35}}\right]+\tan^{-1}\left[\frac{\frac{11}{24}}{1-\frac{1}{24}}\right]$$

$$= \tan^{-1}\left(\frac{12}{34}\right)+\tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left[\frac{\frac{12}{34}+\frac{11}{23}}{1-\frac{12}{34}\times\frac{11}{23}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{276+374}{782}}{1-\frac{132}{782}}\right]$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

21) Prove that: $tan^{-1}(1) + tan^{-1}(2) + tan^{-1}(3) = \pi$

[S-19,W-18,S-17,W-14,S-14,SQP]

Ans.

$$\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi+\tan^{-1}\left(\frac{1+2}{1-1\cdot 2}\right)+\tan^{-1}(3)$$

$$=\pi+\tan^{-1}(-3)+\tan^{-1}(3)$$

$$=\pi-\tan^{-1}(3)+\tan^{-1}(3)$$

$$=\pi$$

22) Prove that: $tan^{-1}(\frac{9}{2}) + tan^{-1}(\frac{11}{7}) = \frac{3\pi}{4}$

Ans.

$$tan^{-1}(\frac{9}{2}) + tan^{-1}(\frac{11}{7}) = tan^{-1}(\frac{\frac{9}{2}+\frac{11}{7}}{1-\frac{9}{2}\cdot\frac{11}{7}})$$

$$tan^{-1}(\frac{9}{2}) + tan^{-1}(\frac{11}{7}) = tan^{-1}(-1) + \pi$$

$$tan^{-1}(\frac{9}{2}) + tan^{-1}(\frac{11}{7}) = -\frac{\pi}{4} + \pi$$

$$tan^{-1}(\frac{9}{2}) + tan^{-1}(\frac{11}{7}) = \frac{3\pi}{4}$$

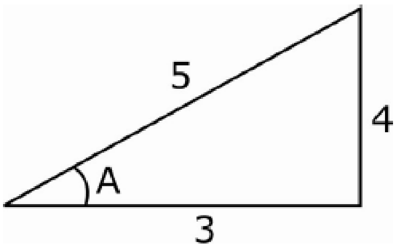
23) Prove that: $2\,cot^{-1}(3) + cosec^{-1}(\frac{5}{4}) = \frac{\pi}{2}$ **[W-14]**

Ans.

$$2\cot^{-1}(3) = 2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$$Let \;\; A = \cos ec^{-1}\left(\frac{5}{4}\right)$$

$$\therefore \cos ecA = \frac{5}{4}$$



$$\therefore 2\cot^{-1}(3) + \cos ec^{-1}\left(\frac{5}{4}\right) = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4}{3}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{4}{3}}{1-\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)}\right)$$

$$= \tan^{-1}(\infty)$$

$$= \frac{\pi}{2}$$

Maratha Vidya Prasarak Samaj's

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MSBTE CODE-1002 DTE CODE:- 5247

DEPARTMENT OF SCIENCE AND HUMANITY

ACADEMIC YEAR-2020-2021

TUTORIAL NO.8

INVERSE TRIGONOMETRIC RATIOS

1) Find the principal value of $tan^{-1}(\sqrt{3})$ **[S-19,S-16]**

2) Using principal value find the value of x if

$tan^{-1}(1) + tan^{-1}(x) = 0$ **[S-17,W-13]**

3) Prove that: $2tan^{-1}(x) = tan^{-1}\left(\frac{2x}{1-x^2}\right)$ **[S-15,W-12]**

4) Prove that: $\cos^{-1}(\frac{4}{5}) + \cos^{-1}(\frac{12}{13}) = \cos^{-1}(\frac{33}{65})$

[S-19,W-18,S-18, W-17,W-16,S-16,W-15,W-14,W-13,SQP]

5) Prove that: $cos^{-1}(\frac{4}{5}) + tan^{-1}(\frac{3}{5}) = tan^{-1}(\frac{27}{11})$

[W-17,W-15,S-15, S-14,W-13,W-12]

6) Prove that: $tan^{-1}(\frac{1}{7}) + tan^{-1}(\frac{1}{13}) = tan^{-1}(\frac{2}{9}) = cot^{-1}(\frac{9}{2})$

[W-18,S-18,S-14,S-13,W-12,SQP]

7) Prove that: $tan^{-1}(\frac{1}{4}) + tan^{-1}(\frac{2}{9}) = tan^{-1}(\frac{1}{2}) = cot^{-1}(2)$

[S-19,S-18]

8) Prove that: $tan^{-1}(\frac{1}{2}) + tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

[W-19,S-19,S-18,W-16,S-16,SQP]

9) Prove that: $tan^{-1}(1) + tan^{-1}(2) + tan^{-1}(3) = \pi$

[S-19,W-18,S-17,W-14,S-14,SQP]

10) Prove that: $2\,cot^{-1}(3) + cosec^{-1}(\frac{5}{4}) = \frac{\pi}{2}$ **[W-14]**

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DEPARTMENT OF SCIENCE AND HUMANITY

ACADEMIC YEAR-2020-2021

STRAIGHT LINE

Position in Question Paper **Total Marks-12**

Q.5. a) i) 3-Marks.

ii) 3- Marks

Q.5.b) i) 3-Marks.

ii) 3- Marks

A. BASIC (GENERAL) TYPE :-

1) **Inclination of line:-** $i)$ Inclination of X-axis = 0^0

$ii)$ Inclination of Y-axis = 90^0

2) **Slope OR Gradient of a line:-** $m = tan\theta$

3) **The slope of a line passing through two points:-** $m = \frac{y_2-y_1}{x_2-x_1}$

Note:- $i)$ Slope of $X -$ axis is 0 $ii)$ Slope of Y- axis is ∞

4) **General equation of line:-**

$i)$ The general equation of line $ax + by + c = 0$ $ii)$ $m = \frac{-Coefficient\ of\ x}{coefficient\ of\ y}$

5) **Parallel and Perpendicular lines:-** Two lines having slopes m_1 & m_2 are

$i)$ parallel if $m_1 = m_2$ $ii)$ perpendicular if $m_1.m_2 = -1$

6) Intercepts of a line:- $i)$ To find X – intercepts put $y = 0$

$ii)$ To find Y – intercepts put $x = 0$

7) Standard form of Equation of line:-

$i)$ **Slope point form:-** $y - y_1 = m(x - x_1)$

$ii)$ **Slope Intercept form:-** $y = mx + c$

$iii)$ **Two point form:-** $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$iv)$ **Double intercept form:-** $\frac{x}{a} + \frac{y}{b} = 1$

8) Intersection of two lines:-

Consider two intersecting lines $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ to find point of intersection solve above equation simultaneously or by crammer's rule.

Examples:-

Que. **Solve the problems with given condition.**

1) Find the equation of the line passing through the point (1 , 7) & having slope 2 units **[S-19,SQP]**

Ans. Point $=(x_1,y_1)=(1,7)$ & slope $=2$

$\therefore$ Equation of line is,

$$y-y_1=m(x-x_1)$$

$\therefore y-7=2(x-1)$

$\therefore 2x-y+5=0$

2) Find the equation of the line passing through the point (2 , 3) & having slope 5 units **[W-19]**

Ans. Point $(x_1,y_1)=(2,3)$ and slope $m=5$

Equation of line is,

$$y-y_1=m(x-x_1)$$

$\therefore y-3=5(x-2)$

$\therefore y-3=5x-10$

$\therefore 5x-y-7=0$

3) Find the equation of line passing through (4 , – 5) and having slope $\frac{-2}{3}$

[W-18,S-15]

Ans.

$\therefore$ the equation is

$$y-y_1=m\left(x-x_1\right)$$

$$\therefore y+5=-\frac{2}{3}(x-4)$$

$$\therefore 3y+15=-2x+8$$

$$\therefore 2x+3y+7=0$$

4) Find the equation of the line passing through (3 , – 4) and having slope $\frac{3}{2}$

[S-18]

Ans. $\text{Point}=(x_1,y_1)=(3,-4)$ & $\text{slope}=\frac{3}{2}$

$\therefore$ equation of line is,

$$y-y_1=m\left(x-x_1\right)$$

$$\therefore y-(-4)=\frac{3}{2}(x-3)$$

$$\therefore 2(y+4)=3(x-3)$$

$$\therefore 3x-2y-17=0$$

5) Find the equation of straight line passing through (5 , 6) and making angle

150° with $X-$ axis **[W-14]**

Ans.

$$Given \quad \theta = 150^\circ$$

$$\therefore slope \quad m = \tan\theta = \tan 150^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

$$\therefore equation\ is$$

$$y - y_1 = m(x - x_1)$$

$$\therefore y - 6 = -\frac{1}{\sqrt{3}}(x - 5)$$

$$\therefore \sqrt{3}y - 6\sqrt{3} = -x + 5$$

$$\therefore x + \sqrt{3}y - 6\sqrt{3} - 5 = 0$$

6) Find the slope of the line passing through the points $(-1, -2)$ and $(-3, 8)$

[S-14]

Ans.

$$slope \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8+2}{-3+1}$$

$$= -5$$

7) Find the equation of straight line passes through the points (– 4 , 6) and (8 , – 3) **[W-18,S-14]**

Ans.

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\therefore \frac{y - 6}{-3 - 6} = \frac{x + 4}{8 + 4}$$

$$\therefore \frac{y-6}{-9} = \frac{x+4}{12}$$

$$\therefore 12(y-6) = -9(x+4)$$

$$\therefore 12y - 72 = -9x - 36$$

$$\therefore 9x + 12y - 36 = 0 \quad or \quad -9x - 12y + 36 = 0$$

$$or \quad 3x + 4y - 12 = 0 \qquad or \qquad -3x - 4y + 12 = 0$$

8) Find the equation of straight line passes through the points (3 , 5) and (4 , 6) **[W-17]**

Ans. Equation of line is

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\frac{y-5}{5-6} = \frac{x-3}{3-4}$$

$$\frac{y-5}{-1} = \frac{x-3}{-1}$$

$$x - y + 2 = 0$$

9) State the condition of two lines is parallel and perpendicular to each other

[W-15]

OR

State the conditions of parallel and perpendicular lines, whose slopes are M_1 and M_2 **[S-17,W-12]**

Ans. Two lines are parallel, if $m_1 = m_2$

Two lines are perpendicular, if

$m_1 m_2 = -1$

10) Prove that $2x + 3y + 7 = 0$ & $4x + 6y + 2 = 0$ are parallel to each other.

Ans.

$L_1 : 2x + 3y + y = 0$

$\therefore$ Slope of L_1 is $= m_1 = \dfrac{-\text{Coefficient of x}}{\text{Coefficient of y}}$

$m_1 = \dfrac{-2}{3}$

$L_2 : 4x + 6y + 2 = 0$

Slope of L_2 is $= m_2 = \dfrac{-\text{Coefficient of x}}{\text{Coefficient of y}} = \dfrac{-4}{6} = \dfrac{-2}{3}$

$m_1 = m_2 = \dfrac{-2}{3}$

$\therefore$ L_1 and L_2 are parallel.

11) Prove that $2x + 3y - 5 = 0$ & $4x + 6y - 1 = 0$ are parallel to each other

Ans.

Let $L_1 : 2x + 3y - 5 = 0$

$\therefore$ Slope of L_1 is $m_1 = \dfrac{-\text{Coefficient of x}}{\text{Coefficient of y}} = \dfrac{-2}{3}$

and $L_2 : 4x + 6y - 1 = 0$

$\therefore$ Slope of L_2 is $m_2 = \dfrac{-\text{Coefficient of x}}{\text{Coefficient of y}} = \dfrac{-4}{6} = \dfrac{-2}{3}$

We observed that $m_1 = m_2 = \dfrac{-2}{3}$

This is the condition for parallel lines.

$\therefore$ $L_1 \,||\, L_2$

$\therefore$ Given lines are parallel.

12) Prove that the lines $3x + 2y = 5$ & $2x - 3y = 6$ are perpendicular lines

[S-19,W-16]

Ans. slope of $3x+2y=5$ is

$$m_1 = -\frac{3}{2}$$

slope of $2x-3y=6$ is

$$m_2 = \frac{-2}{-3} = \frac{2}{3}$$

$$m_1 m_2 = \left(-\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$$

$\therefore$ lines are perpendicular.

13) Show that $2x + y + 3 = 0$ and $x - 2y - 1 = 0$ are perpendicular lines

[W-18]

Ans. Slope of line $2x+y+3=0$ is $m_1 = \frac{-a}{b} = \frac{-2}{1} = -2$

Slope of line $x-2y-1=0$ is $m_2 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$

$$\therefore m_1 \cdot m_2 = -2 \times \frac{1}{2}$$

$$\therefore m_1 \cdot m_2 = -1$$

Lines are perpendicular.

14) Show that the lines $5x + 6y - 1 = 0$ and $6x - 5y + 3 = 0$ are perpendicular lines **[S-15]**

Ans *i) For the line* $5x+6y-1=0$

$$\therefore slope\ m_1 = -\frac{A}{B} = -\frac{5}{6}$$

$ii)\ For\ the\ line\ \ 6x-5y+3=0$

$\therefore slope\ \ m_2=-\frac{A}{B}=-\frac{6}{-5}=\frac{6}{5}$

$\therefore m_1=-\frac{5}{6}=-\frac{1}{6/5}=-\frac{1}{m_2}$

$\therefore m_1\cdot m_2=-\frac{5}{6}\times\frac{6}{5}=-1$

$\therefore$ the lines are perpendicular.

15) Prove that the lines $3x-2y+6=0$ and $2x+3y-1=0$ are perpendicular to each other **[S-18]**

Ans.

$L_1:3x-2y+6=0$

$L_2:2x+3y-1=0$

$m_1=\frac{-3}{-2}=\frac{3}{2}$

$m_2=\frac{-2}{3}$

consider $m_1.m_2=\frac{3}{2}.\frac{-2}{3}=-1$

$\therefore$ Lines are perpendicular to each other.

16) If the lines $3y+4px+8=0$ & $3px-9y+10=0$ are perpendicular, find the value of $'p'$ **[S-17]**

Ans.

$L_1:3x+4py+8=0$ and

$L_2:-9x+3py+10=0$

$\therefore$ slope $=\frac{-a}{b}$

$m_1 = \frac{-3}{4p}$

$m_2 = \frac{9}{3p}$

lines are perpendicular

$\therefore m_1 m_2 = -1$

$\frac{-3}{4p}\frac{9}{3p} = -1$

$\therefore \frac{9}{4p^2} = 1$

$\therefore \frac{9}{4} = p^2$

$\therefore p = \pm\frac{3}{2}$

17) Find intercepts of a line $2x + 3y = 6$ on co-ordinates axes **[W-13]**

Ans. $2x + 3y = 6$

$put\ y = 0$

$x -$ intercept is 3

$put\ x = 0$

$\therefore y -$ intercept is 2

18) Find the slope and $X -$ intercept of the line $\frac{x}{2} - \frac{y}{3} = \frac{1}{4}$ **[W-17]**

Ans. $\frac{x}{2} - \frac{y}{3} - \frac{1}{4} = 0$

$6x - 4y - 3 = 0$

$\text{Slope} = -\frac{a}{b} = -\frac{6}{-4} = \frac{3}{2}$

$\text{X-intercept} = -\frac{c}{a} = -\frac{-3}{6} = \frac{1}{2}$

19) Find the slope and Y −intercept of line $\frac{x}{4} - \frac{y}{3} = 2$ **[W-14]**

Ans.

$$\frac{x}{4} - \frac{y}{3} = 2$$

$$\therefore 3x - 4y - 24 = 0$$

$$\therefore a = 3 \qquad b = -4 \qquad c = -24$$

$$\therefore slope\ \ m = -\frac{a}{b} = -\frac{3}{-4} = \frac{3}{4} \quad or \quad 0.75$$

$$y - \text{int} = -\frac{c}{b} = -\frac{-24}{-4} = -6$$

20) Find the equation of the line whose X −intercept is double its Y −intercept on co-ordinate axes & passing through the point $(4\ ,\ 1)$ **[S-17]**

Ans.

Let x-intercept = a

y-intercept = b

from given $a = 2b$

equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\therefore \frac{x}{2b} + \frac{y}{b} = 1$$

$$\therefore x + 2y = 2b$$

at $(4,1)$

$$6 = 2b$$

$$\therefore b = 3$$

$\therefore$ equation of line is

$$x + 2y = 6$$

21) Find the equation of the straight line passing through $(12\ ,-4)$ and sum of their intercept is 10 **[W-15]**

Ans. Let $x-$intercept is a

$y-$intercept is b

$a+b=10$

Equation of line is,

$$\therefore \frac{x}{a}+\frac{y}{b}=1$$

line passing through $(12,-4)$

$$\therefore \frac{12}{a}+\frac{(-4)}{10-a}=1$$

$$\therefore a^2-26a+120=0$$

$$\therefore a=20, a=6$$

$$\therefore b=-10, b=4$$

When $a=20, b=-10$

Equation of line is,

$$\therefore \frac{x}{20}-\frac{y}{10}=1$$

$i.e. x-2y=20$

When $a=6, b=4$

Equation of line is,

$$\therefore \frac{x}{6}+\frac{y}{4}=1$$

$i.e. 2x+3y=12$

22) Find the equation of the straight line passing through (– 3 , 10) and sum of their intercept is 8 **[W-14]**

Ans. $Let\ x-\text{int}=a$ $\quad y-\text{int}=b$

$\therefore a+b=8$

$\therefore$ *equation is*

$$\frac{x}{a}+\frac{y}{b}=1 \quad or \quad \frac{x}{a}+\frac{y}{8-a}=1$$

$\therefore bx+ay=ab$

$\therefore (8-a)x+ay=a(8-a)$

But passing through $(-3,\ 10)$

$\therefore -3(8-a)+10a=a(8-a)$

$\therefore -24+3a+10a=8a-a^2$

$\therefore a^2+5a-24=0$

$\therefore a=3,\ -8$

$\therefore \frac{x}{3}+\frac{y}{5}=1 \quad or \quad \frac{x}{-8}+\frac{y}{16}=1$

23) Show that the points (6 , 1) , (−1 , 8) , (3 , −2) are the vertices of right angled triangle by using slopes **[W-17]**

Ans. Let $A(6,1)$, $B(-1,8)$, $C(3,-2)$ are the vertices of the ΔABC,

Slope of side $AB = m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{8-1}{-1-6} = -1$

Slope of side $BC = m_2 = \frac{-2-8}{3+1} = \frac{-5}{2}$

Slope of side $AC = m_3 = \frac{-2-1}{3-6} = 1$

We observe that $m_1 \times m_3 = -1$

$\therefore$ side $AB \perp$ side AC

$\therefore \Delta ABC$ is right angled triangle at vertex A.

TUTORIAL NO.9
STRAIGHT LINE

1) Find the equation of line passing through (4 , – 5) and having slope $\frac{-2}{3}$ **[W-18,S-15]**

2) Find the equation of straight line passes through the points (– 4 , 6) and (8 , – 3) **[W-18,S-14]**

3) State the conditions of parallel and perpendicular lines, whose slopes are M_1 and M_2 **[S-17,W12]**

4) Prove that $2x + 3y + 7 = 0$ & $4x + 6y + 2 = 0$ are parallel to each other

5) Prove that the lines $3x + 2y = 5$ & $2x - 3y = 6$ are perpendicular lines **[S-19,W-16]**

6) If the lines $3y + 4px + 8 = 0$ & $3px – 9y + 10 = 0$ are perpendicular, find the value of $'p'$ **[S-17]**

7) Find intercepts of a line $2x + 3y = 6$ on co-ordinates axes **[W-13]**

8) Find the slope and Y –intercept of line $\frac{x}{4} - \frac{y}{3} = 2$ **[W-14]**

9) Find the equation of the line whose X –intercept is double its Y –intercept on co-ordinate axes & passing through the point (4 , 1) **[S-17]**

10) Find the equation of the straight line passing through (12 , −4) and sum of their intercept is 10 **[W-15]**

B. ANGLE BETWEEN LINES:-

If θ is the acute angle between two lines having slopes m_1 and m_2 then

$$\theta = \tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

Examples:-

Que. **Find the Acute angle OR Find the between the lines**

1) $3x - y = 4$ and $2x + y = 3$ **[W-19,S-19,S-18]**

Ans. For $3x - y = 4$,

$$\text{slope } m_1 = -\frac{a}{b} = -\frac{3}{-1} = 3$$

For $2x + y = 3$,

$$\text{slope } m_2 = -\frac{a}{b} = -\frac{2}{1} = -2$$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

$$= \left|\frac{3-(-2)}{1+3\times(-2)}\right|$$

$$= 1$$

2) $3x - y + 4 = 0$ and $2x + y - 3 = 0$ **[S-17,S-13]**

Ans. For $3x - y+4=0$

$$\text{slope}=m_1 = \frac{-a}{b} = \frac{-3}{-1} = 3$$

For $2x+y-3=0$

$$\text{slope}=m_2=\frac{-a}{b}=\frac{-2}{1}=-2$$

3) $2x+3y+5=0$ and $x-2y-4=0$ **[W-17,W-12]**

Ans.

$$slope\ \ m_1=-\frac{a}{b}=-\frac{2}{3}$$

$For\ \ x-2y-4=0,$

$$slope\ \ m_1=-\frac{a}{b}=-\frac{1}{-2}=\frac{1}{2}$$

$$\therefore \tan\theta=\left|\frac{m_1-m_2}{1+m_1\cdot m_2}\right|$$

$$=\left|\frac{-\frac{2}{3}-\frac{1}{2}}{1+\left(-\frac{2}{3}\right)\cdot\left(\frac{1}{2}\right)}\right|$$

$$=\frac{7}{4}\quad or\quad 1.75$$

$$\therefore \theta=\tan^{-1}\left(\frac{7}{4}\right)\quad or\quad \tan^{-1}(1.75)$$

4) $2x+3y=13$ and $2x-5y+7=0$ **[W-14]**

Ans.

$For\ 2x+3y=13,$

$$slope\ \ m_1=-\frac{a}{b}=-\frac{2}{3}$$

$For\ \ 2x-5y+7=0,$

$$slope\ \ m_1=-\frac{a}{b}=-\frac{2}{-5}=\frac{2}{5}$$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$$

$$= \left|\frac{-\frac{2}{3} - \frac{2}{5}}{1 + \left(-\frac{2}{3}\right)\cdot\left(\frac{2}{5}\right)}\right|$$

$$= \frac{16}{11} \quad or \quad 1.455$$

$$\therefore \theta = \tan^{-1}\left(\frac{16}{11}\right) \quad or \quad \tan^{-1}(1.455)$$

5) $y = 5x + 6$ and $y = x$ **[S-19,W-18,S-15,SQP]**

Ans. $For \ y = 5x + 6 \ \ i.e., \ \ 5x - y + 6 = 0 \quad or \quad -5x + y - 6 = 0$

$$slope \ m_1 = -\frac{a}{b} = 5$$

$$For \ \ y = x \quad or \quad x - y = 0,$$

$$slope \ m_1 = -\frac{a}{b} = 1$$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$$

$$= \left|\frac{5 - 1}{1 + (5)\cdot(1)}\right|$$

$$= \frac{2}{3} \quad or \quad 0.667$$

$$\therefore \theta = \tan^{-1}\left(\frac{2}{3}\right) \quad or \quad \tan^{-1}(0.667)$$

6) $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$ **[S-15]**

Ans. $For\ 3x-2y+4=0,$

$$slope\ \ m_1=-\frac{a}{b}=-\frac{3}{-2}=\frac{3}{2}$$

$For\ \ 2x-3y-7=0,$

$$slope\ \ m_2=-\frac{a}{b}=-\frac{2}{-3}=\frac{2}{3}$$

$$\therefore \tan\theta=\left|\frac{m_1-m_2}{1+m_1\cdot m_2}\right|$$

$$=\left|\frac{\frac{3}{2}-\frac{2}{3}}{1+\left(\frac{3}{2}\right)\cdot\left(\frac{2}{3}\right)}\right|$$

$$=\frac{5}{12}\quad or\quad 0.417$$

$$\therefore \theta=\tan^{-1}\left(\frac{5}{12}\right)\quad or\quad \tan^{-1}\left(0.417\right)$$

7) $3x + 2y + 4 = 0$ and $2x - 3y - 7 = 0$ **[W-18]**

Ans. $\text{For }3x+2y+4=0,$

$$\text{slope }\ m_1=\frac{-a}{b}=\frac{-3}{2}$$

$\text{For }\ 2x-3y-7=0,$

$$\text{slope }\ m_2=\frac{-a}{b}=\frac{-2}{-3}=\frac{2}{3}$$

$$\therefore \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$=\left|\frac{\frac{-3}{2}-\frac{2}{3}}{1+\left(\frac{-3}{2}\right)\left(\frac{2}{3}\right)}\right|$$

$$\therefore \tan\theta = \infty$$

$$\therefore \theta = \tan^{-1}(\infty)$$

$$\therefore \theta = 90^0 \quad \text{or} \quad \frac{\pi}{2}$$

8) $3x + 2y = 6$ and $2x - 3y = 5$ **[S-16]**

Ans.

$$m = \frac{-3}{2} \quad m = \frac{2}{3}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{-3}{2} - \frac{2}{3}}{1 + \frac{-3}{2} \times \frac{2}{3}}\right| = \infty$$

$$\theta = \tan^{-1}\infty = 90^0 = \frac{\pi}{2}$$

9) $3x - 4y = 420$ and $4x + 3y = 420$ **[S-18]**

Ans.

For $3x - 4y = 420$

$$\text{slope } m_1 = -\frac{a}{b} = -\frac{3}{-4} = \frac{3}{4}$$

For $4x + 3y = 420$

$$\text{slope } m_2 = -\frac{a}{b} = -\frac{4}{3}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left| \frac{\frac{3}{4} - \left(\frac{4}{-3} \right)}{1 + \frac{3}{4} \times \left(\frac{4}{-3} \right)} \right|$$

$$= \infty$$

$$\therefore \ \theta = \tan^{-1}(\infty)$$

$$\therefore \ \theta = \frac{\pi}{2} \text{ or } 90^0$$

10) Find the acute angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$ **[W-13]**

Ans.

$$m_1 = \sqrt{3} \text{ and } m_2 = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\,\frac{1}{\sqrt{3}}} \right|$$

$$= \left| \frac{2}{2\sqrt{3}} \right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}} \right)$$

$$\theta = \frac{\pi}{6} \text{ or } 30^0$$

11) Find the equation of line passing through (– 1 , 1) and making an angle $\frac{\pi}{4}$ with the line $2x + 3y = 6$ **[W-17]**

Ans. The slope of the given line $2x+3y=6$ is

$m_1=\frac{-2}{3}$

Let the slope of the required line be 'm'

$\therefore\ \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

putting $\theta=45^0$, $m_1=\frac{-2}{3}$, $m_2=m$

$$\therefore\ \tan45^0=\left|\frac{\frac{-2}{3}-m}{1+\left(\frac{-2}{3}\right)m}\right|$$

$$\therefore 1=\left|\frac{-2-3m}{3-2m}\right|$$

$$\therefore \frac{-2-3m}{3-2m}=\pm1$$

$$\therefore \frac{-2-3m}{3-2m}=1 \text{ or } \frac{-2-3m}{3-2m}=-1$$

$$\therefore -2-3m=3-2m \quad\text{or}\quad -2-3m=-3+2m$$

$$\therefore m=-5 \text{ or } m=\frac{1}{5}$$

Hence the equation of lines, in slope-point form is

(i) for $m=-5$

$y-y_1=m(x-x_1)$

$\therefore y-1=-5(x+1)$

$\therefore 5x+y+4=0$

(ii) for $m=\frac{1}{5}$

$y-y_1=m(x-x_1)$

$\therefore y-1=\frac{1}{5}(x+1)$

$\therefore x-5y+6=0$

C. CONDITION OF PARALLEL AND PERPENDICULAR LINES :–

Examples:-

1) Find the equation of line through the point (4 , 5) parallel to the line

$2x - 3y - 5 = 0$

Ans. The slope of the given line $m_1 = \frac{-2}{-3} = \frac{2}{3}$

$m_1 = m_2$

$m_2 = \frac{2}{3}$

By slope point form

$y - y_1 = m_2(x - x_1)$

$y - 5 = \frac{2}{3}(x - 4)$

$3y - 15 = 2x - 8$

$2x - 3y + 7 = 0$

2) Find the equation of line passing through the point (2 , 3) and parallel to the line $x - y + 7 = 0$

Ans. The slope of the given line $m_1 = \frac{-1}{-1} = 1$

$m_1 = m_2$

$m_2 = 1$

By slope point form

$y - y_1 = m_2(x - x_1)$

$y - 3 = 1(x - 2)$

$y - 3 = x - 2$

$x - y + 1 = 0$

3) Find the equation of line passing through the point $(2 , -3)$ and parallel to the line $4x - y + 7 = 0$

Ans. The slope of the given line $m_1 = \frac{-4}{-1} = 4$

$m_1 = m_2$

$m_2 = 4$

By slope point form

$y - y_1 = m_2(x - x_1)$

$y - -3 = 4(x - 2)$

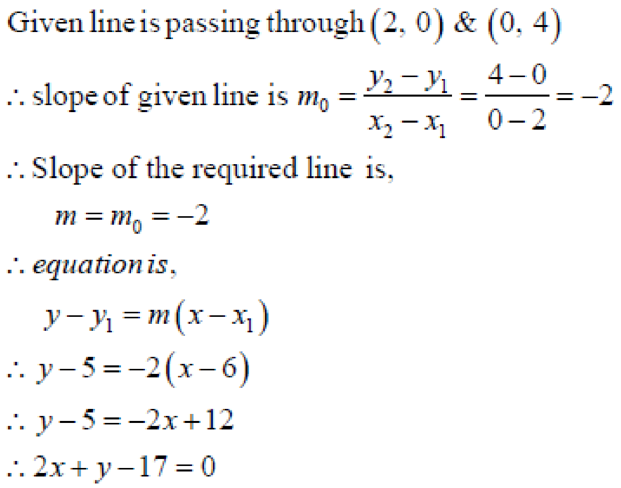
$y + 3 = 4x - 8$

$4x - y - 11 = 0$

4) Find the equations of the lines passing through the point $(6 , \ 5)$ and parallel to the line having intercepts 2 and 4 on X and Y axis respectively

[W-18,S-15]

Ans.



Given line is passing through $(2, 0)$ & $(0, 4)$

$\therefore$ slope of given line is $m_0 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-2} = -2$

$\therefore$ Slope of the required line is,

$m = m_0 = -2$

$\therefore$ equation is,

$y - y_1 = m(x - x_1)$

$\therefore \ y - 5 = -2(x - 6)$

$\therefore \ y - 5 = -2x + 12$

$\therefore 2x + y - 17 = 0$

5) Find the equation of line passing through the point (3 , 4) and perpendicular to the line $2x - 4y + 5 = 0$ **[S-18]**

Ans. Point $=(x_1, y_1)=(3,4)$

Slope of the line $2x-4y+5=0$ is,

$$m=-\frac{a}{b}=-\frac{2}{-4}=\frac{1}{2}$$

$\therefore$ Slope of the required line is,

$$m'=-\frac{1}{m}=-2$$

$\therefore$ equation is,

$$y-y_1=m'(x-x_1)$$

$$\therefore y-4=-2(x-3)$$

$$\therefore 2x+y-10=0$$

6) Find the equation of line passing through the point (3 , 4) and perpendicular to the line $3x + 2y + 5 = 0$ **[W-13]**

Ans. $(x_{1,}y_1)=(3,4)$

slope of the line $3x+2y+5=0$ is $m_1=\frac{-3}{2}$

$$\therefore m_2=\frac{2}{3}$$

$\therefore$ equation of required line is,

$$(y-y_1)=m_2(x-x_1)$$

$$y-4=\frac{2}{3}(x-3)$$

$$2x-3y+6=0$$

7) Find the equation of line passing through the point (4 , 5) and perpendicular to the line $7x - 5y = 420$ **[S-19,SQP]**

Ans. Point $=(x_1,y_1)=(4,5)$

Slope of the line $7x-5y=420$ is,

$$m=-\frac{a}{b}=-\frac{7}{-5}=\frac{7}{5}$$

$\therefore$ Slope of the required line is,

$$m_1=-\frac{1}{m}=\frac{-5}{7}$$

$\therefore$ equation is,

$$y-y_1=m_1(x-x_1)$$

$$\therefore\ y-5=\frac{-5}{7}(x-4)$$

$$\therefore 5x+7y-55=0$$

8) Find the equation of line passing through the point (2 , 3) and perpendicular to the line $3x - 5y = 6$ **[W-19]**

Ans. Point $(x_1,y_1)=(2,3)$

Slope of the line $3x-5y-6=0$ is,

$$m=-\frac{a}{b}=-\frac{3}{-5}=\frac{3}{5}$$

$\therefore$ Slope of the required line is,

$$m'=-\frac{1}{m}=-\frac{5}{3}$$

$\therefore$ equation is,

$$y-y_1=m'(x-x_1)$$

$$\therefore\ y-3=-\frac{5}{3}(x-2)$$

$$\therefore\ 3y-9=-5x+10$$

$$\therefore\ 5x+3y-19=0$$

9) Find the equation of line passing which is perpendicular bisector of the line joining the points $A(8,-1)$ and $B(6,3)$ **[S-17]**

Ans. Let P be midpoint of AB

$$\therefore P \text{ is } \left(\frac{8+6}{2},\frac{-1+3}{2}\right)$$

$$i.e. P(7,1)$$

$$\text{Slope of } AB, m_1 = \frac{3-(-1)}{6-8}$$

$$m_1 = -2$$

$\because$ required line is perpendicular to AB

$$\therefore m_1 m_2 = -1$$

$$\therefore m_2 = \frac{1}{2}$$

$\therefore$ equation of required line is

$$y - y_1 = m_2(x - x_1)$$

$$\therefore y - 1 = \frac{1}{2}(x-7)$$

$$\therefore x - 2y - 5 = 0$$

10) Find the equation of line passing which is perpendicular bisector of the line joining the points $A(-2,3)$ and $B(8,-1)$ **[W-13]**

Ans. mid point of AB is $\left(\frac{-2+8}{2},\frac{3-1}{2}\right)=(3,1)$

slope of AB is, $m_1 = \frac{-1-3}{8+2} = \frac{-2}{5}$

$$y-1=\frac{5}{2}(x-3)$$

$$5x-2y-13=0$$

D. INTERSECTION OF TWO LINES:–

Examples:-

1) Find the equation of line passing through the point (2 , 5) and through the intersection of the lines $x + y = 0$ and $2x - y = 9$

[S-18,W-18,W-16,W-14,S-13, W-12]

Ans.

$$x+y=0$$
$$\underline{2x-y=9}$$
$$\therefore\ 3x=9$$
$$\therefore x=3$$
$$y=-3$$
$$\therefore \text{Point of intersection} = (3,\ -3)$$
$$\therefore \textit{equation is},$$
$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$
$$\therefore \frac{y-5}{-3-5}=\frac{x-2}{3-2}$$
$$\therefore 8x+y-21=0$$

2) Find the equation of line passing through the point (4 , 5) and through the intersection of the lines $x + y = 0$ and $2x - y = 9$ **[S-16]**

Ans.

$$x+y=0,\quad 2x-y=9$$
$$\therefore\quad x+y=0$$
$$\underline{2x-y=9}$$
$$\therefore\quad 3x=9$$

$\therefore \quad x = 3$

$y = -3$

$\therefore$ point of intersection $= (3, -3) = (x_1, y_1)$

and given point $= (4, 5) = (x_2, y_2)$

its equation in two points form is $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

$$\therefore \frac{y - (-3)}{5 - (-3)} = \frac{x - 3}{4 - 3}$$

$$\therefore \frac{y + 3}{8} = x - 3$$

$$\therefore y + 3 = 8(x - 3)$$

$$\therefore y + 3 = 8x - 24$$

$$\therefore -8x + y + 27 = 0 \quad \text{or} \quad 8x - y - 27 = 0$$

3) Find the equation of line passing through the point (3 , 2) and through the intersection of the lines $2x + 3y = 1$ and $3x - 4y = 4$ **[SQP]**

Ans. The point of intersection of the lines $2x + 3y = 1$ and $3x - 4y = 4$ is $\left(\frac{16}{17}, \frac{-5}{17}\right)$

$\therefore$ The required line passing through the point (3 , 2) & $\left(\frac{16}{17}, \frac{-5}{17}\right)$

$\therefore$ by two point form

$$\frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$$

$$\frac{\frac{-5}{17}-2}{\frac{16}{17}-3} = \frac{y-2}{x-3}$$

$$\frac{39}{35} = \frac{y-2}{x-3}$$

$$39x - 117 = 35y - 70$$

$$39x - 35y - 47 = 0$$

4) Find the equation of line passing through the point of intersection of the line $2x + y + 6 = 0$ & $3x + 5y - 15 = 0$ & parallel to the line $5x + 6y + 3 = 0$ **[S-19]**

Ans.

$$2x+y=-6$$
$$3x+5y=15$$

$$\therefore \begin{array}{r} 10x+5y=-30 \\ 3x+5y\ =15 \\ -\quad\ -\quad\ - \\ \hline 7x=-45 \end{array}$$

$$y=\frac{-45}{7}$$

$$\therefore\ 2x-\frac{45}{7}=-6$$

$$\therefore 2x=\frac{45}{7}-6$$

$$\therefore x=\frac{3}{14}$$

$$\therefore \text{Point of intersection}=\left(\frac{3}{14},\frac{-45}{7}\right)$$

Slope of the line $5x+6y+3=0$ is,

$$m=-\frac{a}{b}=-\frac{5}{6}$$

$\therefore$ Slope of the required line is,

$$m=-\frac{5}{6}$$

$\therefore$ equation is,

$$y-y_1=m\left(x-x_1\right)$$

$$\therefore y+\frac{45}{7}=-\frac{5}{6}\left(x-\frac{3}{14}\right)$$

$$\therefore y+\frac{45}{7}=-\frac{5x}{6}+\frac{5}{28} \qquad \text{OR} \qquad \frac{7y+45}{7}=-\frac{5}{6}\left(\frac{14x-3}{14}\right)$$

$$\therefore \frac{5x}{6}+y+\frac{45}{7}-\frac{5}{28}=0 \qquad \therefore 12(7y+45)=-5(14x-3)$$

$$\therefore \frac{5x}{6}+y+\frac{25}{4}=0 \qquad \therefore 70x+84y+525=0$$

5) Find the equation of a line passing through the point of intersection of lines $x-2y-5=0$ and $x+3y=10$ and parallel to the line $3x+4y=0$

[S-18]

Ans. $x-2y=5 \quad \times \; 3$

$x+3y=10 \quad \times \; 2$

$$\begin{array}{rl} \therefore & 3x-6y=15 \\ + & 2x+6y=20 \\ \hline & 5x=35 \\ & x=7 \end{array}$$

$\therefore \; 7-2y=5$

$\therefore -2y=-2$

$\therefore y=1$

$\therefore$ Point of intersection $=(7, 1)$

Slope of the line $3x+4y=0$ is,

$$m_1=-\frac{a}{b}=-\frac{3}{4}$$

$\therefore$ Slope of the required line is,

$$m = m_1 = -\frac{3}{4}$$

$\therefore$ equation of line is ,

$$y - y_1 = m\left(x - x_1\right)$$

$$\therefore y - 1 = -\frac{3}{4}(x - 7)$$

$$\therefore 3x + 4y - 25 = 0$$

6) Find the equation of line passing through the point of intersection of the line $4x + 3y = 8$ & $x + y = 1$ & parallel to the line $5x - 7y = 3$

[W-17,S-17,SQP]

Ans.

$$4x + 3y = 8$$

$$x + y = 1$$

$$4x + 3y = 8$$

$$3x + 3y = 3$$

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$$x = 5$$

$$\therefore y = -4$$

$\therefore$ Point of intersection is $(x, y) = (5, -4)$

line is parllel to the line $5x - 7y = 3$

$\therefore$ slope of required line $m = \frac{-a}{b} = \frac{5}{7}$

$\therefore\ y = -4$

$\therefore$ Point of intersection is $(x, y) = (5, -4)$

line is parllel to the line $5x - 7y = 3$

$\therefore$ slope of required line $m = \frac{-a}{b} = \frac{5}{7}$

$\therefore$ equation of line is

$y - y_1 = m(x - x_1)$

$\therefore\ y + 4 = \frac{5}{7}(x - 5)$

$\therefore\ 7y + 28 = 5x - 25$

$\therefore\ 5x - 7y - 53 = 0$

7) Find equation of line passing through the point of intersection of the lines

$2x + 3y = 13$ and $5x - y = 7$ and perpendicular to the line $3x - y + 7 = 0$

[W-18,S-16,S-15, W-12]

Ans.

$2x + 3y = 13$

$5x - y = 7$

$\therefore\ 2x + 3y = 13$

$15x - 3y = 21$

$\therefore\ 17x = 34$

$\therefore x = 2$

$y = 3$

$\therefore$ Point of intersection $= (2, 3)$

Slope of the line $3x - y + 7 = 0$ is,

$$m_0 = -\frac{a}{b} = -\frac{3}{-1} = 3$$

$\therefore$ Slope of the required line is,

$$m = -\frac{1}{m_0} = -\frac{1}{3}$$

$\therefore$ *equation is*,

$$y - y_1 = m(x - x_1)$$

$$\therefore y - 3 = -\frac{1}{3}(x - 2)$$

$$\therefore x + 3y - 11 = 0$$

8) Find equation of line passing through the point of intersection of the lines $2x + 3y = 13$ and $5x - y = 7$ and perpendicular to the line $3x - y + 17 = 0$

[S-13]

Ans.

$$2x + 3y = 13 \qquad - (1)$$

$$5x - y = 7 \qquad - (2)$$

$$2x + 3y = 13$$

$$\underline{15x - 3y = 21}$$

$$\therefore 17x \quad = 34$$

$$\therefore x = 2$$

$$y = 3$$

$\therefore$ point of intersection=(2,3)

slope of the line $3x - y + 17 = 0$ is

$$m_1 = \frac{-a}{b} = \frac{-3}{-1} = 3$$

$\therefore$ slope of the required line is,

$$m = -\frac{1}{m_1} = -\frac{1}{3}$$

$\therefore$ equation is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{3}(x - 2)$$

$$\therefore x + 3y - 11 = 0$$

9) Find equation of line passing through the point of intersection of the lines

$2x + 3y = 13$ and $5x - y = 7$ and perpendicular to the line

$3x - 2y + 7 = 0$ **[W-16]**

Ans.

$$2x + 3y = 13$$

$$5x - y = 7$$

$$\therefore \begin{array}{r} 2x + 3y = 13 \\ +\ 15x - 3y = 21 \\ \hline 17x = 34 \end{array}$$

$$x = 2$$

$$\therefore \quad 5(2) - y = 7$$

$$\therefore -y = -3$$

$$\therefore y = 3$$

$\therefore$ Point of intersection $=(2,\ 3)$

Slope of the line $3x-2y+7=0$. is,

$$m_0=-\frac{a}{b}=-\frac{3}{-2}=\frac{3}{2}$$

$\therefore$ Slope of the required line is,

$$m=-\frac{1}{m_0}=-\frac{1}{\frac{3}{2}}=-\frac{2}{3}$$

$\therefore$ *equation is,*

$$y-y_1=m(x-x_1)$$

$$\therefore\ y-3=-\frac{2}{3}(x-2)$$

$$\therefore 2x+3y-13=0$$

10) Find equation of line passing through the point of intersection of the lines $2x+3y=13$ and $5x-y=7$ and perpendicular to the line $2x-5y+7=0$ **[S-14]**

Ans.

$$2x+3y=13,\ \ 5x-y=7$$

$$\therefore\ \ 2x+3y=13$$

$$\underline{15x-3y=21}$$

$$\therefore\ \ 17x=34$$

$$\therefore x=2$$

$$y=3$$

$\therefore$ Point of intersection $=(2,\ 3)$

Slope of line $2x-5y+7=0$ *is*

$$m_0=-\frac{A}{B}=-\frac{2}{-5}=\frac{2}{5}$$

$$\therefore Slope\ of\ required\ line is\ m=-\frac{5}{2}$$

$$\therefore the\ equation is,$$

$$y-y_1=m\left(x-x_1\right)$$

$$\therefore y-3=-\frac{5}{2}\left(x-2\right)$$

$$\therefore 2y-6=-5x+10$$

$$\therefore 5x+2y-16=0 \qquad or \qquad 5x+2y=16$$

11) Find equation of line passing through the point of intersection of the lines

$2x+3y=13$ and $5x-y=7$ and perpendicular to the line

$2x-5y+9=0$ **[W-13]**

Ans.

$$2x+3y=13$$

$$5x-y=7 \quad \times 3$$

$$\therefore 2x+3y=13$$

$$+\,15x-3y=21$$

$$17x=34$$

$$\therefore x=2 \ \text{ and } y=3$$

$$\left(x_{1,}y_1\right)=\left(2,3\right)$$

$slope$ of the line $2x-5y+9=0$ is $m_1=\frac{2}{5}$

$$\therefore m_2=\frac{-5}{2}$$

$\therefore$ equation of required line is,

$$\left(y-y_1\right)=m_2\left(x-x_1\right)$$

$$y-3=\frac{-5}{2}(x-2)$$

$$5x+2y-16=0$$

12) Find the co-ordinate of the foot of perpendicular drawn from (3 , 4) to the straight line $4x-2y+9=0$ **[W-17]**

Ans. The slope of the given line $4x-2y+9=0$ is

$$m_1=\frac{-4}{-2}=2$$

For perpendicular lines , $m_1m_2=-1$

$$\therefore m_2=\frac{-1}{2}$$

Equation of perpendicular is

$$(y-y_1)=m(x-x_1)$$

$$\therefore y-4=\frac{-1}{2}(x-3)$$

$$\therefore x+2y-11=0$$

Foot of the perpendicular = Point of intersection of two lines

$\therefore$ Solving the equations

$$4x-2y=-9$$

$$x+2y=11$$

$$\therefore 4x-2y=-9$$

$$+\ x+2y=11$$

$$5x=2$$

$$\therefore x=\frac{2}{5}\quad \text{and } y=\frac{53}{10}$$

$$(x_{1,}y_1)=\left(\frac{2}{5},\frac{53}{10}\right)$$

E. DISTANCE OF A POINT FROM A LINE :–

$$distance = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Examples:-

1) Find the length of perpendicular from the points (2 , 3) on the line

$4x - 6y - 3 = 0$ **[S-19,W-15]**

Ans.

$$p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{4(2) + (-6)(3) - 3}{\sqrt{(4)^2 + (-6)^2}}\right|$$

$$p = \left|\frac{8 - 18 - 3}{\sqrt{16 + 36}}\right|$$

$$= \left|\frac{-13}{\sqrt{52}}\right|$$

$$= \frac{13}{\sqrt{52}}$$

$$p = \frac{\sqrt{13}}{2}$$

2) Find the length of perpendicular from the points (3 , 2) on the line

$4x - 6y - 5 = 0$ **[S-14]**

Ans.

$Given\ \ 4x - 6y - 5 = 0$

$\therefore A = 4,\ \ B = -6,\ \ C = -5$

$\therefore$ *the length of the perpendicular is,*

$$p=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$

$$=\left|\frac{4(3)-6(2)-5}{\sqrt{4^2+(-6)^2}}\right|$$

$$=\frac{5}{\sqrt{52}} \quad or \quad 0.693$$

3) Find the length of perpendicular from the points (2 , 5) on the line

$2x+3y-6=0$ **[W-19]**

Ans.

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$p=\left|\frac{2(2)+3(5)-6}{\sqrt{(2)^2+(3)^2}}\right|$$

$$p=\frac{13}{\sqrt{13}} \quad \text{or} \quad \sqrt{13} \quad \text{or} \quad 3.61$$

4) Find the perpendicular distance between the point (3 , 4) and the line

$3x+4y=5$ **[S-13]**

OR

Find length of perpendicular from the point (3 , 4) on the line

$3x+4y-5=0$ **[W-16,W-13,SQP]**

Ans.

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$= \left|\frac{3(3)+4(4)-5}{\sqrt{(3)^2+(4)^2}}\right|$$

$$= \left|\frac{9+16-5}{\sqrt{9+16}}\right|$$

$$= \left|\frac{20}{\sqrt{25}}\right|$$

$$= \frac{20}{5}$$

$$p = 4 \text{ units}$$

5) Find the length of the perpendicular from the point (5 , 4) on the straight line $2x + y = 34$ **[S-18]**

Ans.

$$p = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$= \left|\frac{2(5)+1(4)-34}{\sqrt{(2)^2+(1)^2}}\right|$$

$$= \left|\frac{10+4-34}{\sqrt{5}}\right|$$

$$= \frac{20}{\sqrt{5}} \quad \text{OR} \quad 8.94$$

6) Find the distance between (-2 , 3) & the line $3x + 2y + 26 = 0$

[S-19,SQP]

Ans. $3x+2y+26=0$

$$d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$\therefore d = \left|\frac{3(-2)+2(3)+26}{\sqrt{(3)^2+(2)^2}}\right|$$

$$\therefore d = \left|\frac{26}{\sqrt{13}}\right| \quad \text{or} \quad 2\sqrt{13} \quad \text{or} \quad 7.21$$

7) If the length of perpendicular from (5 , 4) on the straight line $2x + y + K = 0$ is $4\sqrt{5}$ units. Find the value of K **[W-14]**

Ans.

$$p = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$\therefore 4\sqrt{5} = \left|\frac{2(5)+4+k}{\sqrt{2^2+1^2}}\right|$$

$$\therefore 4\sqrt{5} = \left|\frac{14+k}{\sqrt{5}}\right|$$

$$\therefore 4\sqrt{5}\cdot\sqrt{5} = |14+k|$$

$$\therefore 20 = |14+k|$$

$$\therefore 20 = 14+k \quad or \quad -20 = 14+k$$

$$\therefore \boxed{6 = k} \quad or \quad \boxed{-34 = k}$$

F. DISTANCE BETWEEN TWO PARALLEL LINES :–

$$distance = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

Examples:-

1) Find the distance between two parallel lines $3x - y + 7 = 0$ and $3x - y + 16 = 0$ **[S-18,W-17]**

Ans. For $3x - y + 7 = 0$

$a = 3,\ b = -1,\ c_1 = 7$

For $3x - y + 16 = 0$

$a = 3,\ b = -1,\ c_2 = 16$

$\therefore$ distance between two parallel lines is

$$= \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right| = \left|\frac{16 - 7}{\sqrt{3^2 + (-1)^2}}\right|$$

$$= \left|\frac{9}{\sqrt{10}}\right|$$

$$= \frac{9}{\sqrt{10}} \quad \text{OR } 2.846$$

2) Find the distance between parallel lines $3x + 2y - 6 = 0$ and $3x + 2y - 12 = 0$

Ans. $A = 3, B = 2, C_1 = -6, C_2 = -12$

$$p = \left|\frac{C_1 - C_2}{\sqrt{A^2 + B^2}}\right|$$

$$p = \left|\frac{-6-(-12)}{\sqrt{(3)^2 + (2)^2}}\right|$$

$$p = \left|\frac{6}{\sqrt{9+4}}\right|$$

$$p = \frac{6}{\sqrt{13}} \text{ units.}$$

3) Find the distance between the lines $4x - 3y + 2 = 0$ and $4x + 3y - 9 = 0$

Ans. $4x + 3y + 2 = 0$ and $4x + 3y - 9 = 0$

Here $a = 4,\ b = 3,\ c_1 = 2,\ c_2 = -9$

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{-9-2}{\sqrt{(4)^2 + (3)^2}}\right| = \left|\frac{-11}{\sqrt{16+9}}\right| = \left|\frac{-11}{\sqrt{25}}\right|$$

$$d = \frac{11}{5} \text{ units}$$

4) Find the distance between the lines $3x + 2y = 5$ and $6x + 4y = 6$

[W-18,S-16]

Ans. Let L : $3x + 2y = 5$

and $L_2 : 6x + 4y = 6 \Rightarrow 3x + 2y = 3$

$\therefore\ L_1 :\ 3x + 2y - 5 = 0$

$\therefore\ L_2 :\ 3x + 2y - 3 = 0$

$\therefore\ a = 3,\ b = 2,\ c = -5,\ c = -3$

distance between the lines is given by,

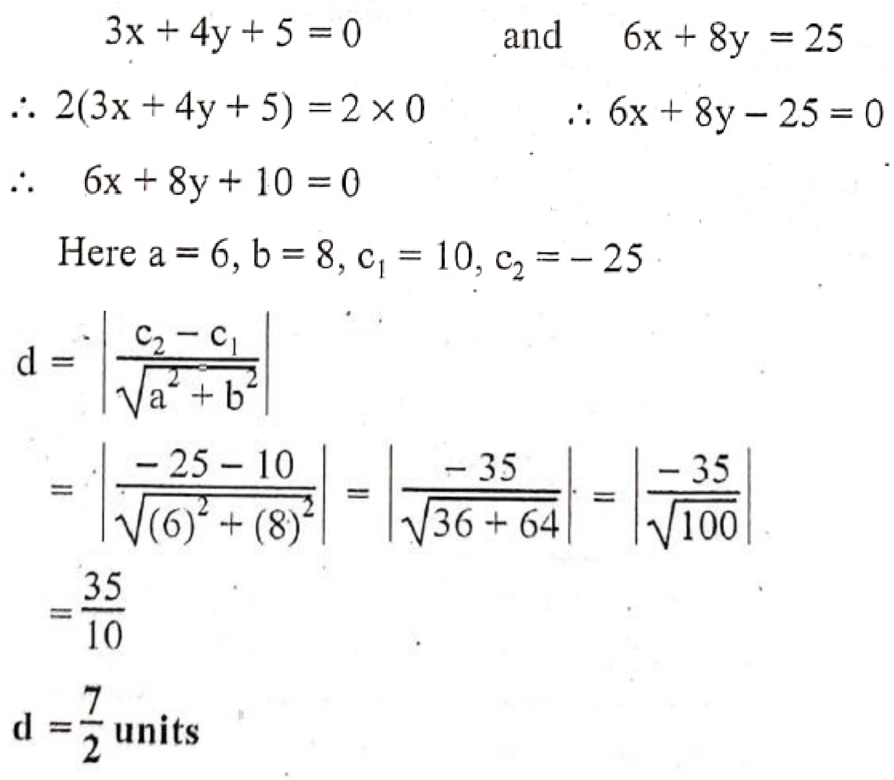
$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{-3-(-5)}{\sqrt{3 \ \ + 2}}\right|$$

$$= \left|\frac{2}{\sqrt{13}}\right| = \frac{2}{\sqrt{13}} \textit{units}$$

5) Find the distance between the lines $3x + 4y + 5 = 0$ and $6x + 8y = 25$

Ans.



$3x + 4y + 5 = 0$ and $6x + 8y = 25$

$\therefore\ 2(3x + 4y + 5) = 2 \times 0$ $\therefore\ 6x + 8y - 25 = 0$

$\therefore\ 6x + 8y + 10 = 0$

Here $a = 6,\ b = 8,\ c_1 = 10,\ c_2 = -25$

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{-25 - 10}{\sqrt{(6)^2 + (8)^2}}\right| = \left|\frac{-35}{\sqrt{36 + 64}}\right| = \left|\frac{-35}{\sqrt{100}}\right|$$

$$= \frac{35}{10}$$

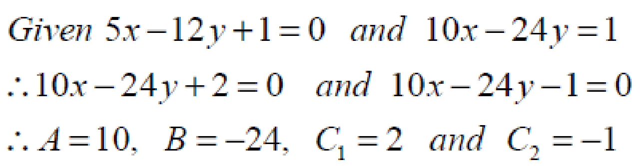
$d = \frac{7}{2}$ **units**

6) Find the perpendicular distance between two parallel lines $5x - 12y + 1 = 0$ and $10x - 24y = 1$ **[S-14]**

OR

Find the perpendicular distance between two parallel lines $5x - 12y + 1 = 0$ and $10x - 24y - 1 = 0$ **[W-13]**

Ans.



Given $5x - 12y + 1 = 0$ *and* $10x - 24y = 1$

$\therefore 10x - 24y + 2 = 0$ *and* $10x - 24y - 1 = 0$

$\therefore A = 10,\ B = -24,\ C_1 = 2$ *and* $C_2 = -1$

$$\therefore p=\left|\frac{C_1-C_2}{\sqrt{A^2+B^2}}\right|$$

$$=\left|\frac{2+1}{\sqrt{10^2+(-24)^2}}\right|$$

$$=\frac{3}{26} \qquad or \qquad 0.115$$

7) If the distance between the lines $12x+5y=K$ and $12x+5y=36$ is 2 then find K.

Ans. $12x+5y=K$ and $12x+5y=36$

$\therefore\ 12x+5y-K=0$ $\quad\therefore\ 12x+5y-36=0$

Here $a=12$, $b=5$, $c_1=-k$, $c_2=-36$

and also given $d=2$.

The distance between two parallel lines are given by

$$d=\left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right|$$

$$2=\left|\frac{-36-(-K)}{\sqrt{(12)^2+(5)^2}}\right|$$

$$2=\left|\frac{-36+K}{\sqrt{144+25}}\right|$$

$$2=\left|\frac{-36+K}{13}\right|$$

$$26=|-36+K|$$

$26=-36+K$ $\qquad -26=-36+K$

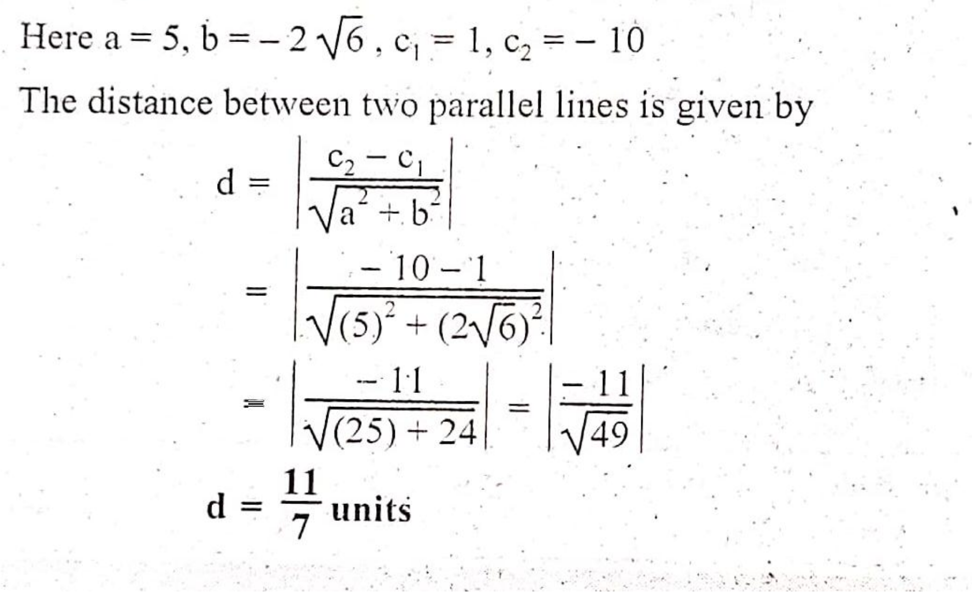
$K=26+36$ $\qquad K=-26+36$

$K=62$ $\qquad K=10$

8) Find the distance between the lines $5x - 2\sqrt{y} + 1 = 0$ &

$5x - 2\sqrt{y} - 10 = 0$

Ans.



Here a = 5, b = – 2 $\sqrt{6}$, c_1 = 1, c_2 = – 10

The distance between two parallel lines is given by

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{-10 - 1}{\sqrt{(5)^2 + (2\sqrt{6})^2}}\right|$$

$$= \left|\frac{-11}{\sqrt{(25) + 24}}\right| = \left|\frac{-11}{\sqrt{49}}\right|$$

$$d = \frac{11}{7} \text{ units}$$

TUTORIAL NO.10
STRAIGHT LINE

1) Find the acute angle between $2x + 3y + 5 = 0$ & $x - 2y - 4 = 0$ **[W-17,W-12]**

2) Find the acute angle between $y = 5x + 6$ & $y = x$ **[S-19,W-18,S-15,SQP]**

3) Find the equations of the lines passing through the point $(6, 5)$ and parallel to the line having intercepts 2 and 4 on X and Y axis respectively **[W-18,S-15]**

4) Find the equation of line passing through the point $(4, 5)$ and perpendicular to the line $7x - 5y = 420$ **[S-19,SQP]**

5) Find the equation of line passing through the point $(2, 5)$ and intersection of the lines $x + y = 0$ & $2x - y = 9$**[S-18,W-18,W-16,W-14,S-13,W-12]**

6) Find equation of line passing through the point of intersection of the lines $2x + 3y = 13$ and $5x - y = 7$ and perpendicular to $3x - y + 7 = 0$ **[W-18,S-16,S-15, W-12]**

7) Find length of perpendicular from the point $(2, 3)$ on the line $4x - 6y - 3 = 0$ **[S-19,W-15]**

8) Find length of perpendicular from the point $(3, 4)$ on the line $3x + 4y - 5 = 0$ **[W-16,W-13,SQP]**

9) Find the distance between two parallel lines $3x - y + 7 = 0$ and $3x - y + 16 = 0$ **[S-18,W-17]**

10) Find the distance between the lines $3x + 2y = 5$ and $6x + 4y = 6$

MENSURATION

Position in Question Paper **Total Marks-10**

Q.1. d) 2-Marks

Q.1. e) 2-Marks

Q.5. c) i) 3-Marks.

ii) 3- Marks

I. AREA:-

A. CIRCLE:-

FORMULAE:-

1) $Area \ = \pi r^2$

2) $Circumference = 2\pi r$

Examples:-

1) Find the area of the circle & circumference whose radius is 7.7 □□

Ans. $r = 7.7 \ cm$

$Area \ = \pi r^2 = \frac{22}{7}\boldsymbol{X}(7.7)^2 = 186.34 \ cm^2$

$Circumference = 2\pi r = 2\boldsymbol{X}\frac{22}{7}\boldsymbol{X}7.7 = 48.4 \ cm$

2) What is the radius of the circle if it's area is $120cm^2$? find it's circumference.

Ans. $A = 120\ cm^2$

$A = \pi r^2$

$120 = \frac{22}{7}\boldsymbol{X}(r)^2$

$38.18 = (r)^2$

$6.1791 = r$

$r = 6.1791\ \ cm$

$Circumference = 2\pi r = 2\boldsymbol{X}\frac{22}{7}\boldsymbol{X}6.1791 = 38.8400\ \ cm$

3) A circle has a diameter of $14\ cm$. Calculate it's area.

Ans. $d = 14\ cm$

$r = \frac{d}{2} = \frac{14}{2} = 7\ \ cm$

$A = \pi r^2 = \frac{22}{7}\boldsymbol{X}(7)^2 = 154\ \ \ cm^2$

4) Find the area of the of ring between two concentric circles whose circumference are $75\ cm$ & $55\ cm$. **[S-19]**

Ans. $2\pi r_1 = 75$

$2\boldsymbol{X}\frac{22}{7}\boldsymbol{X}\ r_1 = 75$

$r_1 = 11.93\ \ cm$

$2\pi r_2 = 55$

$2\boldsymbol{X}\frac{22}{7}\boldsymbol{X}\ r_2 = 55$

$r_2 = 8.75\ cm$

$A = \pi r_1^2 - \pi r_2^2$

$A = \pi(r_1^2 - r_2^2)$

$A = \frac{22}{7}[(11.93)^2 - (8.75)^2] = 206.68 \quad cm^2$

5) The area of the ring formed by two concentric circle is 346.5 cm^2. If the circumference of the inner circle is 88 cm. Find the radius of the outercircle.

Ans. Let r_1 be the radius of inner circle & r_2 be the radius of outer circle.

$Circumference = 2\pi r_1$

$88 = 2X\frac{22}{7}Xr_1$

$14 = r_1$

$r_1 = 14\ cm$

$(A)_{ring} = (A)_{outer} - (A)_{inner}$

$346.5 = \pi r_2^2 - \pi r_1^2$

$346.5 = \pi[r_2^2 - r_1^2]$

$346.5 = \frac{22}{7}[r_2^2 - (14)^2]$

$346.5 = \frac{22}{7}[r_2^2 - 196]$

$110.25 = r_2^2 - 196$

$306.25 = r_2^2$

$17.5 = r_2$

$r_2 = 17.5\ cm$

B. TRIANGLE:-

FORMULAE:-

1) $Area\ = \frac{1}{2}\ \boldsymbol{X}\ Base\ \boldsymbol{X}\ Height$

2) $Area\ = \sqrt{s(s-a)(s-b)(s-c)}$ where

$s = \frac{a+b+c}{2} = semiperimeter$

$Perimeter = 2s$

Examples:-

1) Find the area of the triangular plot whose base is 17.2 cm & height 19.60cm.

Ans. $Area\ = \frac{1}{2}\ \boldsymbol{X}\ Base\ \boldsymbol{X}\ Height$

$Area\ = \frac{1}{2}\ \boldsymbol{X} 17.2\ \boldsymbol{X}\ 19.60$

$Area\ = 168.56\ \ cm^2$

2) In a right angle triangle, the length of one side is 4.5 cm & the length of the hypotenuse is 20.5 cm.Find the area of the right triangle.

Ans. $Area\ = \frac{1}{2}\ \boldsymbol{X}\ Base\ \boldsymbol{X}\ Height$

$Area\ = \frac{1}{2}\ \boldsymbol{X}\ 4.5\ \boldsymbol{X}\ 20$

$Area\ = 45\ \ cm^2$

3) The base of the right angled triangle is 8 m & hypotenuse is 10 m . Find it's area.

Ans.

$$Area = \frac{1}{2} \boldsymbol{X} \; Base \; \boldsymbol{X} \; Height$$

$$Area = \frac{1}{2} \boldsymbol{X} \; 6 \; \boldsymbol{X} \; 8$$

$$Area = 24 \; cm^2$$

4) A park is in the form of a right angled triangle with hypotenuse13 cm. If one of the side is 12 cm ,find the cost of leveling at the rate of ₹ 10 per sq.cm

Ans.

$$Area = \frac{1}{2} \boldsymbol{X} \; Base \; \boldsymbol{X} \; Height$$

$$Area = \frac{1}{2} \boldsymbol{X} \; 5 \; \boldsymbol{X} \; 12$$

$$Area = 30 \; cm^2$$

$$Cost = Area \; \boldsymbol{X} \; Rate$$

$$Cost = 30 \; \boldsymbol{X} \; 10$$

$$Cost = 300 \; Rs.$$

5) Find the area of the triangle whose sides are 4 cm , 6 cm & 8 cm.

Ans. $Area = \sqrt{s(s-a)(s-b)(s-c)} \qquad s = \frac{a+b+c}{2}$

$a = 4\ cm \ , \qquad b = 6\ cm, \qquad c = 8\ cm$

$$s = \frac{4+6+8}{2} = 9\ cm$$

$$Area = \sqrt{9(5)(3)(1)} = \sqrt{135} = 11.60\ cm^2$$

6) Find the perimeter of the triangle whose sides are of lengths $13\ cm$, $14\ cm$ & $15\ cm$.

Ans. $Perimeter = 2s$

$$s = \frac{a+b+c}{2} = semiperimeter$$

$a = 13\ cm \ , \qquad b = 14\ cm, \qquad c = 15\ cm$

$$s = \frac{13+14+15}{2} = 21\ cm$$

$$Perimeter = 2s = 2\ \boldsymbol{X}\ 21 = 42\ cm$$

C. RECTANGLE:-

FORMULAE:-

1) $Area \ = l \ \boldsymbol{X} \ b$

2) $Perimeter = Sum \ of \ all \ side$

Examples:-

1) The area of rectangle with one side 6 cm & other side 8 cm.Find the area.

Ans. $Area \ = l \ \boldsymbol{X} \ b$

$Area \ = 6 \ \boldsymbol{X} \ 8 = 48 \ \ cm^2$

2) The area of rectangle with one side 8 cm is $172cm^2$. Find length of the other side. **[SQP]**

Ans. $Area \ = l \ \boldsymbol{X} \ b$

$172 \ = 8 \ \boldsymbol{X} \ b$

$21.5 \ = \ b$

$b = 21.5 \ \ cm$

3) The area of the floor of a rectangular hall of length 40 m is 960 $sq.m$. Carpets of size $6m \ X \ 4m$ are available. Find how many carpets are requied to cover the hall.

Ans. $(Area)_{Hall} \ = l \ \boldsymbol{X} \ b$

$960 \ = 40 \boldsymbol{X} b$

$24 = b$

$b = 24 \ cm$

$(Area)_{carpet} \ = l \boldsymbol{X} b$

$(Area)_{carpet} \ = 6 \boldsymbol{X} 4$

$(Area)_{carpet} \ = 24 \ Sq.m$

$$Req.^{rd} = \frac{(Area)_{Hall}}{(Area)_{carpet}} = \frac{960}{24} = 40 \ m$$

4) The length of one side of rectangle is twice the length of it's adjacent side. If the perimeter of rectangle is $60 \ cm$. Find the area of the rectangle. **[S-18]**

Ans. $Perimeter = sum \ of \ all \ sides$

Let x be the adjacent(length) side & $2x$ (breadth) be the other side

$Perimeter = 2x + x + 2x + x$

$60 = 6x$

$x = 10 \ cm$

$l = x = 10$

$b = 2x = 2 \boldsymbol{X} 10 = 20cm$

$Area \ = l \boldsymbol{X} b$

$Area \ = 10 \boldsymbol{X} 20$

$Area \ = 200 \ cm^2$

5) The area of rectangular garden(Courtyard) is $3000m^2$. It's sides are in the ratio is $6:5$. Find the perimeter of the garden(Courtyard) **[S-19,W-17]**

Ans. $\frac{l}{b} = \frac{6}{5}$

$l = \frac{6}{5} \boldsymbol{X}\, b$

$Area\ = l\, \boldsymbol{X}\, b$

$3000 = \frac{6}{5} \boldsymbol{X}\, b\, \boldsymbol{X}\, b$

$3000 = \frac{6}{5} \boldsymbol{X}\, b^2$

$2500 = b^2$

$50 = b$

$b = 50\ \ m$

$l = \frac{6}{5} \boldsymbol{X}\, b$

$l = \frac{6}{5} \boldsymbol{X}\, 50 = 60\ \ m$

$Perimeter = sum\ of\ all\ sides$

$Perimeter = 60 + 50 + 60 + 50 = 220\, m$

D. SQUARE:-

FORMULAE:-

1) $Area = (side)^2$

2) $Perimeter = Sum\ of\ all\ side = 4\ \boldsymbol{X}\ Side$

Examples:-

1) Find the area of the square whose side is 4 cm.

Ans. $Area = (side)^2$

$Area = (4)^2 = 16\ \ cm^2$

2) The perimeter of the square is 24 cm .Find area of square.

Ans. $Perimeter = 4\ \boldsymbol{X}\ Side$

$24 = 4\ \boldsymbol{X}\ Side$

$6 = Side$

$Side = 6\ \ cm$

$Area = (side)^2$

$Area = (6)^2 = 36\ \ cm^2$

3) A square grassy plot is of side 100 $meter$. It has a gravel path 10 $meters$ wide all round it on the inside. Find the area of path. **[W-18,SQP]**

Ans. $Area = (side)^2$

$(Area)_{outer} = (100)^2 = 10000$

$(Area)_{inner} = (80)^2 = 6400$

$(Area)_{path} = (Area)_{outer} - (Area)_{inner}$

$(Area)_{path} = 10000 - 6400 = 3600\ m^2$

4) On rectangular field of $25\ \boldsymbol{X}\ 12\ m$, a square shaped house was built it's side $9m$. Find the remaining area of the field?

Ans. $(Area)_{rectangle} = 25\ \boldsymbol{X}\ 12 = 300\ m^2$

$(Area)_{square} = (side)^2 = (9)^2 = 81\ m^2$

$(Area)_{Remaining} = (Area)_{rectangle} - (Area)_{square}$

$(Area)_{Remaining} = 300 - 81 = 219\ m^2$

5) A square flowerbed is surrounded by a path $10\ cm$ wide around it.If the area of the path is $2000\ cm^2$. Find the area of the square flowerbed.

Ans. Let x be the side of square flower bed

$(Area)_{outer} = (side)^2 = (x+20)^2$

$(Area)_{inner} = (side)^2 = (x)^2$

$(Area)_{path} = (Area)_{outer} - (Area)_{inner}$

$2000 = (x+20)^2 - (x)^2$

$2000 = x^2 + 40x + 400 - x^2$

$2000 = 40x + 400$

$1600 = 40x$

$40 = x$

$x = 40\ cm$

$(Area)_{flower\ bed} = (side)^2 = (40)^2 = 1600\ cm^2$

E. TRAPEZIUM:-

FORMULAE:-

1) $Area = \frac{1}{2}\boldsymbol{X}(Sum\ of\ parallel\ side)\ \boldsymbol{X}\ Height$

Examples:-

1) Find the area of trapezoid with base of $10\ cms$, $14\ cm$ & height of $5\ cm$

Ans.

$$Area = \frac{1}{2}\boldsymbol{X}(Sum\ of\ parallel\ side)\ \boldsymbol{X}\ Height$$

$$Area = \frac{1}{2}\boldsymbol{X}(10+14)\ \boldsymbol{X}\ 5$$

$$Area = \frac{1}{2}\boldsymbol{X}(24)\ \boldsymbol{X}\ 5$$

$$Area = 60\ \ cm^2$$

2) Find the area of the trapezium whose parallel sides are $10\ cm$ & $8\ cm$ where the perpendicular distance between the side is $4\ cm$

Ans.

$$Area = \frac{1}{2}\boldsymbol{X}(Sum\ of\ parallel\ side)\ \boldsymbol{X}\ Height$$

$$Area = \frac{1}{2}\boldsymbol{X}(10+8)\ \boldsymbol{X}\ 4$$

$$Area = \frac{1}{2}\boldsymbol{X}(18)\ \boldsymbol{X}\ 4$$

$$Area = 36\ \ cm^2$$

3) The area of the trapezoid is $24\ \ sq.cm$ & the bases are $9cm$ & $7\ cm$, find the height.

Ans. $Area = \frac{1}{2}\boldsymbol{X}(Sum\ of\ parallel\ side)\ \boldsymbol{X}\ Height$

$24 = \frac{1}{2}\boldsymbol{X}(9+7)\ \boldsymbol{X}\ Height$

$24 = \frac{1}{2}\boldsymbol{X}(16)\ \boldsymbol{X}\ Height$

$24 = \ 8\ \boldsymbol{X}\ Height$

$3 = \ Height$

$Height = 3\ \ cm$

4) A wall is of the form of trapezium with height 4 m & parallel side being 3 m & 5 m . Find the cost of painting of wall if it has rate of paining as Rs. 25 per $sq.m$

Ans. $Area = \frac{1}{2}\boldsymbol{X}(Sum\ of\ parallel\ side)\ \boldsymbol{X}\ Height$

$Area = \frac{1}{2}\boldsymbol{X}(3+5)\ \boldsymbol{X}\ 4$

$Area = \frac{1}{2}\boldsymbol{X}(8)\ \boldsymbol{X}\ 4$

$Area = \ 16\ \ m^2$

$Cost = Area\ \boldsymbol{X}\ Rate$

$Cost = 16\ \boldsymbol{X}\ 25 = 400\ \ Rs.$

F. RHOMBUS:-

FORMULAE:-

1) $Area = \frac{Product\ of\ diagonal}{2}$

Examples:-

1) Calculate the area of rhombus whose diagonals are 30 cm & 16 cm

Ans.

$$Area = \frac{Product\ of\ diagonal}{2}$$

$$Area = \frac{30\ \boldsymbol{X}\ 16}{2} = 240\ \ cm^2$$

2) Find the area of rhombus whose diagonals are 6 cm and 9 cm.**[W-18]**

Ans.

$$Area = \frac{Product\ of\ diagonal}{2}$$

$$Area = \frac{6\ \boldsymbol{X}\ 9}{2} = 27\ \ cm^2$$

3) Find the area of the rhombus whose diagonal is 10 cm and 8.2 cm. **[W-17]**

Ans.

$$Area = \frac{Product\ of\ diagonal}{2}$$

$$Area = \frac{10\ \boldsymbol{X}\ 8.2}{2} = 41\ \ cm^2$$

TUTORIAL NO.11

MENSURATION

1) Find the area of the of ring between two concentric circles whose circumference are $75\ cm$ & $55\ cm$. **[S-19]**

2) A park is in the form of a right angled triangle with hypotenuse $13\ cm$. If one of the side is $12\ cm$,find the cost of leveling at the rate of ₹ 10 per sq.cm

3) The length of one side of rectangle is twice the length of it's adjacent side. If the perimeter of rectangle is $60\ cm$. Find the area of the rectangle.**[S-18]**

4) The area of rectangular garden(Courtyard) is $3000m^2$. It's sides are in the ratio is $6:5$. Find the perimeter of the garden(Courtyard) . **[S-19,W-17]**

5) The area of rectangle with one side $8\ cm$ is $172cm^2$. Find length of the other side. **[SQP]**

6) A square grassy plot is of side $100\ meter$. It has a gravel path $10\ meters$ wide all round it on the inside. Find the area of path. **[W-18,SQP]**

7) Find the area of rhombus whose diagonals are $6\ cm$ and $9\ cm$.**[W-18]**

8) Find the area of the rhombus whose diagonal is $10\ cm$ and $8.2\ cm$. **[W-17]**

II. VOLUME AND SURFACE AREA

A. SPHERE :-

FORMULAE:-

1) $Volume = \frac{4}{3}\pi r^3$

2) $Volume\ of\ hemisphere. = \frac{2}{3}\pi r^3$

3) $S.A. = 4\pi r^2$

4) $S.A.of\ hemisphere = 2\pi r^2$

Examples:-

1) Find the volume & Surface area of a sphere of radius 4.2 cm

Ans. $Volume = \frac{4}{3}\pi r^3$

$Volume = \frac{4}{3}\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (4.2)^3 = 310.464\ \ cm^3$

$S.A. = 4\pi r^2 = 4\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (4.2)^2 = 221.76\ \ cm^2$

2) Find the volume of a hemisphere having radius 2 m

Ans. $Volume = \frac{2}{3}\pi r^3$

$Volume = \frac{2}{3}\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ 2 = 16.7619\ \ cm^3$

3) If the volume of a sphere is $\frac{4\pi}{3}\ cm^3$.Find it's surface area. **[W-17]**

Ans.

$$Volume = \frac{4}{3}\pi r^3$$

$$\frac{4\pi}{3} = \frac{4}{3}\pi r^3$$

$$1 = r^3$$

$$1 = r$$

$$r = 1\ \ cm$$

$$S.A. = 4\pi r^2 = 4\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (1)^2 = 12.5714\ \ cm^2$$

4) The volume of a sphere is $\frac{88}{21}$ cube meters. Find it's surface area. **[S-19]**

Ans.

$$Volume = \frac{4}{3}\pi r^3$$

$$\frac{88}{21} = \frac{4}{3}\boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (r)^3$$

$$\frac{88}{21} = \frac{88}{21}\ \boldsymbol{X}\ (r)^3$$

$$1 = r^3$$

$$1 = r$$

$$r = 1\ \ m$$

$$S.A. = 4\pi r^2 = 4\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (1)^2 = 12.5714\ \ m^2$$

5) Total volume of 21 steel balls in a bearing is 88 *cubic cm*. Find the diameter of each ball

Ans.

$$Volume = \frac{4}{3}\pi r^3$$

$$Volume = \frac{88}{21}$$

$$\frac{88}{21} = \frac{4}{3}\boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (r)^3$$

$$\frac{88}{21} = \frac{88}{21}\ \boldsymbol{X}\ (r)^3$$

$$1 = r^3$$

$$1 = r$$

$$r = 1\ \ m$$

$$diameter = 2\ \boldsymbol{X}\ r = 2\ \boldsymbol{X}\ 1 = 2\ \ cm$$

6) A hemispherical tank has the diameter 4.2 m , then find the capacity of the tank in liter

Ans. $d = 4.2\ \ m$

$$r = \frac{d}{2} = \frac{4.2}{2} = 2.1\ \ m$$

$$Capacity = Volume\ of\ hemispherical$$

$$Capacity = \frac{2}{3}\pi r^3$$

$$Capacity = \frac{2}{3}\boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (2.1)^3 = 19.404\ \ m^3$$

$$Capacity = 19.404\ \ \boldsymbol{X}\ 1000 = 19404\ \ liter$$

7) The surface area of the sphere is 616 $sq.m.$ Find the

$\boldsymbol{i}$) Volume **[W-19]** $\boldsymbol{ii}$) diameter

Ans. $S.A. = 4\pi r^2$

$$616 = 4\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (r)^2$$

$$616 = \frac{88}{7}\ \boldsymbol{X}\ (r)^2$$

$$49 = (r)^2$$

$$7 = r$$

$$r = 7\ m$$

$\boldsymbol{i})$

$$Volume = \frac{4}{3}\pi r^3$$

$$\text{Volume} = \frac{4}{3}\boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (7)^3 = 1437.33\ m^3$$

$\boldsymbol{ii})$

$$diameter = 2\ \boldsymbol{X}\ r = 2\ \boldsymbol{X}\ 7 = 14\ m$$

8) A metal sphere of a diameter 16 cm is melted & small spheres of radius 2 cm each are cast from the molten metal. How many such spheres will be formed?

Ans. $d = 16\ c\, m$

$$r = \frac{d}{2} = 16 = 8\ cm$$

$$(Volume)_{large} = \frac{4}{3}\pi r^3$$

$$Volume = \frac{4}{3}\boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (8)^3 = 2145.5238\ m^3$$

$$(Volume)_{small} = \frac{4}{3}\pi r^3$$

$$Volume = \frac{4}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (2)^3 = 33.5238 \ \ m^3$$

$$No. of\ Sphere = \frac{(Volume)_{large}}{(Volume)_{small}} = \frac{2145.5238}{33.5238} = 64$$

OR

$$No. of\ Sphere = \frac{(Volume)_{large}}{(Volume)_{small}} = \frac{\frac{4}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (8)^3}{\frac{4}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (2)^3} = 64$$

9) The volume of two spheres is in the ratio $64: 27$ .Find their radii if the sum of their radii is $21\ cm$.

Ans. Let V_1 be the volume of sphere having radius r_1 & V_2 be the volume of sphere having radius r_2

$$\frac{V_1}{V_2} = \frac{64}{27}$$

$$\frac{\frac{4}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (r_1)^3}{\frac{4}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (r_2)^3} = \frac{64}{27}$$

$$\frac{(r_1)^3}{(r_2)^3} = \frac{64}{27}$$

$$(\frac{r_1}{r_2})^3 = (\frac{4}{3})^3$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

$$r_1 = \frac{4}{3} \boldsymbol{X} r_2$$

But $r_1 + r_2 = 21$

$$\frac{4}{3}\,\boldsymbol{X}\,r_2 + r_2 = 21$$

$$\frac{4}{3}\,\boldsymbol{X}\,2r_2 = 21$$

$$r_2 = 9 \;\; cm$$

$$r_1 + 9 = 21$$

$$r_1 = 12 \;\; cm$$

B. CYLINDER:-

FORMULAE:-

1) $Volume = \pi r^2 h$

2) $T.S.A. = 2\pi r(h + r)$

3) $C.S.A. = 2\pi rh$

Examples:-

1) Find the volume of a cylinder having radius 7 cm & height 12 cm

Ans. $Volume = \pi r^2 h$

$$Volume = \frac{22}{7}\ \boldsymbol{X}\ (7)^2\ \boldsymbol{X}\ 12 = 1848\ cm^3$$

2) Volume of the cylinder is 462 cm^3 & it's diameter is $7cm$. Find the height of the cylinder.

Ans. $d = 7\ cm$

$$r = \frac{d}{2} = \frac{7}{2} = 3.5\ cm$$

$Volume = \pi r^2 h$

$$462 = \frac{22}{7}\ \boldsymbol{X}\ (3.5)^2\ \boldsymbol{X}\ h$$

$12 = h$

$h = 12\ cm$

3) The area of the base of a right circular cylinder is 154 cm^2 & it's height is $15cm$. Find the volume of the cylinder.

Ans. $Volume = \pi r^2 h$

$Volume = Area\ \boldsymbol{X}\ h$

$Volume = 154\ \boldsymbol{X}\ 15$

$Volume = 2310\ \ cm^3$

4) The circumference of the base of cylinder is 132 cm & it's height 25 cm .

Find the volume of the cylinder.

Ans. $Circumference = 132\ \ cm$

$2\pi r = 132$

$2\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ r = 132$

$r = 21\ \ cm$

$Volume = \pi r^2 h$

$Volume = \frac{22}{7}\ \boldsymbol{X}\ (21)^2\ \boldsymbol{X}\ 25 = 34650\ \ cm^3$

5) Find the capacity of a cylindrical water tank whose radius is 2.1 m and height is 5 m. **[S-18]**

Ans. $Volume = \pi r^2 h$

$Volume = \frac{22}{7}\ \boldsymbol{X}\ (2.1)^2\ \boldsymbol{X}\ 5 = 69.3\ \ m^3$

$Capacity = 69.3\ \boldsymbol{X}\ 1000 = 69300\ \ liter$

6) Find the curved surface area & total surface area of a right circular cylinder whose height is 15 cm & the radius of the base is 7 cm

Ans. $C.S.A. = 2\pi rh = 2\,\boldsymbol{X}\,\frac{22}{7}\,\boldsymbol{X}\,7\,\boldsymbol{X}\,15 = 660\ \ cm^2$

$T.S.A. = 2\pi r(h+r) = 2\,\boldsymbol{X}\,\frac{22}{7}\,\boldsymbol{X}\,7\,\boldsymbol{X}(\mathbf{15}+\mathbf{7}) = 968\ \ cm^2$

7) A cylinder has hemispherical ends having radius $14\ cm$ and height $50\ cm$.

Find the total surface area. **[W-19]**

Ans. $T.S.A. = (C.S.A.)_{cylinder} + (C.S.A.)_{hemisphere} = \pi r(h+r) + 2\pi r^2$

$T.S.A. = 2\,\boldsymbol{X}\,\frac{22}{7}\,\boldsymbol{X}\,14\,\boldsymbol{X}(\mathbf{50}+\mathbf{14}) + 2\,\boldsymbol{X}\,\frac{22}{7}\,\boldsymbol{X}\,(14)^{\mathbf{2}} = 6864\ \ cm^2$

C. CONE:-

FORMULAE:-

1) $Volume = \frac{1}{3}\pi r^2 h$

2) $T.S.A. = \pi r(l + r)$

3) $C.S.A. = \pi rl$

4) $Slant\ height = l = \sqrt{h^2 + r^2}$ where r is radius of circular base

Examples:-

1) Find the volume of the cone having radius 10 cm & height 20 cm.

Ans.

$$Volume = \frac{1}{3}\pi r^2 h$$

$$Volume = \frac{1}{3}\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (10)^2\ \boldsymbol{X}\ 20 = 2095.2380\ cm^3$$

2) The radius of the base of a right circular cone is 6 cm & the slant height is 6.5 cm . Find the volume.

Ans.

$$Volume = \frac{1}{3}\pi r^2 h$$

$$l = \sqrt{h^2 + r^2}$$

$$l^2 = h^2 + r^2$$

$$(6.5)^2 = h^2 + (6)^2$$

$$6.25 = h^2$$

$$2.5 = h$$

$h = 2.5 \ cm$

$Volume = \frac{1}{3} \boldsymbol{X} \pi \boldsymbol{X} r^2 \boldsymbol{X} h = \frac{1}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (6)^2 \boldsymbol{X} 2.5 = 94.2857 \ cm^3$

3) The height & slant height of a cone are $12 \ cm$ & $20 \ cm$ respectively. Find it's volume.

Ans. $Volume = \frac{1}{3}\pi r^2 h$

$l = \sqrt{h^2 + r^2}$

$l^2 = h^2 + r^2$

$(20)^2 = (12)^2 + r^2$

$256 = r^2$

$16 = r$

$r = 16 \ cm$

$Volume = \frac{1}{3} \boldsymbol{X} \pi \boldsymbol{X} r^2 \boldsymbol{X} h = \frac{1}{3} \boldsymbol{X} \frac{22}{7} \boldsymbol{X} (16)^2 \boldsymbol{X} 12 = 3218.2857 \ cm^3$

4) The curved surface area of a cone is $4070 \ sq.cm$ & it's diameter is $70 \ cm$. What is it's slant height?

Ans. $r = \frac{d}{2} = \frac{70}{2} = 35 \ cm$

$C.S.A. = \pi r l$

$4070 = \frac{22}{7} \boldsymbol{X} 35 \boldsymbol{X} l$

$37 = l$

$l = 37 \ cm$

5) Find the volume of the right circular cone whose diameter is $6\ cm$ slant height $5\ cm$.

Ans.

$$r = \frac{d}{2} = \frac{6}{2} = 3\ \ cm$$

$$l = \sqrt{h^2 + r^2}$$

$$l^2 = h^2 + r^2$$

$$(5)^2 = h^2 + (3)^2$$

$$16 = h^2$$

$$4 = h$$

$$h = 4\ \ cm$$

$$Volume = \frac{1}{3}\ \boldsymbol{X}\ \pi\ \boldsymbol{X}\ r^2\ \boldsymbol{X}\ h = \frac{1}{3}\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (3)^2\ \boldsymbol{X}\ 4 = 37.71\ \ cm^3$$

6) A cone has a circular base of radius $10\ cm$ and slant height of $30\ cm$. Calculate the total surface area. **[SQP]**

Ans.

$$T.S.A. = \pi r(l + r) = \frac{22}{7}\ \boldsymbol{X}\ 10\ \boldsymbol{X}\ (30 + 10) = 1257.1428\ \ cm^2$$

7) A circus tent is cylindrical to a high of $3\ m$ and conical above it. If it's diameter is $105\ m$ & slant height of cone is $5\ m$. Calculate the area of total canvas required. **[W-17]**

Ans.

$$r = \frac{d}{2} = \frac{105}{2} = 52.5\ \ cm$$

$$Req.^{rd}\ Canvas = (C.S.A.)_{Cylinder} + (C.S.A.)_{Cone}$$

$$(C.S.A.)_{Cylinder} = 2\pi rh = 2\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ 52.5\ \boldsymbol{X}\ 3 = 990\ \ m^2$$

$$(C.S.A.)_{Cone} = \pi rl = \frac{22}{7}\ \boldsymbol{X}\ 52.5\ \boldsymbol{X}\ 5 = 825\ \ m^2$$

$$Req.^{rd}\ Canvas = (C.S.A.)_{Cylinder} + (C.S.A.)_{Cone}$$

$$Req.^{rd}\ Canvas = 990 + 825 = 1815\ \ m^2$$

8) A solid right circular cone of radius $2\ m$ and height $27\ m$ melted and re-casted into a sphere. Find the volume and surface area of the sphere. **[W-19]**

Ans.

$$Volume = \frac{1}{3}\pi r^2 h = \frac{1}{3}\ \boldsymbol{X}\ \frac{22}{7}\ \boldsymbol{X}\ (2)^2\ \boldsymbol{X}\ 27 = 113.14\ \ \ m^3$$

$$(Volume)_{Sphere} = (Volume)_{cone}$$

$$(Volume)_{Sphere} = 113.14\ \ \ m^3$$

$$\frac{4}{3}\pi r^3 = 113.14$$

$$\frac{4}{3}\boldsymbol{X}\frac{22}{7}\boldsymbol{X}\ r^3 = 113.14$$

$$r^3 = 27$$

$$r = 3\ \ \ m$$

$$(S.A.)_{Sphere} = 4\pi r^2 = 4\ \boldsymbol{X}\frac{22}{7}\boldsymbol{X}\ 3^2 = 113.14\ \ m^2$$

TUTORIAL NO.12

MENSURATION

1) If the volume of a sphere is $\frac{4\pi}{3}\ cm^3$.Find it's surface area. **[W-17]**

2) The volume of a sphere is $\frac{88}{21}$ cube meters. Find it's surface area. **[S-19]**

3) The circumference of the base of cylinder is $132\ cm$ & it's height $25\ cm$. Find the volume of the cylinder.

4) Find the capacity of a cylindrical water tank whose radius is $2.1\ m$ and height is $5\ m$. **[S-18]**

5) Find the volume of the cone having radius $10\ cm$ & height $20\ cm$.

6) A cone has a circular base of radius $10\ cm$ and slant height of $30\ cm$. Calculate the surface area. **[SQP]**

7) A circus tent is cylindrical to a high of $3\ m$ and conical above it. If it's diameter is $105\ m$ & slant height of cone is $5\ m$. Calculate the area of total canvas required. **[W-17]**

8) A solid right circular cone of radius $2\ m$ and height $27\ m$ melted and re-casted into a sphere. Find the volume and surface area of the sphere. **[W-19]**

D. CUBOID:-

FORMULAE:-

1) $Volume. = l\ X\ b\ X\ h$

2) $S.A. = 2\ (lb + bh + hl)$

3) $Longest\ distance = length\ of\ diagonal = \sqrt{l^2 + b^2 + h^2}$

Examples:-

1) Find the surface area of a cuboid of dimensions 26 cms , 20 cms and 12 cms. **[S-18]**

Ans. $S.A. = 2\ (lb + bh + hl)$

$S.A. = 2\ (26\ \boldsymbol{X}\ 20 + 20\ \boldsymbol{X}\ 12 + 12\ \boldsymbol{X}\ 26) = 2144\ \ cm^2$

2) The length, breadth and height of a cuboid are 8 cm , 11 cm and 15 cm respectively. Find the surface area.**[W-18]**

Ans. $S.A. = 2\ (lb + bh + hl)$

$S.A. = 2\ (8\ \boldsymbol{X}\ 11 + 11\ \boldsymbol{X}\ 15 + 15\ \boldsymbol{X}\ 8) = 746\ \ cm^2$

3) A metal strip having sides 17 $\boldsymbol{X}$ 7 $\boldsymbol{X}$ 5 cm is melted down & minted (recast) into coins each of diameter 1.4 cm & thickness 0.08 cm. Assuming no wastage, how many coins can be minted (recast)? **[S-19]**

Ans. $(Volume)_{strip} = l \, \boldsymbol{X} \, b \, \boldsymbol{X} \, h = 17 \, \boldsymbol{X} \, 7 \, \boldsymbol{X} \, 5 = 595 \;\; cm^3$

$d = 1.4 \, cm$

$r = \frac{d}{2} = \frac{1.4}{2} = 0.7 \;\; cm$

$(Volume)_{coin} = \pi r^2 h = \frac{22}{7} \, \boldsymbol{X} \, (0.7)^2 \, \boldsymbol{X} \, 0.08 = 0.1232 \;\; cm^3$

$No. of \; Coin = \frac{(Volume)_{strip}}{(Volume)_{coin}} = \frac{595}{0.1232} = 4829.5454 = 4830$

4) External dimensions of a wooden cuboid are $30 \, cm \, \boldsymbol{X} \, 25 \, cm \, \boldsymbol{X} \, 20 \, cm$. If the thickness of wood is $2 \, cm$ all round. Find the volume of the wood contained in the cuboid formed. **[S-18]**

Ans. $(Volume)_{external} = l \, \boldsymbol{X} \, b \, \boldsymbol{X} \, h = 30 \, \boldsymbol{X} \, 25 \, \boldsymbol{X} \, 20 = 15000 \;\; cm^3$

$(Volume)_{internal} = l \, \boldsymbol{X} \, b \, \boldsymbol{X} \, h = 26 \, \boldsymbol{X} \, 21 \, \boldsymbol{X} \, 16 = 8736 \;\; cm^3$

$(Volume)_{wood} = (Volume)_{external} - (Volume)_{internal}$

$(Volume)_{wood} = 15000 - 8736 = 6264 \;\; cm^3$

5) The outer dimensions of a closed wooden boxes are $42 \, \boldsymbol{X} \, 30 \, \boldsymbol{X} \, 27 \, cm$ .If the box is made of wood of $1 \, cm$ thickness. Determine the capacity of box. Also find the volume of the wood.

Ans. $(Volume)_{external} = l \, \boldsymbol{X} \, b \, \boldsymbol{X} \, h = 42 \, \boldsymbol{X} \, 30 \, \boldsymbol{X} \, 27 = 34020 \;\; cm^3$

$(Volume)_{internal} = l \, \boldsymbol{X} \, b \, \boldsymbol{X} \, h = 40 \, \boldsymbol{X} \, 28 \, \boldsymbol{X} \, 25 = 28000 \;\; cm^3$

$(Volume)_{wood} = (Volume)_{external} - (Volume)_{internal}$

$$(Volume)_{wood} = 34020 - 28000 = 6020\ \ cm^3$$

6) The internal measures of a cuboidal room are $12\ m\ \boldsymbol{X}\ 8\ m\ \boldsymbol{X}\ 4\ m$. Find the total cost of whitewashing all four walls of a room, if the cost of white washing is Rs. 8 per meter. What will be the cost of white washing if the ceiling of the room is also whitewashed? **[SQP]**

Ans. $(Area)_{fourwall} = Perimeter\ of\ the\ base\ \boldsymbol{X}\ H$

$(Area)_{fourwall} = 2\ (l + b)\ \boldsymbol{X}\ H$

$(Area)_{fourwall} = 2\ (12 + 8)\ \boldsymbol{X}\ 4 = 160\ \ m^2$

$(Cost)_{fourwall} = Area\ \boldsymbol{X}\ Rate = 160\ \boldsymbol{X}\ 8 = 1280\ \ Rs.$

$(Area)_{ceiling} = l\ \boldsymbol{X}\ b = 12\ \boldsymbol{X}\ 8 = 96\ \ m^2$

$(Cost)_{ceiling} = Area\ \boldsymbol{X}\ Rate = 96\ \boldsymbol{X}\ 8 = 768\ \ Rs.$

$(Cost)_{total} = (Cost)_{fourwall} + (Cost)_{ceiling}$

$(Cost)_{total} = 1280 + 768 = 2048\ \ Rs.$

7) Find the length of the longest pole that can be placed in a room $12m$ long $9m$ broad and $8m$ high. **[W-19]**

Ans. $Longest\ distance = length\ of\ diagonal = \sqrt{l^2 + b^2 + h^2}$

$Longest\ distance = \sqrt{(12)^2 + (9)^2 + (8)^2} = \sqrt{289} = 17\ \ m$

E. CUBE:-

FORMULAE:-

1) $S.A. = 6\ \boldsymbol{X}\ (Side)^2$

2) $Volume = (Side)^3$

Examples:-

1) Calculate the volume of a cube whose length of one side is 5 cm.

Ans. $Volume = (Side)^3 = (5)^3 = 125\ \ cm^3$

2) Calculate the surface area of the cube having length of one side as 5.3 cm

Ans. $S.A. = 6\ \boldsymbol{X}\ (Side)^2 = 6\ \boldsymbol{X}\ (5.3)^2 = 168.54\ \ cm^2$

3) A cube having surface area $96cm^2$. Find it's edge length.

Ans. $S.A. = 6\ \boldsymbol{X}\ (Side)^2$

$96 = 6\ \boldsymbol{X}\ (Side)^2$

$16 = (Side)^2$

$4 = Side$

$Side = 4\ \ cm$

4) The total S.A. of a cube is 294 cm^2.Find the volume.

Ans. $S.A. = 6\ \boldsymbol{X}\ (Side)^2$

$294 = 6\ \boldsymbol{X}\ (Side)^2$

$49 = (Side)^2$

$7 = Side$

$Side = 7\ \ cm$

$Volume = (Side)^3 = (7)^3 = 343\ \ cm^3$

5) The volume of cube is 1000 cm^3. Find its total surface area.**[S-19,W-18]**

Ans. $Volume = (Side)^3$

$1000 = (Side)^3$

$10 = Side$

$Side = 10\ \ cm$

$S.A. = 6\ \boldsymbol{X}\ (Side)^2$

$S.A = 6\ \boldsymbol{X}\ (10)^2 = 600\ \ cm^2$

TUTORIAL NO.13

MENSURATION

1) Find the surface area of a cuboid of dimensions $26\ cms$, $20\ cms$ and $12\ cms$. **[S-18]**

2) The length, breadth and height of a cuboid are $8\ cm$, $11\ cm$ and $15\ cm$ respectively. Find the total surface area. **[W-18]**

3) A metal strip having sides $17\ \boldsymbol{X}\ 7\ \boldsymbol{X}\ 5\ cm$ is melted down & minted (recast) into coins each of diameter $1.4\ cm$ & thickness $0.08\ cm$. Assuming no wastage, how many coins can be minted (recast)? **[S-19]**

4) External dimensions of a wooden cuboid are $30\ cm\ \boldsymbol{X}\ 25\ cm\ \boldsymbol{X}\ 20\ cm$. If the thickness of wood is $2\ cm$ all round. Find the volume of the wood contained in the cuboid formed. **[S-18]**

5) The internal measures of a cuboidal room are $12\ m\ \boldsymbol{X}\ 8\ m\ \boldsymbol{X}\ 4\ m$. Find the total cost of whitewashing all four walls of a room, if the cost of white washing is Rs. 8 per meter. What will be the cost of white washing if the ceiling of the room is also whitewashed? **[SQP]**

6) Calculate the surface area of the cube having length of one side as $5.3\ cm$

7) A cube having surface area $96cm^2$ .Find it's edge length.

8) The volume of cube is $1000\ cm^3$. Find its total surface area. **[S-19,W-18]**

STATISTICS

Position in Question Paper **Total Marks-20**

Q.1. f) 2-Marks.

Q.1. g) 2-Marks.

Q.2. d) 4-Marks.

Q.6. a) i) 3-Marks.

ii) 3- Marks

Q.6. b) i) 3-Marks.

ii) 3- Marks

I. RANGE :-

A. FOR RAW DATA :-

FORMULAE:-

$$Range = Largest\ value - Smallest\ value = L - S$$

$$Coefficient\ of\ Range = \frac{L - S}{L + S}$$

Examples:-

Que. Find **Range , Coefficient of Range** of the following observations.

1) 5 , 7 , 9 , 13 , 11 , 5 , 3 **[S-19]**

Ans.

$$\text{Range} = L - S$$
$$= 13 - 3$$
$$= 10$$
$$\text{Ceifficient of range} = \frac{L-S}{L+S}$$
$$= \frac{13-3}{13+3}$$
$$\text{Ceifficient of range } = \frac{10}{16} \quad \text{or} \quad \frac{5}{8} \quad \text{or} \quad 0.625$$

2) 5, 25, 65, 55, 35, 45, 15 **[W-17]**

Ans.

$$\text{Range } = \text{Largest value} - \text{Smallest value} = L - S$$
$$= 65 - 5$$
$$= 60$$
$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{60}{70} = 0.857$$

3) 2, 3, 1, 10, 6, 31, 17, 20, 24 **[W-14,SQP]**

Ans.

$$L = 31 \quad S = 1$$
$$\therefore Range = L - S$$
$$= 31 - 1$$
$$= 30$$
$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{31-1}{31+1} = \frac{30}{32} = 0.9375$$

4) 3, 6, 10, 1, 15, 16, 21, 19, 18 **[S-13,SQP]**

Ans. Range =Largest value-Smallest value

=21-1

=20

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{21-1}{21+1} = \frac{20}{22} = 0.9090$$

5) 40,52,47,28,45,36,47,50 **[S-19]**

Ans. Range $= L - S$

$=52 - 28$

$= 24$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$= \frac{52-28}{52+28}$$

$$= 0.3$$

6) 14, 18, 22, 35, 42, 44, 8, 7, 5 , 2 **[W-18]**

Ans. Range $= L - S$

$= 44 - 2$

$= 42$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{44-2}{44+2} = \frac{42}{46} = 0.9130$$

7) 120, 50, 90, 100, 180, 200, 150, 40, 80 **[S-18]**

Ans. $Range = L - S = 150 - 50 = 100$

$$\text{coefficient of range} = \frac{L-S}{L+S}$$

$$= \frac{150-50}{150+50}$$

$$= \frac{100}{200} = \frac{1}{2} \quad \text{or} \quad 0.5$$

8) 120, 100, 130, 50, 150 **[S-18,S-14]**

Ans. $Smallest\ \ Value\ \ S=50, \qquad Largest\ \ Value\ \ L=150$

$\therefore Range = L - S = 150 - 50$

$= 100$

$$Coeff.\ of\ \ Range = \frac{L-S}{L+S} = \frac{150-50}{150+50}$$

$$= \frac{1}{2} \quad or \quad 0.5$$

9) 50, 90, 120, 40, 180, 200, 80 **[W-17,W-16,W-13]**

Ans. i) Range = L-S = 200-40

=160

ii)Co-efficient of range = $\frac{L-S}{L+S}$

$$= \frac{200-40}{200+40}$$

$$= \frac{160}{240}$$

=0.666 or 0.66

10) 45, 42, 39, 40, 48, 41, 45, 44 **[S-17]**

Ans. 45,42,39,40,48,41,45,44

Range = Largest value – Smallest value

= 48 – 39

= 9

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{48-39}{48+39} = \frac{9}{87} = 0.1034$$

11) 800, 725, 750 , 900, 925 , 910 , 1000 , 790 , 870 , 920 **[S-16]**

Ans. $\text{Range} = \text{Largest Value} - \text{Smallest value}$

$= 1000 - 725$

$= 275$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{1000-725}{1000+725} = \frac{275}{1725} = 0.1594$$

12) 70, 75, 69, 80, 85, 83, 65, 89, 73, 84, 90 **[W-15]**

Ans. $\text{Range} = \text{Largest Value} - \text{Smallest value}$

$\text{Range} = \text{L} - \text{S} = 90 - 65$

$= 25$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{900-65}{900+65} = \frac{835}{965} = 0.8652$$

13) 3,7,11,2,16,17,22,20,19 **[W-19]**

Ans. $\text{Range} = L - S$

$= 22 - 2$

$\therefore \text{Range} = 20$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$= \frac{22-2}{22+2}$$

$= 0.833$

B. FOR UNGROUPD DATA:-

FORMULAE:-

$$Range = Largest\ value\ in\ x_i - Smallest\ value\ in\ x_i = L(x_i) - S(x_i)$$

$$Coefficient\ of\ Range = \frac{L - S}{L + S}$$

Examples:-

Que. Find **Range , Coefficient of Range** of the following.

1)

x_i	10	20	30	40	50
f_i	7	5	3	2	1

[W-18,S-15,W-12]

Ans.

$$Range = \text{Largest Value} - \text{Smallest Value}$$
$$= 50 - 10$$
$$= 40$$

$$Coefficient\ of\ Range = \frac{\text{Largest Value} - \text{Smallest Value}}{\text{Largest Value} + \text{Smallest Value}}$$
$$= \frac{50-10}{50+10}$$
$$= \frac{2}{3}$$

2)

x_i	2	3	4	5	6
f_i	2	15	10	5	5

Ans. $Range = Largest\ value\ in\ x_i - Smallest\ value\ in\ x_i = L(x_i) - S(x_i)$

$Range = 6 - 2 = 4$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{6-2}{6+2} = \frac{4}{8} = 0.5$$

3)

Weight in kgs.	35	40	45	50	55
Number of boys	8	14	19	12	7

Ans. $Range = Largest\ value\ in\ x_i - Smallest\ value\ in\ x_i = L(x_i) - S(x_i)$

$Range = 55 - 35 = 20$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{55-35}{55+35} = \frac{20}{90} = 0.222$$

4)

Age in years	13	14	15	16	17	18
No. of students	10	12	20	14	9	3

Ans. $Range = Largest\ value\ in\ x_i - Smallest\ value\ in\ x_i = L(x_i) - S(x_i)$

$Range = 18 - 13 = 5$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{18-13}{18+13} = \frac{5}{31} = 0.1612$$

5)

No. of students	7	5	16	7	8	2
Marks	19	10	2	50	40	80

Ans. $Range = Largest\ value\ in\ x_i - Smallest\ value\ in\ x_i = L(x_i) - S(x_i)$

$Range = 16 - 2 = 14$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{16-2}{16+2} = \frac{14}{18} = 0.777$$

C. FOR GROUPD DATA:-

FORMULAE:-

$$Range = Upper\ limit\ of\ highest\ class - Lower\ limit\ lowest\ class$$

$$= L(x_i) - S(x_i)$$

$$Coefficient\ of\ Range = \frac{L-S}{L+S}$$

Examples:-

Que. Find **Range , Coefficient of Range** of the following.

1)

Marks	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
No. of students	6	10	16	14	8	4

[W-19,S-19,S-18,S-16]

Ans.

$$\text{Range=Upper boundary of the last class} - \text{lower boundary of first class}$$

$$= 69.5 - 9.5$$

$$= 60$$

$$\text{coefficient of range} = \frac{\text{Range}}{\text{sum of the highest and lowest value}}$$

$$= \frac{60}{69.5 + 9.5}$$

$$= \frac{60}{79} \text{ or } 0.759$$

2)

Marks	10 –19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70-79
No. of students	03	61	223	137	53	19	04

[S-13]

Ans. Range=Upper boundary of the last class − lower boundary of first class

$= 79.5 - 9.5$

$= 70$

$$\text{coefficient of range} = \frac{\text{Range}}{\text{sum of the highest and lowest value}}$$

$$= \frac{70}{79.5 + 9.5}$$

$$= \frac{70}{89} \text{ or } 0.787$$

3)

C.I	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Frequency	15	25	13	17	10

[S-19]

Ans.

Marks:	9.5-19.5	19.5-29.5	29.5-39.5	39.5-49.5	49.5-59.5	59.5-69.5
No. of students:	06	10	16	14	08	04

$\text{Range} = L - S$

$= 69.5 - 9.5$

$= 60$

$$\text{Coefficient of range } = \frac{L - S}{L + S}$$

$$= \frac{69.5 - 9.5}{69.5 + 9.5}$$

$= 0.76$

4)

C.I	21-25	26-30	31-35	36-40	41-45
Frequency	4	16	38	12	10

[W-17]

Ans.

C.I.	20.5-25.5	25.5-30.5	30.5-35.5	35.5-40.5	40.5-45.5
f_i	4	16	38	12	10

$$\text{Range} = L - S = 45.5 - 20.5$$
$$= 25$$

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$
$$= \frac{45.5-20.5}{45.5+20.5}$$
$$= \frac{25}{66} \quad \text{OR} \quad 0.379$$

5) The weight of the students is given below **[SQP]**

C.I	60-62	63-65	66-68	69-71	72-74
Frequency	5	18	42	27	8

Ans. $Range = 74.5 - 59.5 = 15$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{74.5-59.5}{74.5+59.5} = \frac{15}{134} = 0.111$$

6)

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of students	10	15	16	20	21	22	09	8

[S-17,W-14]

Ans. $L = 99.5 \qquad S = 19.5$

$$\therefore Range = L - S$$
$$= 99.5 - 19.5$$
$$= 80$$

$$Coeff.\ of\ \ Range = \frac{L-S}{L+S}$$
$$= \frac{99.5-19.5}{99.5+19.5}$$
$$= \frac{80}{119} \quad or \quad 0.672$$

7)

Temperature	25-26	27-28	29-30	31-32	33-34	35-36
No. of students	2	11	12	10	4	1

[SQP]

Ans. $Range = 36.5 - 24.5 = 12$

$$Coefficient\ of\ Range = \frac{L-S}{L+S} = \frac{36.5-24.5}{36.5+24.5} = \frac{12}{61} = 0.196$$

II. MEAN DEVIATION (M.D) FROM MEAN:-

A. FOR RAW DATA :-

FORMULAE:-

$M.D. about\ \ mean\ = \frac{\sum|x_i - \overline{x}|}{N}$ Where

$\overline{x} = \frac{\sum x_i}{N}\ \ N = Total\ No.Observation$

Examples:-

Que. 1) Calculate the **Mean Deviation (M.D) about mean.**

3, 6, 5, 7, 10, 12, 15, 18.**[S-18]**

Ans.

x_i	$\lvert x_i - \overline{x}\rvert$
3	6.5
5	4.5
6	3.5
7	2.5
10	0.5
12	2.5
15	5.5
18	8.5
$\sum = 76$	$\sum = 34$

$\overline{x} = \frac{\sum x_i}{N} = \frac{76}{8} = 9.5$

$M.D. = \frac{\sum|x_i - \overline{x}|}{N} = \frac{34}{8} = 4.25$

2) 3,4,5,7,8,9

Ans.

x_i	$\|x_i - \overline{x}\|$
3	3
4	2
5	1
7	1
8	2
9	3
$\sum = 36$	$\sum = 12$

$$\overline{x} = \frac{\sum x_i}{N} = \frac{36}{6} = 6$$

$$M.D. = \frac{\sum|x_i - \overline{x}|}{N} = \frac{12}{6} = 2$$

3) 5, 15, 25, 35, 45.

Ans.

x_i	$\|x_i - \overline{x}\|$
5	20
15	10
25	0
35	10
45	20
$\sum = 125$	$\sum = 60$

$$\overline{x} = \frac{\sum x_i}{N} = \frac{125}{5} = 25$$

$$M.D. = \frac{\sum|x_i - \overline{x}|}{N} = \frac{60}{5} = 12$$

B. FOR UNGROUPED DATA:-

FORMULAE:-

$$M.D. about\ mean = \frac{\sum f_i|x_i - \overline{x}|}{\sum f_i} \quad \text{Where } \overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Examples:-

Que. Calculate the **mean deviation (M.D) about the mean.**

1) Find the mean deviation from of the following data **[S-19,S-17]**

OR

Find the mean deviation from mean of the following data **[W-15]**

Marks	3	4	5	6	7	8
No. of students	1	3	7	5	2	2

Ans.

x_i	f_i	$f_i x_i$	$\|x_i - \overline{x}\|$	$f_i\|x_i - \overline{x}\|$
3	1	3	2.5	2.5
4	3	12	1.5	4.5
5	7	35	0.5	3.5
6	5	30	0.5	2.5
7	2	14	1.5	3
8	2	16	2.5	5
	$\sum = 20$	$\sum = 110$		$\sum = 21$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{110}{20} = 5.5$$

$$M.D.=\frac{\sum f_i|x_i-\overline{x}|}{\sum f_i}=\frac{21}{20}=1.05$$

2) Find the mean deviation of the following data **[S-15]**

Marks	3	4	5	6	7	8
No. of students	4	9	10	8	6	3

Ans.

x_i	f_i	f_ix_i	$\lvert x_i-\overline{x}\rvert$	$f_i\lvert x_i-\overline{x}\rvert$
3	4	12	2.3	9.2
4	9	36	1.3	11.7
5	10	50	0.3	3
6	8	48	0.7	5.6
7	6	42	1.7	10.2
8	3	24	2.7	8.1
	$\sum=40$	$\sum=212$		$\sum=47.8$

$$\overline{x}=\frac{\sum f_ix_i}{\sum f_i}=\frac{212}{40}=5.3$$

$$M.D.=\frac{\sum f_i|x_i-\overline{x}|}{\sum f_i}=\frac{47.8}{40}=1.195$$

3)

x_i	3	4	5	6	7
f_i	8	14	19	12	7

Ans.

x_i	f_i	f_ix_i	$\lvert x_i-\overline{x}\rvert$	$f_i\lvert x_i-\overline{x}\rvert$

3	8	24	1.93	15.44
4	14	56	0.93	13.02
5	19	95	0.07	1.33
6	12	72	1.07	12.84
7	7	49	2.07	14.49
	$\sum = 60$	$\sum = 296$		$\sum = 57.12$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{296}{60} = 4.93$$

$$M.D. = \frac{\sum f_i |x_i - \overline{x}|}{\sum f_i} = \frac{57.12}{60} = 0.952$$

C. FOR GROUPED DATA:-

FORMULAE:-

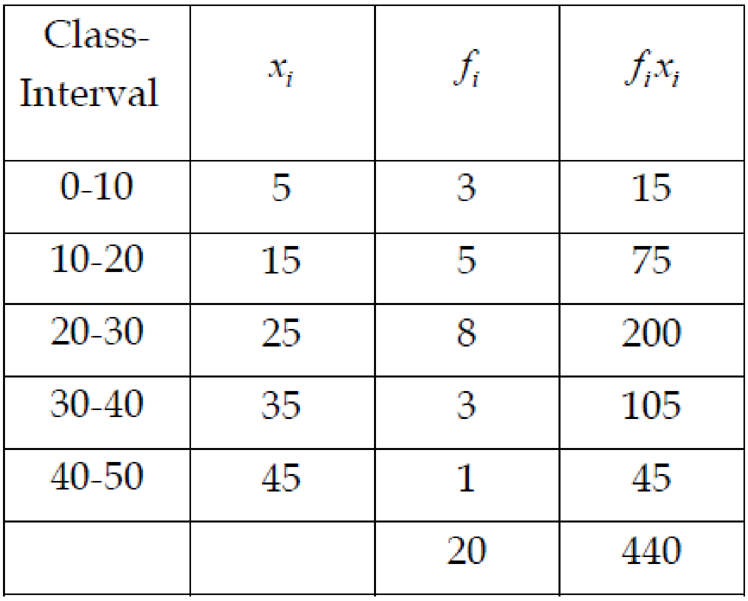
$$M.D.\,about\ mean=\frac{\sum f_i|x_i-\overline{x}|}{\sum f_i}\quad \text{Where}\ \ \overline{x}=\frac{\sum f_i x_i}{\sum f_i}\quad where\ x_i = mid-value$$

Examples:-

1) Find mean of the following data : **[W-18]**

Class-Interval	0 – 10	0 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	5	8	3	1

Ans.



Class-Interval	x_i	f_i	$f_i x_i$
0-10	5	3	15
10-20	15	5	75
20-30	25	8	200
30-40	35	3	105
40-50	45	1	45
		20	440

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N}$$

$$\therefore \bar{x} = \frac{440}{20}$$

$$\therefore \bar{x} = 22$$

2) Find mean for the following data :**[W-18,SQP]**

Class-Interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	6	10	18	9	3

OR

Find the mean deviation from mean for the following data: [**W-18]**

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of student	4	6	10	18	9	3

Ans.

C.I.	f_i	x_i	$f_i x_i$	$d_i = \quad \overline{x}\|$	$f_i \quad \|x_i - \overline{x}\|$
10-20	4	15	60	26.2	104.8
20-30	6	25	150	16.2	97.2
30-40	10	35	350	6.2	62
40-50	18	45	810	3.8	68.4
50-60	9	55	495	13.8	124.2
60-70	3	65	195	23.8	71.4
	$\sum f_i = 50$		$\sum f_i x_i = 2060$		$\sum f_i \quad = 528$

$$Mean\ \overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore \overline{x} = \frac{2060}{50}$$

$$\therefore \overline{x} = 41.2$$

$$M.D. = \frac{\sum f_i |x_i - \overline{x}|}{\sum f_i}$$

$$\therefore M.D = \frac{528}{50}$$

$$\therefore M.D. = 10.56$$

3) Find mean deviation (M.D)from mean of the following distribution

[S-19,S-18,S-14,W-13,S-13,W-12]

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No of students	05	08	15	16	06

Ans.

Marks	x_i	f_i	f_ix_i	$x_i-\overline{x}$	$\left\|x_i-\overline{x}\right\|$	$f_i\left\|x_i-\overline{x}\right\|$
0-10	5	5	25	-22	22	110
10-20	15	8	120	-12	12	96
20-30	25	15	375	-2	2	30
30-40	35	16	560	8	8	128
40-50	45	06	270	18	18	108
		50	1350			472

$$\text{Mean} = \frac{\sum f_ix_i}{\sum f_i}$$

$$= \frac{1350}{50}$$

$$= 27$$

$$\text{M.D.} = \frac{\sum f_i\left|x_i-\overline{x}\right|}{\sum f_i}$$

$$= \frac{472}{50}$$

$$= 9.44$$

4) Calculate the mean deviation for the following data:**[W-14]**

Expenditure (Rs.)	40-59	60-79	80-99	100-119	120-139
No. of families	50	300	500	200	60

Ans.

Class	xi	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \ \lvert x_i - \bar{x} \rvert$
40-59	49.5	50	2475	38.559	1927.95
60-79	69.5	300	20850	18.559	5567.7
80-99	89.5	500	44750	1.441	720.5
100-119	109.5	200	21900	21.441	4288.2
120-139	129.5	60	7770	41.441	2486.46
		1110	**97745**		**14990.81**

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{97745}{1110} = 88.059$$

$$M.D. = \frac{\sum f_i \ \lvert x_i - \bar{x} \rvert}{N}$$

$$= \frac{14990.81}{1110}$$

$$= 13.505$$

TUTORIAL NO.14

STATISTICS

1) Find **Range , Coefficient of Range** of the following observations
50, 90, 120, 40, 180, 200, 80 **[W-17,W16,W-13]**

2) Find the range and co-efficient of range of the following **[W-18,S-15,W-12]**

x_i	10	20	30	40	50
f_i	7	5	3	2	1

3) Calculate range & coefficient of range of the following data

[W-19,S-19,S-18,S-16]

Marks	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
No. of students	6	10	16	14	8	4

4) Calculate the **mean deviation about mean.** 3, 6, 5, 7, 10, 12, 15, 18 **[S-18]**

5) Find the mean deviation from of the following data **[S-19,S-17]**

Marks	3	4	5	6	7	8
No. of students	1	3	7	5	2	2

6) Find the M.D from mean for the following data:[**W-18,W-17]**

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of student	4	6	10	18	9	3

7) Find mean deviation (M.D)from mean & median of the following distribution
[S-19,S-18,S-14,W-13,S-13,W-12]

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No of students	05	08	15	16	06

8) Find the mean deviation from mean of the following distribution[**W-16,S-16]**

Weight (in gms)	10-15	15-20	20-25	25-30	30-35	35-40	40-45
No. of items	7	12	16	25	19	15	6

III. STANDARD DEVIATION (S.D) , VARIANCE AND COEFFICIENT OF VARIANCE(C.V) & COEFFICIENT OF STANDARD

DEVIATION (C.S.D) :-

A. FOR RAW DATA :-

FORMULAE:-

$$\sigma = \sqrt{\frac{\sum x_i^2}{\sum f_i} - (\overline{x})^2} \qquad \text{Where} \qquad \overline{x} = \frac{\sum x_i}{N}$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\overline{x}}$$

$$vaiance = \sigma^2$$

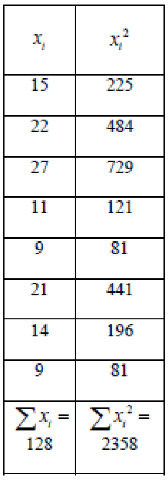
$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\overline{x}} \boldsymbol{X}\ 100$$

Examples:-

Que. 1) **Compute S.D, Variance, C.V & Coefficient of Standard Deviation.**

15, 22 , 27 , 11 , 9 , 21 , 14 , 9 **[W-17]**

Ans.



x_i	x_i^2
15	225
22	484
27	729
11	121
9	81
21	441
14	196
9	81
$\sum x_i =$ 128	$\sum x_i^2 =$ 2358

$$\text{Mean } \bar{x} = \frac{\sum x_i}{n}$$

$$\bar{x} = \frac{128}{8} = 16$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$= \sqrt{\frac{2358}{8} - (16)^2}$$

$$= 6.22$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\bar{x}} = \frac{6.22}{16} = 0.3887$$

$$vaiance = \sigma^2 = (6.22)^2 = 38.6884$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\bar{x}} \boldsymbol{X}\ 100 = \frac{6.22}{16} \boldsymbol{X}\ 100 = 38.87$$

2) 19, 23, 16, 07, 18, 35, 14, 24 **[SQP]**

Ans.

x_i	x_i^2
19	361
23	529
16	256
7	49
18	324
35	1225
14	196
24	576
$\sum x_i$ =156	$\sum x_i^2$ =3516

$$Mean = \bar{x} = \frac{\sum x_i}{N} = \frac{156}{8} = 19.5$$

$$\sigma = \sqrt{\frac{\sum {x_i}^2}{\text{N}} - (\ \bar{x}\ )^2} = \sqrt{\frac{3516}{8} - (\ 19.5\ )^2} = 7.6974$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\bar{x}} = \frac{7.6974}{19.5} = 0.3947$$

$$vaiance = \sigma^2 = (7.6974)^2 = 59.2499$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\overline{x}} \boldsymbol{X}\ 100 = \frac{7.6974}{19.5} \boldsymbol{X}\ 100 = 39.4738$$

3) 1,2,3,4,5,6,7 **[W-19]**

Ans.

x_i	1	2	3	4	5	6	7	$\sum x_i = 28$
x_i^2	1	4	9	16	25	36	49	$\sum x_i^2 = 140$

$$\text{Mean } \overline{x} = \frac{\sum x_i}{n} = \frac{28}{7} = 4$$

$$S.D. = \sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\overline{x}\right)^2}$$

$$\therefore \sigma = \sqrt{\frac{140}{7} - (4)^2}$$

$$\therefore \sigma = 2$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\overline{x}} = \frac{2}{4} = 0.5$$

$$vaiance = \sigma^2 = (2)^2 = 4$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\overline{x}} \boldsymbol{X}\ 100 = \frac{2}{4} \boldsymbol{X}\ 100 = 50$$

B. FOR UNGROUPED DATA:-

FORMULAE:-

$$\sigma = \sqrt{\frac{\sum f_i {x_i}^2}{\sum f_i} - (\overline{x})^2} \quad \text{where} \quad \overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\overline{x}}$$

$$vaiance = \sigma^2$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\overline{x}} \boldsymbol{X}\ 100$$

Examples:-

Que. Compute S.D, Variance, C.V & Coefficient of Standard Deviation.

1)

x_i	2	3	4	5	6
f_i	8	14	19	12	7

Ans.

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
2	8	16	4	32
3	14	42	9	126
4	19	76	16	304
5	12	60	25	300
6	7	42	36	252
	$\sum f_i = 60$	$\sum f_i x_i = 236$		$\sum f_i x_i^2 = 1014$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{236}{60} = 3.9333$$

$$\sigma = \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2} = \sqrt{\frac{1014}{60} - (3.9333)^2} = 1.1964$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\bar{x}} = \frac{1.1964}{3.9333} = 0.3041$$

$$vaiance = \sigma^2 = (1.1964)^2 = 3.8572$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\bar{x}}\boldsymbol{X}\ 100 = \frac{1.1964}{3.9333}\boldsymbol{X}\ 100 = 30.4172$$

2)

x_i	5	15	25	35	45
f_i	3	5	8	3	1

Ans.

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
5	3	15	25	75
15	5	75	225	1125
25	8	200	625	5000
35	3	105	1225	3675
45	1	45	2025	2025
	N=20	$\sum f_i x_i =$ 440		$\sum f_i x_i^2 =$ 11900

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{440}{20} = 22$$

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$$

$$S.D. = \sqrt{\frac{11900}{20} - (22)^2}$$

$$S.D. = 10.54$$

$$\text{Coefficient of variance} = \frac{S.D.}{Mean}\times 100$$
$$= \frac{10.54}{22}\times 100$$
$$= 47.91\%$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\overline{x}} = \frac{10.54}{22} = 0.4790$$

$$vaiance = \sigma^2 = (10.54)^2 = 111.0916$$

3)

Age in years	13	14	15	16	17	18
No. of students	9	3	20	14	10	12

Ans.

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
13	9	117	169	1521
14	3	42	196	588
15	20	300	225	4500
16	14	224	256	3584
17	10	170	289	2890
	$\sum f_i = 69$	$\sum f_i x_i = 1069$		$\sum f_i x_i^2 = 16971$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1069}{68} = 15.7205$$

$$\sigma = \sqrt{\frac{\sum f_i {x_i}^2}{\sum f_i} - (\overline{x})^2} = \sqrt{\frac{16971}{68} - (15.7205)^2} = 1.5618$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\overline{x}} = \frac{1.5618}{15.7205} = 0.0993$$

$$vaiance = \sigma^2 = (1.5618)^2 = 2.4392$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\overline{x}}\boldsymbol{X}\ 100 = \frac{1.5618}{15.7205}\boldsymbol{X}\ 100 = 9.9347$$

C. FOR GROUPED DATA:-

FORMULAE:-

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - (\overline{x})^2} \quad \text{Where} \quad \overline{x} = \frac{\sum f_i x_i}{\sum f_i} \quad where \quad x_i = mid - value$$

$$Coefficient\ of\ S.D = \frac{\sigma}{\overline{x}}$$

$$vaiance = \sigma^2$$

$$Coefficient\ of\ vaiance(C.V) = \frac{\sigma}{\overline{x}} \boldsymbol{X}\ 100$$

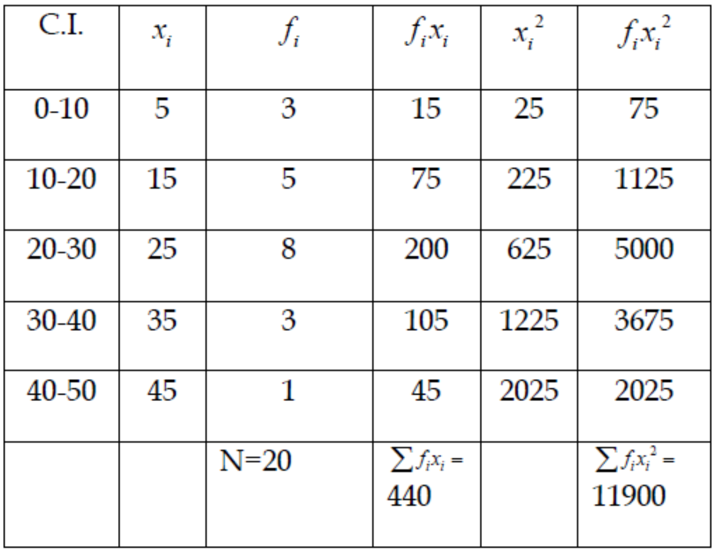
Examples:-

1) Find mean ,S.D , coefficient of variance of the following data

[W-18,S-18,W-17,S-14]

Class-Interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	5	8	3	1

Ans.



C.I.	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	3	15	25	75
10-20	15	5	75	225	1125
20-30	25	8	200	625	5000
30-40	35	3	105	1225	3675
40-50	45	1	45	2025	2025
		N=20	$\sum f_i x_i =$ 440		$\sum f_i x_i^2 =$ 11900

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{440}{20} = 22$$

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$S.D. = \sqrt{\frac{11900}{20} - (22)^2}$$

$$S.D. = 10.54$$

$$\text{Coefficient of variance} = \frac{S.D.}{Mean} \times 100$$

$$= \frac{10.54}{22} \times 100$$

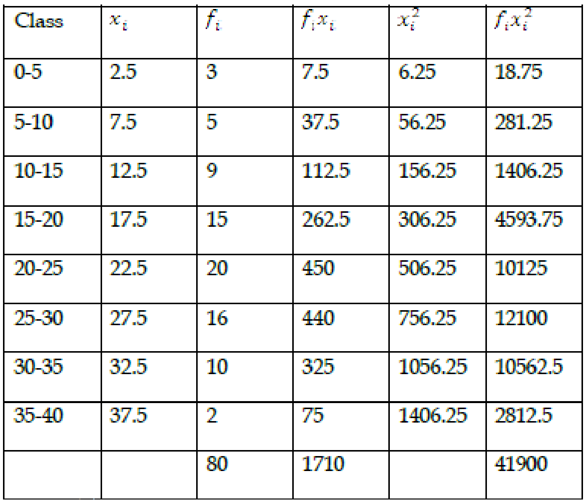
$$= 47.91\%$$

2) Find the standard deviation, variance & coefficient variance from following:

[W-18,S-17,W-13,S-13,W-12]

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	3	5	9	15	20	16	10	2

Ans.



Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-5	2.5	3	7.5	6.25	18.75
5-10	7.5	5	37.5	56.25	281.25
10-15	12.5	9	112.5	156.25	1406.25
15-20	17.5	15	262.5	306.25	4593.75
20-25	22.5	20	450	506.25	10125
25-30	27.5	16	440	756.25	12100
30-35	32.5	10	325	1056.25	10562.5
35-40	37.5	2	75	1406.25	2812.5
		80	1710		41900

$$\text{Mean} = \frac{\Sigma f_i x_i}{N} = \frac{1710}{80} = 21.375$$

$$\text{S.D.} = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{41900}{80} - \left(\frac{1710}{80}\right)^2}$$

$$= 8.177$$

$$\text{Coefficient of variance} = C.V. = \frac{S.D.}{Mean} \times 100$$

$$\therefore C.V. = 38.25$$

$$vaiance = \sigma^2 = (8.177)^2 = 66.8633$$

3) Find the Mean, S.D ,variance and co-efficient of variance for the following:

[W-19,S-19,S-18,W-18,W-17,S-16,S-14,W-12,SQP]

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15

Ans.

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	14	70	25	350
10-20	15	23	345	225	5175
20-30	25	27	675	625	16875
30-40	35	21	735	1225	25725
40-50	45	15	675	2025	30375
			2500		**78500**

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{2500}{100} = 25$$

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{78500}{100} - \left(\frac{2500}{100}\right)^2}$$

$$= 12.649$$

$$\therefore Variance = (S.D.)^2$$
$$= 12.649^2$$
$$= 159.997$$

$$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$$
$$= \frac{12.649}{25} \times 100$$
$$= 50.596$$

4) Find variance and the coefficient of variance for the following distribution.

[S-18]

Class-Interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	6	10	18	9	3

Ans.

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
10-20	15	4	60	225	900
20-30	25	6	150	625	3750
30-40	35	10	350	1225	12250
40-50	45	18	810	2025	36450
50-60	55	9	495	3025	27225
60-70	65	3	195	4225	12675
		50	2060		93250

$$\text{mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{2060}{50}$$
$$\bar{x} = 41.2$$

$$\text{S.D.}\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$= \sqrt{\frac{93250}{50} - (41.2)^2}$$

$$= \sqrt{1865 - 1697.44}$$

$$= \sqrt{167.56}$$

$$\sigma = 12.94$$

$$\text{Variance} = \sigma^2 = (12.94)^2 = 167.44$$

$$\text{C.V.} = \frac{\text{S.D.}}{\text{Mean}} \times 100$$

$$= \frac{12.94}{41.2} \times 100 = 31.41$$

5) Find the variance and co-efficient of variance for the following distribution.

[W-14]

C.I	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	25	30	50	90	75	60	35	25	15

Ans.

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
20-25	22.5	25	562.5	506.25	12656.3
25-30	27.5	30	825	756.25	22687.5
30-35	32.5	50	1625	1056.25	52812.5
35-40	37.5	90	3375	1406.25	126563
40-45	42.5	75	3187.5	1806.25	135469
45-50	47.5	60	2850	2256.25	135375
50-55	52.5	35	1837.5	2756.25	96468.8
55-60	57.5	25	1437.5	3306.25	82656.3
60-65	62.5	15	937.5	3906.25	58593.8
		405	**16637.5**		**723281**

$$\bar{x}=\frac{\sum f_i x_i}{N}=\frac{16637.5}{405}=41.08$$

$$S.D.=\sqrt{\frac{\sum f_i x_i^2}{N}-\left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$=\sqrt{\frac{723281}{405}-\left(\frac{16637.5}{405}\right)^2}$$

$$=9.914$$

$$\therefore Variance=(S.D.)^2$$

$$=9.914^2$$

$$=98.287$$

$$Coeff.\ of\ Variance=\frac{S.D.}{\bar{x}}\times 100$$

$$=\frac{9.914}{41.08}\times 100$$

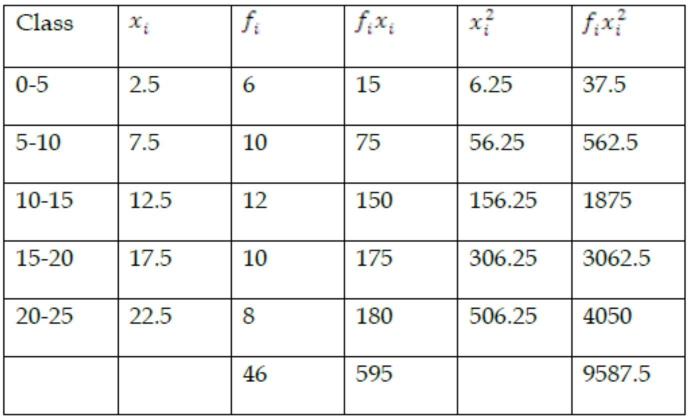
$$=24.133$$

6) Calculate S.D & Coefficient of variance of the following table:

[S-19,S-15,W-13]

Marks Below/ Expenditure/ Weekly Expenditure below	5	10	15	20	25
No.of Student	6	16	28	38	46

Ans.



Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-5	2.5	6	15	6.25	37.5
5-10	7.5	10	75	56.25	562.5
10-15	12.5	12	150	156.25	1875
15-20	17.5	10	175	306.25	3062.5
20-25	22.5	8	180	506.25	4050
		46	595		9587.5

$$\text{mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{595}{46}$$

$$\bar{x} = 12.93$$

$$\text{S.D.}\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$= \sqrt{\frac{9587.5}{46} - (12.93)^2}$$

$$= \sqrt{208.42 - 167.18}$$

$$= \sqrt{41.24}$$

$$\sigma = 6.42$$

$$\text{Coefficient of variance} = \frac{\sigma}{\bar{x}} \times 100$$

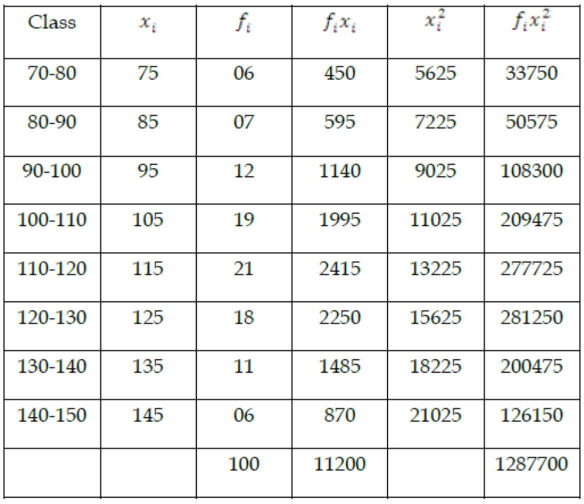
$$= \frac{6.412}{12.935} \times 100$$

$$= 49.57$$

7) Calculate S.D, Co-efficient of variation of the following data **[W-16]**

Rain fall	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of places	06	07	12	19	21	18	11	06

Ans.



Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
70-80	75	06	450	5625	33750
80-90	85	07	595	7225	50575
90-100	95	12	1140	9025	108300
100-110	105	19	1995	11025	209475
110-120	115	21	2415	13225	277725
120-130	125	18	2250	15625	281250
130-140	135	11	1485	18225	200475
140-150	145	06	870	21025	126150
		100	11200		1287700

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{11200}{100} = 112$$

$$\text{i) S.D.}\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$= \sqrt{\frac{1287700}{100} - (112)^2}$$

$$= \sqrt{12877 - 12544}$$

$$= \sqrt{333}$$

$$\sigma = 18.25$$

$$ii)\ \text{Co-efficient of variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{18.25}{112} \times 100 = 16.29.$$

8) Find the standard deviation of the following :**[W-15]**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	20	130	220	70	60

Ans.

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-20	10	20	200	100	2000
20-40	30	130	3900	900	117000
40-60	50	220	11000	2500	550000
60-80	70	70	4900	4900	343000
80-100	90	60	5400	8100	486000
		500	25400		1498000

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{25400}{500}$$

$$\bar{x} = 50.8$$

$$\text{S.D.}\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\bar{x}\right)^2}$$

$$= \sqrt{\frac{1498000}{500} - (50.8)^2}$$

$$= \sqrt{2996 - 2580.64}$$

$$= \sqrt{415.36}$$

$$\sigma = 20.38$$

9) Find variance from the following series :**[W-15]**

Age under	10	20	30	40	50	60	70	80
No. of person	15	30	53	75	100	110	115	125

Ans.

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	15	75	25	375
10-20	15	15	225	225	3375
20-30	25	23	575	625	14375
30-40	35	22	770	1225	26950
40-50	45	25	1125	2025	50625
50-60	55	10	550	3025	30250
60-70	65	5	325	4225	21125
70-80	75	10	750	5625	56250
		125	4395		203325

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{N} = \frac{4395}{125}$$

$$\bar{x} = 35.16$$

$$\text{S.D.} \sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - (\bar{x})^2}$$

$$= \sqrt{\frac{203325}{125} - (35.16)^2}$$

$$\sigma = 19.75$$

10) Find variance and coefficient for the following data :**[S-15]**

C.I	55-65	65-75	75-85	85-95	95-105	105-115	115-125
No. of workers	10	12	15	20	14	7	2

Ans.

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
55-65	60	10	600	3600	36000
65-75	70	12	840	4900	58800
75-85	80	15	1200	6400	96000
85-95	90	20	1800	8100	162000
95-105	100	14	1400	10000	140000
105-115	110	7	770	12100	84700
115-125	120	2	240	14400	28800
		80	**6850**		**606300**

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{6850}{80} = 85.625$$

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{606300}{80} - \left(\frac{6850}{80}\right)^2}$$

$$= 15.7197$$

$$\therefore Variance = (S.D.)^2 = 15.7197^2 = 247.109$$

$$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$$

$$= \frac{15.7197}{82.625} \times 100$$

$$= 18.359$$

11) If the mean is 82.5, standard deviation is 7.2. Find co-efficient of variance.

[W-18, S-15,SQP]

Ans.

$$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$$

$$= \frac{7.2}{82.5} \times 100$$

$$= 8.727$$

12) If the mean is 82, standard deviation is 7. Find co-efficient of variance.**[W-19]**

Ans.

$$\text{Coefficient of variation } = \frac{\sigma}{\bar{x}} \times 100$$

$$\text{Coefficient of variation} = \frac{7}{82} \times 100$$

$$= 8.537$$

13) If mean is 34.5 and S.D is 5 find the coefficient of variance.**[W-18]**

Ans.

$$\text{Coefficient of variance} = \frac{\sigma}{\bar{x}}\times 100$$

$$= \frac{5}{34.5}\times 100$$

$$= 14.493$$

14) If coefficient of variation of a distribution is 75% and S.D is 24, find its mean.

[S-18]

Ans.

$$\text{coefficient of variation} = \frac{\sigma}{\bar{x}}\times 100$$

$$75 = \frac{24}{\bar{x}}\times 100$$

$$\bar{x} = \frac{24\times 100}{75}$$

$$\bar{x} = 32$$

15) If the C.V. of certain data is 5 and mean is 60. Find S.D **[W-17]**

Ans.

$$\text{Coefficient of variation} = \frac{S.D.}{Mean}\times 100$$

$$\therefore 5 = \frac{S.D.}{60}\times 100$$

$$\therefore \frac{5\times 60}{100} = S.D.$$

$$\therefore S.D. = 3$$

TUTORIAL NO.15

STATISTICS

1) Compute S.D**, Variance, C.V & Coefficient of Standard Deviation.**
15, 22 , 27 , 11 , 9 , 21 , 14 , 9 **[W-17]**

2) Calculate **S.D, Variance, C.V &Coefficient of Standard Deviation.**

x_i	2	3	4	5	6
fi	8	14	19	12	7

3) Find mean ,S.D , coefficient of variance of the following data
[W-18,S-18,W-17,S-14]

Class-Interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	5	8	3	1

4) Find the standard deviation variance & coefficient variance from following:
[W-18,S-17,W-13,S-13,W-12]

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	3	5	9	15	20	16	10	2

5) Find the Mean, S.D ,variance and co-efficient of variance for the following:
[W-19,S-19,S-18,W-18,W-17,S-16,S-14,W-12,SQP]

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	14	23	27	21	15

6) If mean is 34.5 and S.D is 5 find the coefficient of variance.**[W-18]**

7) If coefficient of variation of a distribution is 75% and standard deviation is 24, find its mean. **[S-18]**

8) If the coefficient of variation of certain data is 5 and mean is 60. Find S.D
[W-17]

IV. COMPARISONS OF TWO SETS OF OBSERVATIONS:-

FORMULAE:-

If $(C.V)_A < (C.V)_B$ then **set A is More consistent &**

If $(C.V)_A < (C.V)_B$ then **set B has greater variability**

Examples:-

1) The two sets of observations are given below. Which of two sets is more consistent ? **[W-19,S-19,W-18, W-17,S-14, W-13, S-13,W-12]**

Set I	Set II
$\overline{x} = 82.5$	$\overline{x} = 48.75$
$\sigma = 7.3$	$\sigma = 8.35$

Ans. For Set I:

$$\text{Coefficient of variance } = \frac{\sigma}{\overline{x}} \times 100$$

$$= \frac{7.3}{82.5} \times 100$$

$$= 8.848$$

For Set II:

$$\text{Coefficient of variance } = \frac{\sigma}{\overline{x}} \times 100$$

$$= \frac{8.35}{48.75} \times 100$$

$$= 17.128$$

Set I is more consistent

2) The two sets of observations are given below. Which of them is more consistent? **[S-15]**

Set-I	Set-II
$\overline{X}$ = 82.5	$\overline{X}$ = 98.75
σ = 7.3	σ = 8.35

Ans.

$$C.V.(I)=\frac{\sigma}{\overline{x}}\times 100=\frac{7.3}{82.5}\times 100=8.848$$

$$C.V.(II)=\frac{\sigma}{\overline{x}}\times 100=\frac{8.35}{98.75}\times 100=8.456$$

$$\therefore C.V.(II)<C.V.(I)$$

$\therefore$ $Set\ II$ is more consistent.

3) From the following data investigate which set is more consistent: **[S-17]**

Set	a.m. = $\overline{x}$	S.D. = σ
Set I	83.4	5.9
Set II	51.85	7.45

Ans.

Let V_1 and V_2 be coefficient of variations for set I and set II respectively

$$\therefore V_1=\frac{\sigma}{\overline{x}}\times 100$$

$$=\frac{5.9}{83.4}\times 100$$

$$=7.07$$

$$V_2=\frac{7.45}{51.85}\times 100$$

$$=14.36$$

$$\because V_1<V_2$$

$\therefore$ set I is more consistent

4) The data of run scored by two batsmen A and B in five one day matches is given below. State which batsman is more consistent? **[S-18]**

Batsman	Average run scored	S.D.
A	44	5.1
B	54	6.31

Ans. For Batsman A

$$C.V=\frac{\sigma}{\bar{x}}\times 100$$

$$=\frac{5.1}{44}\times 100$$

$$=11.59$$

For Batsman B

$$C.V=\frac{\sigma}{\bar{x}}\times 100$$

$$=\frac{6.31}{54}\times 100$$

$$=11.69$$

$C.V$ of A < $C.V$ of B

$\therefore$ Batsman A is more consistent.

5) In the two factories P & Q engaged in the same industries the average weekly wages & S.D are as follows .Which factory P & Q has greater variability in individual . OR Which factory P or Q is more consistent ?

[S-19,W-16,S-16,SQP]

Factories	average wages	S.D
P	34.5	5.0
Q	28.5	4.5

Ans. Which factory is more consistent?

For factory A

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{5.0}{34.5} \times 100$$

$$= 14.49\%$$

For factory B

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{4.5}{28.5} \times 100$$

$$= 15.79\%$$

$C.V$ of A $<$ $C.V$ of B

$\therefore$ Factory A is more consistent

6) The runs scored by two batsmen A and B in 5 one day matches are given below.

Who is more consistent ? Why ? **[S-18]**

A	48	50	39	46	37
B	50	52	60	55	53

Ans. For Batsman A

x_i	x_i^2
48	2304
50	2500
39	1521
46	2116
37	1369
$\sum x_i = 220$	$\sum x_i^2 = 9810$

$$\therefore \quad Mean, \bar{x} = \frac{\sum x_i}{N} = \frac{220}{5} = 44$$

$$\therefore \quad S.D. = \sqrt{\frac{\sum x_i^2}{N} - \left(\bar{x}\right)^2} = \sqrt{\frac{9810}{5} - 44^2} = 5.099$$

For Batsman B

x_i	x_i^2
50	2500
52	2704
60	3600
55	3025
53	2809
$\sum x_i = 270$	$\sum x_i^2 = 14638$

$$\therefore \quad Mean, \bar{x} = \frac{\sum x_i}{N} = \frac{270}{5} = 54$$

$$\therefore \quad S.D. = \sqrt{\frac{\sum x_i^2}{N} - \left(\bar{x}\right)^2} = \sqrt{\frac{14638}{5} - 54^2} = 3.406$$

For Batsman A

$$C.V.(A)=\frac{\sigma}{\bar{x}}\times100$$

$$=\frac{5.099}{44}\times100$$

$$=11.589\%$$

For Batsman B

$$C.V.(B)=\frac{\sigma}{\bar{x}}\times100$$

$$=\frac{3.406}{54}\times100$$

$$=6.307\%$$

$$C.V.(B)<C.V.(A)$$

$\therefore$ Batsman B is more consistent.

7) Following are the marks obtained by two students A and B.

Marks obtained by A	44	80	76	48	52	72	68	56	60	64
Marks obtained by B	48	75	54	60	63	69	72	51	57	56

which of the two students is more consistent? **[W-17]**

Ans. **For Student A:**

x_i	x^2_i
44	1936
80	6400
76	5776
48	2304
52	2704
72	5184
68	4624
56	3136
60	3600
64	4096
620	**39760**

$$\text{Mean } \bar{x} = \frac{\sum x_i}{N} = \frac{620}{10} = 62$$

$$\text{S.D.} = \sigma = \sqrt{\frac{\sum x_i^2}{N} - (\bar{x})^2} = \sqrt{\frac{39760}{10} - (62)^2} = 11.49$$

$$\text{Coefficient of Variance} = \frac{S.D.}{\bar{x}} \times 100 = \frac{11.49}{62} \times 100 = 18.53$$

For Student B:

x_i	x_i^2
48	2304
75	5625
54	2916
60	3600
63	3969
69	4761
72	5184
51	2601
57	3249
56	3136
605	37345

$$\text{Mean } \bar{x} = \frac{\sum x_i}{N} = \frac{605}{10}$$

$$\bar{x} = 60.5$$

$$\text{S.D.} = \sigma = \sqrt{\frac{\sum x_i^2}{N} - (\bar{x})^2}$$

$$= \sqrt{\frac{37345}{10} - (60.5)^2}$$

$$\sigma = 8.62$$

$$\text{Coefficient of Variance} = \frac{S.D.}{\bar{x}} \times 100$$

$$= \frac{8.62}{60.5} \times 100$$

$$= 14.25$$

$$\therefore CV(B) < CV(A)$$

$\therefore$ Student B is more consistent.

8) The scores of two batsmen A and B in ten innings during a certain season as under. Find which of two batsmen is more consisting in scoring **[W-14]**

A	32	28	47	63	71	39	10	60	96	14
B	19	31	48	53	67	90	10	62	40	80

Ans. **For Batsman A:**

x_i	x_i^2
32	1024
28	784
47	2209
63	3969
71	5041
39	1521
10	100
60	3600
96	9216
14	196
460	**27660**

$$\bar{x} = \frac{460}{10} = 46$$

$$\sigma = \sqrt{\frac{27660}{10} - \left(\frac{460}{10}\right)^2} = 25.495$$

$$CV(A) = \frac{25.495}{46} \times 100 = 55.424$$

For Batsman B:

x_i	x_i^2
19	361
31	961
48	2304
53	2809
67	4489
90	8100
10	100
62	3844
40	1600
80	6400
500	**30968**

$$\bar{x} = \frac{500}{10} = 50$$

$$\sigma = \sqrt{\frac{30968}{10} - \left(\frac{500}{10}\right)^2} = 24.429$$

$$CV(B) = \frac{24.429}{50} \times 100 = 48.858$$

$$\therefore CV(B) < CV(A)$$

$\therefore$ B is more consistent.

TUTORIAL NO.16

STATISTICS

1) The two sets of observations are given below. Which of two sets is more consistent ? **[W-19,S-19,W-18, W-17,S-14, W-13, S-13,W-12]**

Set I	Set II
$\overline{x} = 82.5$	$\overline{x} = 48.75$
$\sigma = 7.3$	$\sigma = 8.35$

2) The data of run scored by two batsmen A and B in five one day matches is given below . State which batsman is more consistent? **[S-18]**

Batsman	Average run scored	S.D.
A	44	5.1
B	54	6.31

3) In the two factories P & Q engaged in the same industries the average weekly wages & S.D are as follows .Which factory P & Q has greater variability in individual . OR Which factory P or Q is more consistent ?

[S-19,W16,S-16,SQP]

Factories	average wages	S.D
P	34.5	5.0
Q	28.5	4.5

4) The runs scored by two batsmen A and B in 5 one day matches are given below. Who is more consistent ? Why ? **[S-18]**

A	48	50	39	46	37
B	50	52	60	55	53

5) Following are the marks obtained by two students A and B.

Marks obtained by A	44	80	76	48	52	72	68	56	60	64
Marks obtained by B	48	75	54	60	63	69	72	51	57	56

which of the two students is more consistent? **[W-17]**

IMPORTANT TRIGONOMETRIC FORMULAE:-

1) $$sin\theta = \frac{Opposite\ Side}{Hypoteneous}$$

2) $$cos\theta = \frac{Adjacent\ Side}{Hypoteneous}$$

3) $$tan\theta = \frac{Opposite\ Side}{Adjacent\ Side}$$

4) $$180^0 = \pi^c$$

5) $$1^0 = (\frac{\pi}{180})^c$$

6) $$1^c = (\frac{180}{\pi})^0$$

7) **Trigonometric Ratios of (-θ):-**

$sin(-\theta)$=$-sin\theta$	$cot(-\theta)$=$-cot\theta$
$cos(-\theta)$= $cos\theta$	$sec(-\theta)$= $sec\theta$
$tan(-\theta)$=$-tan\theta$	$cosec(-\theta)$=$-cosec\theta$

8) **General Formulae :-**

$sinn\theta = 0 \quad for\ n = 1,2,3,4 \ldots$
$cosn\theta = 1 \quad for\ n = 0,2,4,6,,8 \ldots (i.e\ n\ is\ even)$
$cosn\theta = -1 \quad for\ n = 1,3,5,7,9, \ldots (i.e\ n\ is\ odd)$

9) Fundamental Identities:-

$\sin^2\theta + \cos^2\theta = 1$
$\sec^2\theta = 1 + \tan^2\theta$
$\mathrm{cosec}^2\theta = 1 + \cot^2\theta$

10) Trigonometric Ratios of various angles:-

$\frac{Angles}{Tri.Ratios \downarrow}$	0^0	$30^0 \ (\frac{\pi}{6})^c$	$45^0 \ (\frac{\pi}{4})^c$	$60^0 \ (\frac{\pi}{3})^c$	$90^0 \ (\frac{\pi}{2})^c$	$180^0 (\pi)^c$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1
$cosec$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞

11) The sign of Trigonometric Ratios in tabular form:-

$Quadrant \rightarrow$ $Trigonometric\ Ratios \downarrow$	I^{st}	II^{nd}	III^{rd}	IV^{th}
$sin\theta$	+	+	-	-
$cosec\theta$	+	+	-	-
$tan\theta$	+	-	+	-
$cot\theta$	+	-	+	-
$cos\theta$	+	-	-	+
$sec\theta$	+	-	-	+

12) Some Trigonometric Ratios of allied angles in tabular form:-

$Angle \rightarrow$ $Tri.\ Ratios \downarrow$	$90+\theta$ $(\frac{\pi}{2}+\theta)$	$90-\theta$ $(\frac{\pi}{2}-\theta)$	$180+\theta$ $(\pi+\theta)$	$180-\theta$ $(\pi+\theta)$	$270+\theta$ $(\frac{3\pi}{2}+\theta)$	$270-\theta$ $(\frac{3\pi}{2}-\theta)$	$360+\theta$ $(2\pi+\theta)$	$360-\theta$ $(2\pi-\theta)$
$sin\theta$	$cos\theta$	$cos\theta$	$-sin\theta$	$sin\theta$	$-cos\theta$	$-cos\theta$	$sin\theta$	$-sin\theta$
$cosec\theta$	$sec\theta$	$sec\theta$	$-cosec\theta$	$cosec\theta$	$-sec\theta$	$-sec\theta$	$cosec\theta$	$-cosec\theta$
$tan\theta$	$-cot\theta$	$cot\theta$	$tan\theta$	$-tan\theta$	$-cot\theta$	$cot\theta$	$tan\theta$	$-tan\theta$
$cot\theta$	$-tan\theta$	$tan\theta$	$cot\theta$	$-cot\theta$	$-tan\theta$	$tan\theta$	$cot\theta$	$-cot\theta$
$cos\theta$	$-sin\theta$	$sin\theta$	$-cos\theta$	$-cos\theta$	$sin\theta$	$-sin\theta$	$cos\theta$	$cos\theta$
$sec\theta$	$-cosec\theta$	$cosec\theta$	$-sec\theta$	$-sec\theta$	$cosec\theta$	$-cosec\theta$	$sec\theta$	$sec\theta$

13) List of Sum & Difference Formulae:-

$sin(A+B) = sinA.cosB + cosA.sinB$
$sin(A-B) = sinA.cosB - cosA.sinB$
$cos(A+B) = cosA.cosB - sinA.sinB$
$cos(A-B) = cosA.cosB + sinA.sinB$
$tan(A+B) = \frac{tanA + tanB}{1 - tanA.tanB}$
$tan(A-B) = \frac{tanA - tanB}{1 + tanA.tanB}$

14) Trigonometric Ratios of multiple & sub multiple Angles :-

Multiple Angle Formulae	Sub- multiple Angle Formulae
$sin2\theta = 2sin\theta.cos\theta$ $sin2\theta = \frac{2tan\theta}{1+tan^2\theta}$	$sin\theta = 2sin\frac{\theta}{2}.cos\frac{\theta}{2}$ $sin\theta = \frac{2tan\frac{\theta}{2}}{1+tan^2\frac{\theta}{2}}$
$cos2\theta = 1 - 2\,sin^2\theta$ $cos2\theta = 2\,cos^2\theta - 1$ $cos2\theta = cos^2\theta - sin^2\theta$ $cos2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}$	$cos\theta = 1 - 2\,sin^2\frac{\theta}{2}$ $cos\theta = 2\,cos^2\frac{\theta}{2} - 1$ $cos\theta = cos^2\frac{\theta}{2} - sin^2\frac{\theta}{2}$ $cos\theta = \frac{1-tan^2\frac{\theta}{2}}{1+tan^2\frac{\theta}{2}}$
$tan2\theta = \frac{2tan\theta}{1-tan^2\theta}$	$tan\theta = \frac{2tan\frac{\theta}{2}}{1-tan^2\frac{\theta}{2}}$

15) List of Triple Angle Formulae:-

$$sin3\theta = 3sin\theta - 4\,sin^3\theta$$

$$cos3\theta = 4cos^3\theta - 3cos\theta$$

$$tan3\theta = \frac{3tan\theta - tan^3\theta}{1 - 3tan^2\theta}$$

16) List of Product to Addition Formulae:-

$$sin(A+B).sin(A-B) \; = \; sin^2A - sin^2B \; = \; cos^2B - cos^2A$$

$$cos(A+B).cos(A-B) = cos^2A - sin^2B$$

17) FACTORIZATION &DEFACTORIZATION FORMULAE :-

DEFACTORIZATION FORMULAE:-

$$2sin\,A.cosB = sin(A+B) + sin(A-B)$$

$$2cos\,A.sinB = sin(A+B) - sin(A-B)$$

$$2cos\,A.cosB = cos(A+B) + cos(A-B)$$

$$2sin\,A.sinB = cos(A-B) - cos(A+B)$$

18) FACTORIZATION FORMULAE:-

$$sin\,C + sinD = 2\,sin\left(\frac{C+D}{2}\right).cos(\frac{C-D}{2})$$

$$sin\,C - sinD = 2\,cos\left(\frac{C+D}{2}\right).sin(\frac{C-D}{2})$$

$$cos\,C + cosD = 2\,cos\left(\frac{C+D}{2}\right).cos(\frac{C-D}{2})$$

$$cosC - cosD = -2\,sin\left(\frac{C+D}{2}\right).sin\left(\frac{C-D}{2}\right) = 2sin\left(\frac{C+D}{2}\right).sin\left(\frac{D-C}{2}\right)$$

19) **Inverse Trigonometric Functions :-**

$2tan^{-1}(x) = tan^{-1}\left(\frac{2x}{1-x^2}\right)$
$2tan^{-1}(x) = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

20)

Property No.1:-		
$sin^{-1}(sin\theta) = \theta$	***OR***	$sin(sin^{-1}\theta) = \theta$
$cos^{-1}(cos\theta) = \theta$	***OR***	$cos(cos^{-1}\theta) = \theta$
$tan^{-1}(tan\theta) = \theta$	***OR***	$tan(tan^{-1}\theta) = \theta$
$cot^{-1}(cot\theta) = \theta$	***OR***	$cot(cot^{-1}\theta) = \theta$
$sec^{-1}(sec\theta) = \theta$	***OR***	$sec(sec^{-1}\theta) = \theta$
$cosec^{-1}(cosec\theta) = \theta$	***OR***	$cosec(cosec^{-1}\theta) = \theta$

21)

Property No. 2:-	**Property No. 3:-**
$sin^{-1}(x) = cosec^{-1}\left(\frac{1}{x}\right)$	$sin^{-1}(-x) = -sin^{-1}(x)$
$cos^{-1}(x) = sec^{-1}\left(\frac{1}{x}\right)$	$cos^{-1}(-x) = \pi - cos^{-1}(x)$
$tan^{-1}(x) = cot^{-1}\left(\frac{1}{x}\right)$	$tan^{-1}(-x) = -tan^{-1}(x)$
$cosec^{-1}(x) = sin^{-1}\left(\frac{1}{x}\right)$	$sec^{-1}(-x) = \pi - sec^{-1}(x)$
$sec^{-1}(x) = cos^{-1}\left(\frac{1}{x}\right)$	$cot^{-1}(-x) = -cot^{-1}(x)$
$cot^{-1}(x) = tan^{-1}\left(\frac{1}{x}\right)$	$cosec^{-1}(-x) = -cosec^{-1}(x)$

22)

Property No. 4:-	**Property No. 5:-**
$sin^{-1}(x) + cos^{-1}(x) = \frac{\pi}{2}$	$If\ x > 0\,, y > 0\ \&\ x.y < 1\quad then$ $tan^{-1}(x) + tan^{-1}(y) = tan^{-1}\left(\frac{x+y}{1-x.y}\right)$
$tan^{-1}(x) + cot^{-1}(x) = \frac{\pi}{2}$	$If\ x > 0\,, y > 0\ \&\ x.y > 1\quad then$ $tan^{-1}(x) + tan^{-1}(y) = \pi + tan^{-1}\left(\frac{x+y}{1-x.y}\right)$
$cosec^{-1}(x) + sec^{-1}(x) = \frac{\pi}{2}$	$If\ x > 0\,, y > 0\quad then$ $tan^{-1}(x) - tan^{-1}(y) = tan^{-1}\left(\frac{x-y}{1+x.y}\right)$

IMPORTANT DERIVATIVES FORMULAE:-

1) $\frac{d}{dx}(x^n) = nx^{n-1}$

2) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

3) $\frac{d}{dx}(logx) = \frac{1}{x}$

4) $\frac{d}{dx}(e^x) = e^x$

5) $\frac{d}{dx}(a^x) = a^x.loga$

6) $\frac{d}{dx}(Constant) = 0$

	Trigonometry	**Inverse Trigonometry**
7)	$\frac{d}{dx}(sinx) = cosx$	$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
8)	$\frac{d}{dx}(cosx) = -sinx$	$\frac{d}{dx}(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$
9)	$\frac{d}{dx}(tanx) = sec^2x$	$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$
10)	$\frac{d}{dx}(cotx) = -cosec^2x$	$\frac{d}{dx}(cot^{-1}x) = \frac{-1}{1+x^2}$
11)	$\frac{d}{dx}(secx) = secx.tanx$	$\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$
12)	$\frac{d}{dx}(cosecx) = -cosecx.cotx$	$\frac{d}{dx}(cosec^{-1}x\,x) = \frac{-1}{x\sqrt{x^2-1}}$

13) **ADDITION RULE:-** $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

14) **SUBTRACTION RULE:-** $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

15) **MULTIPLICATION RULE:-** $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}$

16) **DIVISION(QUOTIENT) RULE:-** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$

IMPORTANT INTEGRATION FORMULAE:-

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ with $n+1 \neq 0$ $i.e$ $n \neq 1$

2) $\int e^x dx = e^x + c$

3) $\int a^x dx = \frac{a^x}{loga} + c$

4) $\int (Constant) dx = (Constant).x + c$

5) $\int 1 dx = 1.x + c$ **OR** $\int dx = x + c$ **OR** $\int x^0 dx = x + c$

6) $\int \frac{1}{x} dx = logx + c$

	Trigonometry	**Inverse Trigonometry**
7)	$\int sinxdx = -cosx + c$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$
8)	$\int cosxdx = sinx + c$	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + c$
9)	$\int sec^2xdx = tanx + c$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
10)	$\int cosec^2xdx = -cotx + c$	$\int \frac{1}{1+x^2} dx = -\cot^{-1} x + c$
11)	$\int seex.tanxdx = secx + c$	$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$
12)	$\int cosecx.cotxdx = -cosecx + c$	$\int \frac{dx}{x\sqrt{x^2-1}} = -\mathrm{cosec}^{-1} x + c$

13) $\int tanxdx = log|sec\ x| + c$

14) $\int secxdx = log|sec\ x + tanx| + c$

15) $\int cotxdx = log|sin\ x| + c$

16) $\int cosecxdx = log|cosecx - cotx| + c$

17) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\frac{x}{a}) + c$

18) $\int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x + \sqrt{x^2 - a^2}\right| + c$

19) $\int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x + \sqrt{x^2 + a^2}\right| + c$

20) $\int \frac{1}{a^2+ x^2} dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + c$

21) $\int \frac{1}{a^2- x^2} dx = \frac{1}{2a} log \left|\frac{a+x}{a-x}\right| + c$

22) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} log \left|\frac{x-a}{x+a}\right| + c$

23) $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}(\frac{x}{a}) + c$

24) $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + c$

25) $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + c$

26) $\int \frac{f'(x)}{f(x)} dx = log| f(x)| + c$

27) $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

28) $\int [f(x)]^n . f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$

29) $\int a^{f(x)} . f'(x) dx = \frac{a^{f(x)}}{loga} + c$

30) $\int e^{f(x)} . f'(x) dx = e^{f(x)} + c$

31) $\int e^x [f(x) + f'(x)] = e^x f(x) + c$